Earth Resistances

By G. F. TAGG., B.Sc., Ph.D.

EARTH RESISTANCES

By

G. F. Tagg, B.Sc., Ph.D., M.I.E.E., F.Inst.P.

This is the first book in the English language to give complete coverage of the electrical resistance of the earth and earth connections and their measurements. The information, given in readily-usable form, is intended for those concerned with the practical aspects either of resistivity surveying or the provision of earth electrodes. It will, therefore, be of interest to civil engineers faced with site problems (such as depth of bed rock, water table, etc.), archaeologists locating buried structures and old burial sites, companies concerned with soil examination by geophysical methods, electrical engineers designing and testing earth electrodes and electrical contractors installing earth electrodes for lightning conductors.

The first part of the book deals with the specific resistance or resistivity of the soil itself, which is of importance in two ways. Measurements of this quantity under specified conditions can give valuable information as to the nature of the soil beneath the surface, and can also detect the presence of underground features, such as caverns, tunnels, faults, etc. This subject is a large one and a summary of the most useful information is given together with reference to the most important papers. Resistivity of the soil also plays an important part in determining the resistance of any earth electrode, and a knowledge of this quantity is of great value in designing any system of earth electrodes.

The second part of the book deals with the design of earth electrodes, the various instruments used to make the required measurements and the special problems encountered in the measurements.

Although a fairly advanced grade of mathematics is necessarily employed, the final and practical information is given in the form of curves which can be used by those who perhaps cannot follow analysis in detail.

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EARTH RESISTANCES

By

G. F. TAGG, B.Sc., Ph.D., M.I.E.E., F.Inst.P.

With 195 *illustrations*

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PREFACE

The subject of earth resistances and their measurement is one in which I have been interested for a number of years, and during that time have collected a considerable amount of information.

When talking to various engineers, I have been surprised that so little still seemed to be known generally about this subject, which from the viewpoint of safety alone is a very important one. It was frequently pointed out to me that information was not readily available, and naturally engineers engaged in practical work do not have the time or perhaps the facilities to search through published literature for the particular piece of information they require. In view of this I have collected together details of those parts of the subject which are of importance and now present them in the form of this book. A lot of the work is original but I have drawn freely on the works of others, and where necessary, have given the source of more detailed information.

In addition I hope this book will dispel some of the mystery which seems to surround this subject, and that some of the preconceived and erroneous ideas I have met from time to time will be proven wrong.

I should like to express my thanks to Evershed and Vignoles Limited, London, for permission to include the original work already mentioned, and to the following firms for providing me with details and photographs of their instruments:

Associated Research Inc., Chicago
Chauvin-Arnoux, Paris
R. H. Cole (Overseas) Limited, (Siemens-Halske)
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Nash Thompson Limited, and
Norma Fabrik electrischer Meszgerate Gesellschaft, Vienna.

I must also thank Professor Atkinson and the Fondazione Lerici, Milan for providing me with details of their archaeological investigations.

I hope this book will prove of service to engineers, those concerned in geophysical surveying, archaeologists and students.

G. F. TAGG

INTRODUCTION

Earth resistance means the resistance of the earth to the passage of an electrical current. Consideration of such resistance arises in two ways. The earth is not a good conductor of electricity and, in comparison with the normal conductors, the metals, it is extremely poor. It so happens that, in general, the cross-section of the path taken by the current is very large, and this means that, despite the poor conductive qualities of the soil, the actual resistance may be quite small. The earth has long been used as a conductor, very often for safety reasons and its resistance is a matter of considerable importance.

There are a number of cases where it is desired to know exactly what lies beneath the surface of the soil and many methods have been developed by which investigations can be made on the surface. These are usually based on the measurement of one of the physical properties of the soil, and one such property is the electrical resistance. It is usually expressed in terms of the resistivity of the soil which is defined as the resistance between opposite faces of a unit cube. Now, this resistivity varies with the type of soil and in particular with the moisture content and by measurements made on the surface information can be obtained, from which deductions can be made as to the nature of the underlying soil. This method can be used, for example, to determine the depth to bed rock in civil engineering problems and to determine the depth to the water table when finding suitable places to drill for water. Thus, this method of resistivity surveying can be of use to geologists.

It is particularly suitable for the detection and location of buried objects and discontinuities below the surface, and so can be used for the location of mineral lodes. One very interesting application is to archaeological investigations and a number of interesting and important discoveries have been made by its aid. Much work has been done on the theory of the flow of current in a semi-infinite conductor such as the earth, and this in itself is an indication of the great interest shown on this subject. In the first five chapters of the book the theory and practice of this method are described.

In electricity-supply systems it is common practice to connect the system to "earth" at suitable points, the idea being that in the event of a fault, sufficient current will flow through the fault path so that the protective gear will operate and isolate the faulty circuit. It is therefore essential that this "connection to earth" is of a sufficiently low resistance,

and it is not always realised that the design of such earth connections is often far from easy and the practical difficulties can be formidable. In Chapters 6 to 10 the design of various forms of electrodes which can form such connections to earth, is described in detail. The forms include rods, wires, strips and plates. Formulae and curves are given from which the resistance of any particular form can readily be determined. It is hoped that this part will be of assistance to those engineers in the electricity-supply industry concerned with the design of such electrodes, and to those engineers concerned with the design and construction of lightning conductors.

A number of instruments have been designed and manufactured particularly to measure these earth resistances, which have a number of special characteristics. A brief description of a number of these is given in Chapter 11.

Again on the question of safety, a description is given in Chapter 12 of the earth-loop resistance test, now called for by the I.E.E. Wiring Rules. The difficulties involved and the special instruments required are given in detail.

Much of the information contained in this book is original, but a large amount of other material is to be found in the journals and proceedings of various learned societies throughout the world. Many of these are not readily accessible and bringing all this together in one place, thus serves a useful purpose. It is believed that this is the first book on this general subject in the English language.

1

THE SOIL AS A CONDUCTOR OF ELECTRICITY

Conduction through the Soil

Since the earliest days of electric supply the soil has been used as a conductor of electricity. In some cases it was deliberately used as a return conductor but this practice is very seldom adopted nowadays since the earth has many failings as a conductor. It was thought for some time that, because the dimensions of any current path through the earth would be very large, the resistance of any such path would be negligible. The point which was overlooked was that means had to be provided to pass current into and out of the earth, and that these means, in the form of plates, rods or pipes, will have a definite resistance to the passage of current from them into the earth. It is in fact quite difficult, in a large number of cases, to produce such earth electrodes, as they are called, of sufficiently low resistance. The main use of the earth, so far as the electricity supply industry is concerned, is connected with safety and this will be discussed further in later chapters.

The electrical properties of the soil are in themselves of interest and importance, particularly the specific resistance or resistivity. The resistivity is one of the factors in determining the resistance of any earth electrode and is also of importance in connection with the problems of interference between power lines and telecommunication circuits. It is also of importance in itself and studies of the resistivity and the manner in which it varies can give useful information as to the nature of the sub-soil and the presence of buried objects. The resistivity method of surveying has become of considerable importance and has been the subject of a great amount of investigation.

Most soils and rocks when completely dry are non-conductors of electricity, exceptions to this are certain mineral bodies which are conductors because of their metallic content. Sands, loams, and rocks are, however, in themselves of such high resistance that they can be considered as non-conductors. When they contain water, the resistivity drops considerably and they must then be considered as conductors, although very poor ones when compared with metal. For instance, the resistivity of pure copper is 1·6 microhm-cms, when a quite usual value

for a normal soil would be 10,000 ohm-cms. The resistivity of soil would be determined by the quantity of water held in the soil, and on the resistivity of the water itself. In other words conduction through the soil becomes conduction through the water held in the soil and so the conduction must be electrolytic.

Thus the main factors which determine the resistivity of soil are:

(a) Type of soil.
(b) Chemical composition of salts dissolved in the contained water.
(c) Concentration of the salts dissolved in the contained water.
(d) Moisture content.
(e) Temperature.
(f) Grain size of the material and distribution of grain size.
(g) Closeness of packing and pressure.

Effect of the Type of Soil

The type of soil is very important in determining the resistivity of the soil. Unfortunately types of soil are not too clearly defined, for instance, the word clay can cover a wide variety of soils. For this reason it is quite impossible to state that clay, or any other soil for that matter, has a resistivity of so many ohm-cms. Again, the same general type of soil occurs in various localities and it is often found that the resistivity in one locality is different from that in another.

TABLE 1.1. TYPICAL VALUES OF RESISTIVITY OF SOME SOILS

Type of soil	*Resistivity in ohm-cms*
Loams, garden soils, etc.	500–5000
Clays	800–5000
Clay, sand and gravel mixtures	4000–25,000
Sand and gravel	6000–10,000
Slates, shale, sandstone, etc.	1000–50,000
Crystalline rocks	20,000–1,000,000

A number of investigators have from time to time measured the resistivity of various types of soil both by taking samples and measuring them in special apparatus and by measurement of the undisturbed mass of the soil. Neither measurement is easy but the latter is more likely to give accurate results. It is very difficult to ensure that any sample taken from the soil is in the same condition when measured as it was

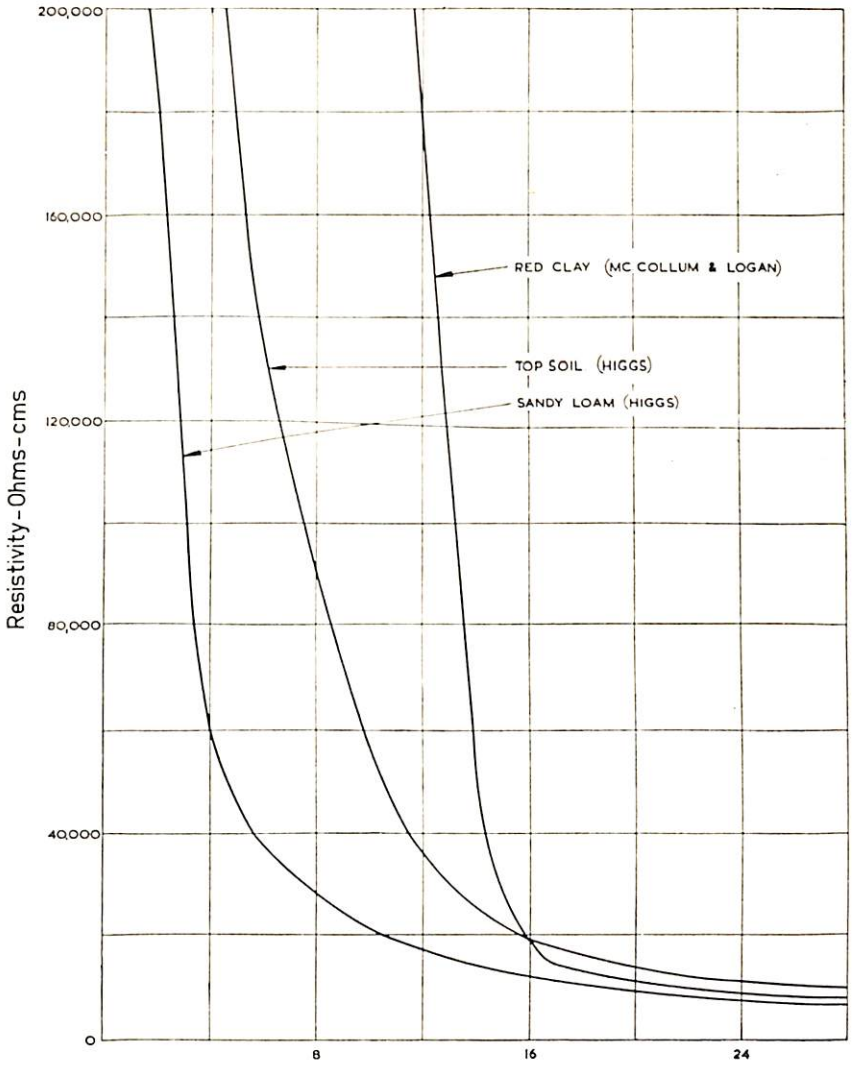

Fig. 1.1. *Variation in soil resistivity with moisture content.*

on site. The figures in Table 1.1 have been collected from a number of sources, but can only be taken as an approximation to the order of resistivity to be expected.

Effect of Moisture Content and of Dissolved Salts in the Water

Since the conduction of current is largely electrolytic, it follows that the quantity of water and the nature and amount of the dissolved salts will play a most important part in determining the resistivity. The actual

amount of water is dependent on a number of factors and is likely to be a variable quantity. It will vary with the weather, the time of year, the nature of the sub-soil and the depth to the permanent water table, if any. It is very seldom that soil is really dry, though possibly desert sands would be the nearest approximation. On the other hand, soil with moisture content greater than 40 per cent does not occur very often.

The manner in which the resistivity of soil varies with the moisture content has been the subject of a number of investigations. Among these may be mentioned those by McCollum and Logan[1], and Higgs[2]. The first two used a red clay soil which has a moisture content of about 5 per cent when air-dried, and a resistivity of $2{\cdot}34 \times 10^6$ ohm-cms. On increasing the moisture content to 22 per cent, the resistivity was reduced to 6,800 ohm-cms. Higgs used two kinds of soil found near the National Physical Laboratory and called respectively "top soil" and "sandy loam". The former was greyish black in colour like garden loam and occurred to a depth of about 2 ft, while the sandy loam was a yellow colour and was found below the top soil.

The manner in which the resistivity of these soils varies with the moisture content is illustrated in Fig. 1.1. It will be noted that the resistivity at first falls rapidly as the moisture content is increased, but after a value of 14–18 per cent, the rate of decrease becomes much less. It is possible that this figure is approaching the maximum value which can be held in the material, and, of course, as this value is increased the effect on the resistivity will be comparatively small.

Moisture content is likely to increase with increasing depth in most localities, but it must not be assumed that because a lot of water is present, i.e. the moisture content is high, the resistivity is necessarily low. A simple experiment with dry sand and distilled water will soon show that this is not the case. Frozen ground will necessarily have a high resistivity since ice has a high value.

Effect of Dissolved Salts in the Water

Since the amount of water present in the soil is a major factor in determining the resistivity, it follows that this resistivity is dependent on that of the water itself and it is a well-known fact that the resistivity of water is governed by the amount of salts dissolved in it. The curves in Fig. 1.2 illustrate this; these show quite clearly that quite a small quantity of dissolved salts can reduce the resistivity very considerably from the infinite value of really pure water. It will also be noted that different salts have different effects and this is probably part of the explanation why the resistivities of similar soils from different localities

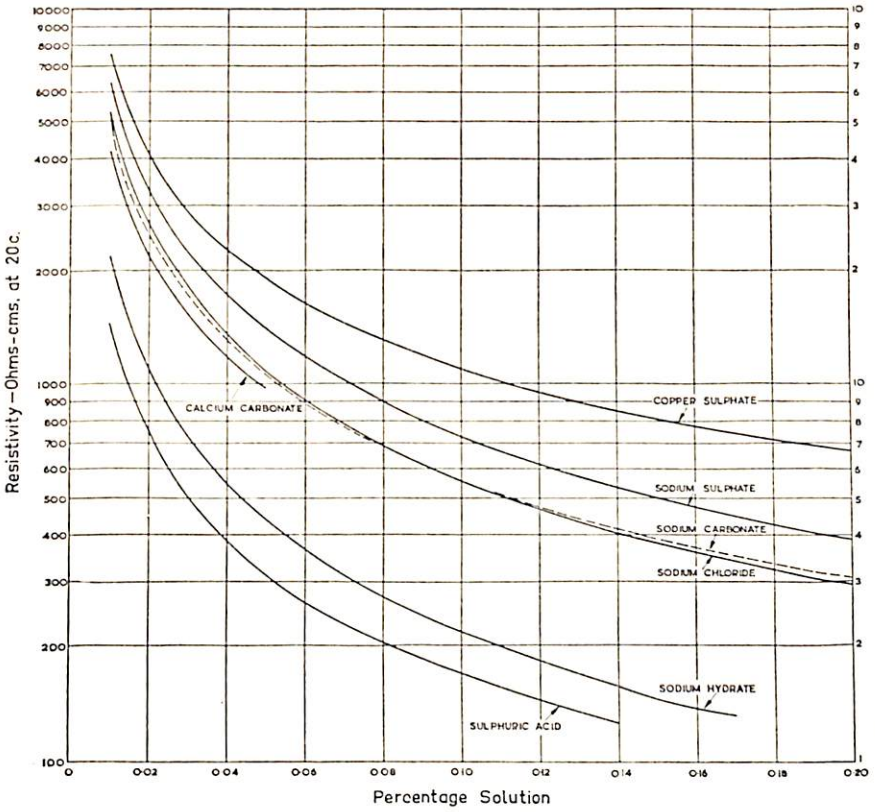

Fig. 1.2. *Typical resistivity curves of solutions.*

vary considerably. First the moisture content may be different, and again the quantity and nature of the dissolved salts may not be the same.

Effect of Grain Size and its Distribution

Grain size and the presence of grains of various sizes undoubtedly plays an important part in determining the resistivity. The grain size and its distribution undoubtedly has an effect on the manner in which the moisture is held. With large grains, moisture is probably held by surface tension at the points of contact with the grains. If, however, grains of various sizes are present, the spaces between the large grains may be filled with smaller ones and the resistivity will be reduced as a result.

Grains of soil are of all shapes and sizes, but it is possible to obtain some idea of the order of volume of the free space which can be filled with water, if the grains are assumed to be spherical in shape. The most compact arrangement of these is obtained when the lines joining the

centres of the spheres form an equilateral parallelopiped having face angles of 60° and 120°. For such an arrangement the pore volume amounts to 25·95 per cent of the total and is independent of the grain size.

If the spheres are arranged in the least compact manner, the lines joining the centre form cubes and in this case the pore volume is 47·64 per cent. Work has been done on various rocks and values have been given of 0·2 to 2 per cent for igneous rocks and crystalline schists, 8 to 15 per cent for ordinary clay and sand, and 25 per cent and more for porous conglomerates and cellular limestone.

Effect of Temperature and Pressure

Very little experimental work appears to have been done on the effect of either temperature or pressure. Since the resistivity is largely determined by the water present and it is known that the resistivity of water has a large temperature coefficient, it is reasonable to assume that the resistivity will increase as the temperature is decreased. The small amount of experimental work which has been done confirms this. If the temperature falls low enough for the water to freeze then, as mentioned previously, the resistivity will rise sharply due to the high value for ice.

Similarly, there is not much experimental evidence on the effect of pressure, but it is only to be expected that higher pressures resulting in a more compact or denser body of earth will result in lower values of resistivity. The small amount of experimental evidence available, however, does not entirely support this theory and it is probable that from a practical point of view the effect of pressure can safely be neglected.

Variation of Resistivity with Location

It has already been explained that it is not possible to allocate a definite value of resistivity to a given type of soil, as this varies with the location in which the soil is found. Furthermore, the resistivity can change very rapidly with location, due partly to change of moisture content and partly to change in the type of soil. This is illustrated in a striking manner in Figs. 1.3 and 1.4 which are resistivity maps of England and Wales and of Southern Scotland respectively. These maps show the variation in the average resistivity to a depth of 500 feet, and were prepared for the Electrical Research Association in 1932. The information was derived from measurements made in certain selected places

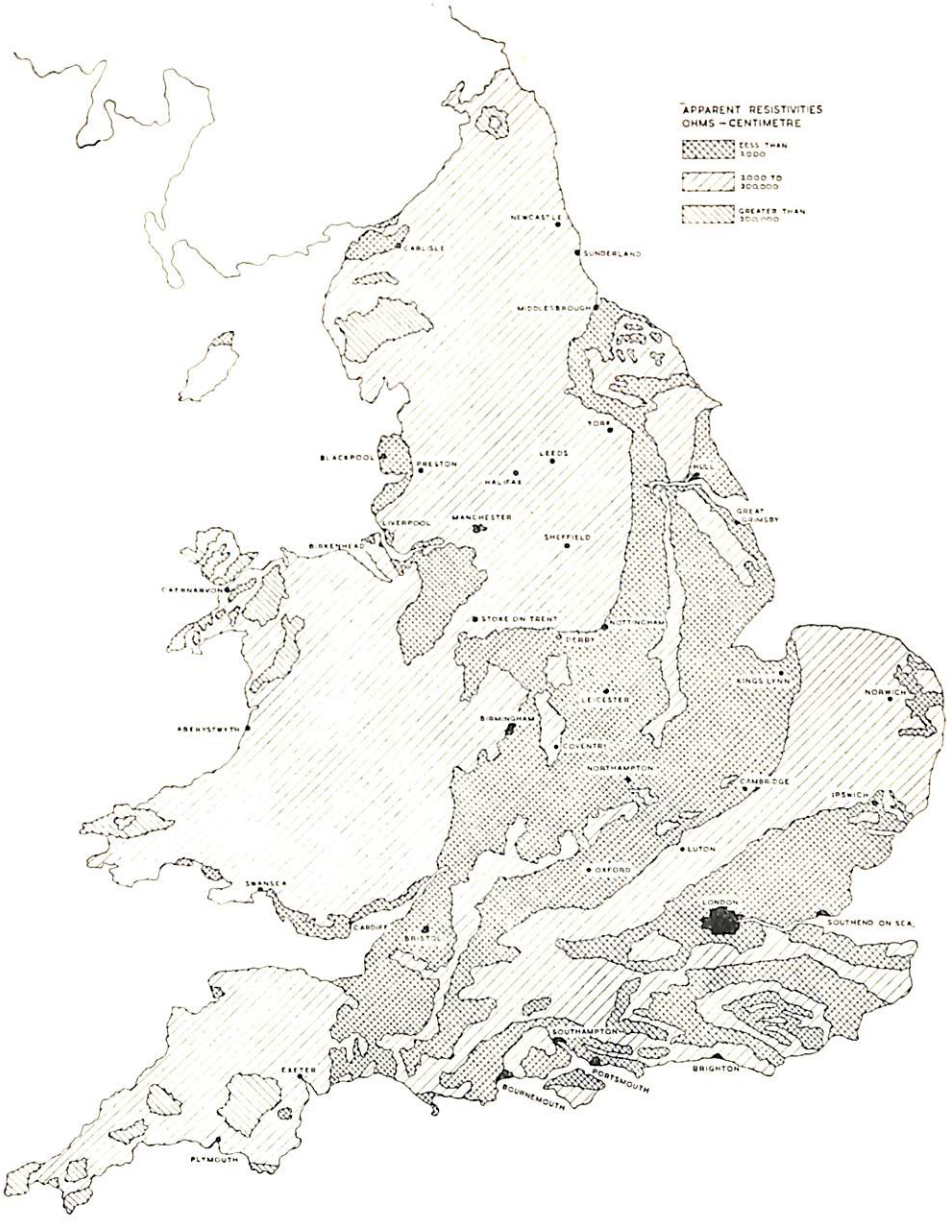

Fig. 1.3. *Resistivity map of England and Wales.*

and from a careful study of all the available geological information. It is interesting to note what a wide range of resistivities can be encountered even in a country the size of England.

Fig. 1.4. Resistivity map of Scotland.

Natural and Stray Currents in the Ground

It is an observed fact that natural currents are to be found flowing in the soil in most parts of the world. This may be due partly to electrolytic action and partly to other causes which are not fully understood. They do not in any way affect considerations of earth resistivity, but they must be borne in mind when dealing with methods of measuring earth resistances *in situ*. Again, in most industrial countries there will also be stray currents in the soil, caused by leakage from power lines and traction systems or by direct induction from alternating current systems. These also have to be considered when measurements of earth resistivity are made.

Anisotropy

One characteristic of soil which can be troublesome in the interpretation is that some formations may not be isotropic, i.e. the resistivity may not be the same in all directions. It is perhaps natural to expect this to occur in a stratified medium and it has been confirmed by actual measurement. A few investigators have looked into this matter, among them Pirson;[3] he quotes the case of a slightly stratified outcrop of hard broken clay in which the resistivity parallel with the bedding surface was 2·1 times the resistivity perpendicular to the bedding surface. Another case quoted was that of a hard laminated shale interbedded with sandstone of the Benton formation (Tertiary). In this case the resistivity parallel with the bedding surface, and with the electrodes in the sandstone seam, was 14·7 times the resistivity perpendicular to the bedding plane.

It is not possible to generalise on this matter and anisotropy can be caused by factors other than stratification, but the anisotropic coefficient α of a formation, which is defined as the ratio of the apparent parallel resistivity to the apparent normal resistivity may range from zero to ten. In homogenous isotropic conditions $\alpha = 1$.

Summary

It will be obvious from the preceding paragraphs that the resistivity of the soil is a very variable quantity and, if it is desired to know the value at any given location, the only safe way is to measure it. This will give the value at the time of the measurement under the conditions prevailing at that time.

There are two main reasons why the value of the resistivity may be required. The first is in the design of earth electrodes for electrical power systems, lightning arrestors and so on, where the resistance to earth of

such electrodes is, as explained in later chapters, directly proportional to the resistivity. In this case a reasonably accurate measurement is required. The second is connected with what is known as resistivity surveying. Here use is made of the fact that non-homogeneity of the soil will cause a change in the resistivity and suitable deductions can be made from these variations. In this case the absolute value is not so important as the variations. All these matters are dealt with in the following chapters.

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2

MEASUREMENT OF SPECIFIC RESISTANCE OR RESISTIVITY

Nature of the Problem

In order to measure the resistivity of the earth it is necessary to pass current through it. This can only be done by inserting electrodes into it so that the current can be fed in and out. As will be shown in a later chapter such electrodes always possess resistance, and this value may be considerably higher than that which it is required to measure. In this way it is analogous to the problem of measuring very low metallic resistances where the contact resistances are comparable with that to be measured. The solution is the same in both cases and a four-terminal method is used.

Practically all the various electrode arrangements used or suggested at different times are variations of this four-terminal connection and are based on the theory originally put forward by Frank Wenner in the Scientific Paper of the Bureau of Standards No. 258 of October 11, 1915, entitled "A Method of Measuring Earth Resistivity".

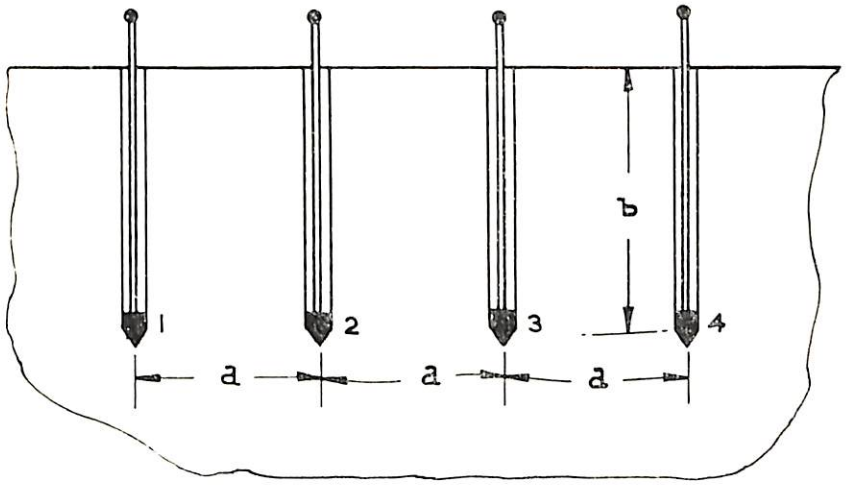

Fig. 2.1. *The four-terminal connection.*

The Four-terminal Connection

The four-terminal connection is shown in Fig. 2.1. Four holes are made in the earth, uniformly spaced in a straight line. The diameter of the holes is not more than 10 per cent of the distance between them, and all extend to approximately the same depth. An electrode is placed in each hole, making electrical contact with the earth only at the bottom as shown in Fig. 2.1. These make up a four-terminal conductor the resistance of which depends upon the distance between the electrodes and the resistivity, but does not depend appreciably upon the size of the electrodes nor the kind of electrical connection they make with the earth.

If the depth of the holes, the distance between them and the resistance (using 1 and 4 as current and 2 and 3 as potential terminals or 2 and 3 as current and 1 and 4 as potential terminals) are measured, it is possible to calculate the effective resistivity of the earth.

If a = distance between the holes (1 to 2, 2 to 3, and 3 to 4)
b = depth of the holes
ρ = resistivity
R = measured resistance
then

$$\rho = \frac{4\pi aR}{1 + \dfrac{2a}{\sqrt{(a^2 + 4b^2)}} - \dfrac{2a}{\sqrt{(4a^2 + 4b^2)}}} = \frac{4\pi aR}{n} \qquad (2.1)$$

where n has a value between 1 and 2 depending on the ratio b/a. Where $b = a$, $n = 1{\cdot}187$; $b = 2a$, $n = 1{\cdot}038$; $b = 4a$, $n = 1{\cdot}003$.

In the same case when b is large compared with a

$$\rho = 4\pi aR \qquad (2.2)$$

and if b is very small compared with a

$$\rho = 2\pi aR \qquad (2.3)$$

If the holes are not in a straight line, or are not of a uniform depth or spacing, the resistivity is easily calculated when all the necessary dimensions are known.

Derivation of the Equations

The above equations are derived as follows. In Fig. 2.2, which represents part of an infinite conductor of uniform resistivity, suppose a unit current enters at the point marked 1. This current will flow away radially from the point of entry and at a distance r from the point 1

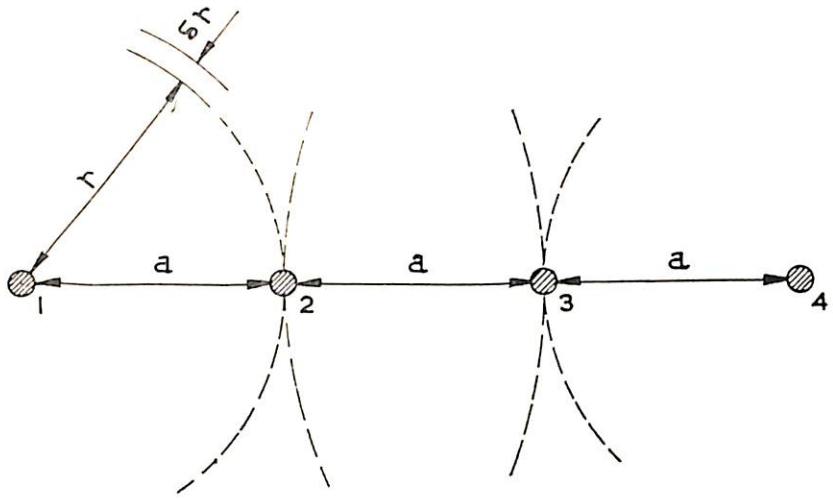

Fig. 2.2. Diagram used in showing the relation between resistivity, resistance and distances between terminals in an infinite conductor.

the current density will be $1/4\pi r^2$. This follows from the fact that at a radius r, the current will be uniformly distributed over a sphere of radius r and hence of area $4\pi r^2$. Now the potential gradient is equal to the current density multiplied by the resistivity. So

$$-\frac{\delta e}{\delta r} = \frac{\rho}{4\pi r^2} \tag{2.4}$$

where e is the potential at a distance r from the point 1.

The difference in potential $(e_1 - e_2)$ between two points distance r_1 and r_2 from 1 is obtained by integrating the potential gradient from $r = r_1$ to $r = r_2$, i.e.

$$e_1 - e_2 = \frac{\rho}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{\rho}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{2.5}$$

If e_x is the potential difference between the points 2 and 3 distant a and $2a$ from 1, caused by unit current flowing radially outward from 1, then equation (2.5) gives

$$e_x = \frac{\rho}{8\pi a} \tag{2.6}$$

Again if e_y is the difference in potential between 2 and 3 caused by unit current flowing radially towards 4,

$$e_y = \frac{\rho}{8\pi a} \tag{2.7}$$

If unit current enters the conductor at 1 and leaves at 4, the current density at any point is the vector sum of that due to unit current flowing from 1 and that due to unit current flowing towards 4. Also,

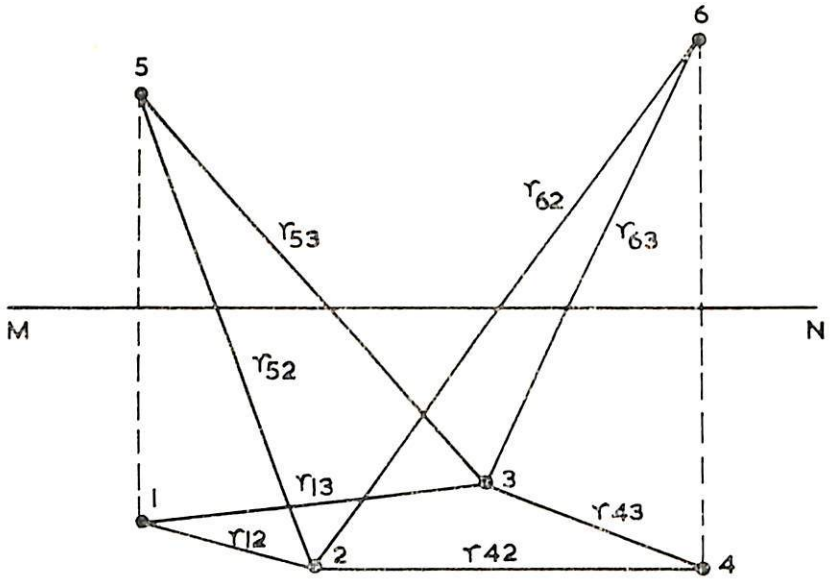

Fig. 2.3. *Diagram used in showing relation between resistivity, resistance and distances between terminals or electrodes in a semi-infinite conductor.*

the potential difference between any two points is the sum of that which results from the flow of unit current from 1 and that which results from the flow of unit current towards 4.

So the potential difference between the points 2 and 3 due to unit current entering at 1 and leaving at 4 is

$$e_x + e_y = \frac{\rho}{4\pi a} \tag{2.8}$$

Since the difference in potential for unit current using 1 and 4 as current terminals and 2 and 3 as potential terminals, is the resistance R,

$$R = \frac{\rho}{4\pi a} \tag{2.9}$$

R is actually the resistance between the equi-potential surfaces on which the potential electrodes are placed.

In the practical case, however, it is not possible to assume an infinite conductor, unless the distance between the electrodes is small compared with their distance below the surface, and so equation (2.9) is not applicable.

To deal with the practical case consider the arrangement of Fig. 2.3 which represents an infinite conductor again. If now e_1 is the potential difference between the points 2 and 3 caused by unit current entering the conductor at 1, then from equation (2.5)

$$e_1 = \frac{\rho}{4\pi}\left(\frac{1}{r_{12}} - \frac{1}{r_{13}}\right) \tag{2.10}$$

Also if the potential difference between 2 and 3 caused by unit current leaving at 4 is e_4, that caused by unit current entering at 5 is e_5, and that caused by unit current leaving at 6 is e_6, then

$$\begin{aligned} e_4 &= \frac{\rho}{4\pi}\left(\frac{1}{r_{43}} - \frac{1}{r_{42}}\right) \\ e_5 &= \frac{\rho}{4\pi}\left(\frac{1}{r_{52}} - \frac{1}{r_{53}}\right) \\ e_6 &= \frac{\rho}{4\pi}\left(\frac{1}{r_{63}} - \frac{1}{r_{62}}\right) \end{aligned} \tag{2.11}$$

Now, if a current I enters at 1 and leaves at 4 and at the same time an equal current enters at 5 and leaves at 6, the potential difference E, between 2 and 3 is $I(e_1 + e_4 + e_5 + e_6)$ or

$$E = \frac{I\rho}{4\pi}\left(\frac{1}{r_{12}} - \frac{1}{r_{13}} + \frac{1}{r_{43}} - \frac{1}{r_{42}} + \frac{1}{r_{52}} - \frac{1}{r_{53}} + \frac{1}{r_{63}} - \frac{1}{r_{62}}\right) \tag{2.12}$$

In deriving this equation it has been assumed that 1, 2, 3, 4, 5 and 6 represent points. The equation can however be used without appreciable error where these six points represent metallic electrodes or terminals on a conductor of relatively high resistivity, provided that the dimensions of the electrodes are small in comparison with the distances between them.

If the points 5 and 6 are so placed that it is possible to choose a plane (represented by the line *mn* in Fig. 2.3) through the conductor, such that the lines joining 1 and 5, and 4 and 6 are normal to and bisected by it,

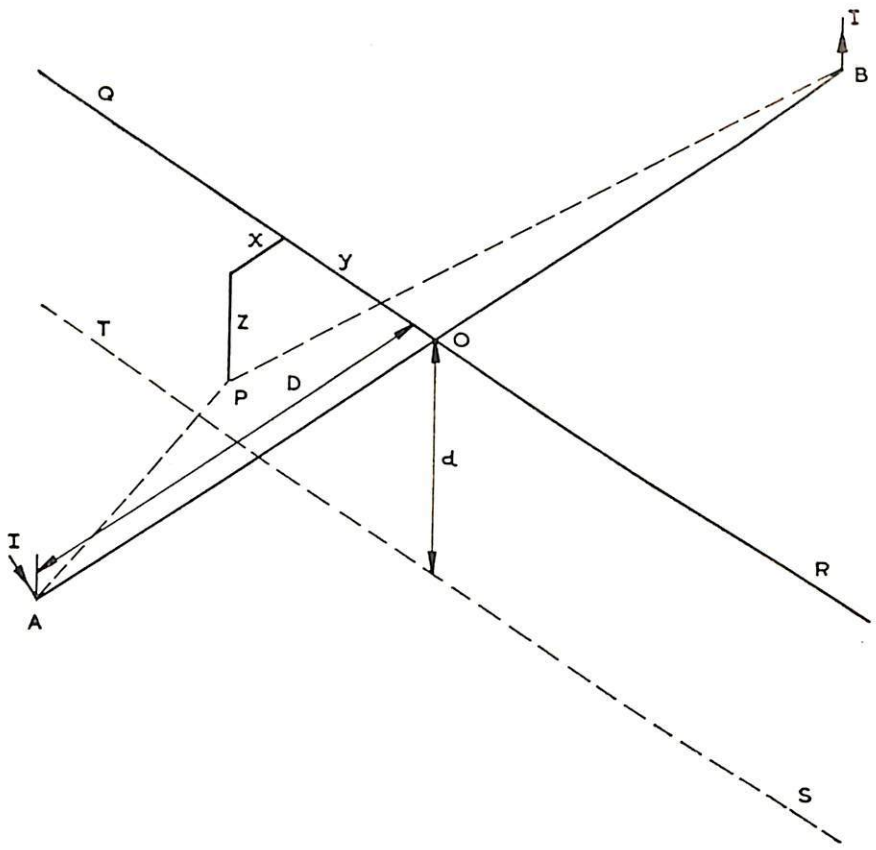

Fig. 2.4. Calculation of current density.

it will be evident on account of the symmetrical arrangement, that no current passes through this plane. So the part of the conductor on one side of the plane can be removed without changing the conditions on the other side.

Thus, the equation applies to a semi-infinite conductor having four terminals, providing the current terminals are considered as having images, and the distances from the potential terminals to the images are taken into account as well as the distance from the potential terminals to the current terminals. This does not, however, require that the potential terminals be in the same plane as the current terminals and their images, as they are in Fig. 2.3.

Since the drop in potential E between 2 and 3 divided by the current I

entering at 1 and leaving at 4 is the resistance R, it follows from equation (2.12) that

$$R = \frac{\rho}{4\pi}\left(\frac{1}{r_{12}} - \frac{1}{r_{13}} + \frac{1}{r_{43}} - \frac{1}{r_{42}} + \frac{1}{r_{52}} - \frac{1}{r_{53}} + \frac{1}{r_{63}} - \frac{1}{r_{62}}\right) \quad (2.13)$$

It will, therefore, be evident that the equation gives the relation between the resistivity, resistance, depth and distance between small electrodes in the earth, as shown in Fig. 1.1, or in the more general case where the electrodes are not in a straight line.

If the electrodes are all at a uniform depth b and at a uniform distance apart a in a straight line, then $r_{12} = a$, $r_{13} = 2a$, $r_{43} = a$, $r_{42} = 2a$, $r_{52} = \sqrt{(4b^2 + a^2)}$, $r_{53} = \sqrt{(4b^2 + 4a^2)}$, $r_{63} = \sqrt{(4b^2 + a^2)}$ and $r_{62} = \sqrt{(4b^2 + 4a^2)}$. So

$$R = \frac{\rho}{4\pi}\left[\frac{2}{a} - \frac{1}{a} + \frac{2}{\sqrt{(a^2 + 4b^2)}} - \frac{2}{\sqrt{(4a^2 + 4b^2)}}\right] \quad (2.14)$$

Current Distribution

It is interesting and important in the practical application of resistivity measurements to examine the current distribution, when it is led into the earth through one electrode and out at another. In Fig. 2.4, the current is led into the earth at A and leaves it at B. The plane $QRST$ is midway between A and B and is at right-angles to the line AB. It is evident from symmetry that the flow of current at all points on this plane will be normal to it. The value of the current density at any point in this plane will show how the current is distributed.

Consider any point P in the soil. This is at a depth z from the surface, at a perpendicular distance x from the plane $QRST$, and at a perpendicular distance y from the line AB. The distance AP is

$$\{(D - x)^2 + y^2 + z^2\}^{\frac{1}{2}},$$

and the distance BP is

$$\{(D + x)^2 + y^2 + z^2\}^{\frac{1}{2}}.$$

Hence the potential at P due to a current I entering at A is

$$\frac{I\rho}{2\pi}\frac{1}{\{(D - x)^2 + y^2 + z^2\}^{\frac{1}{2}}}$$

and that due to a current I leaving at B is

$$-\frac{I\rho}{2\pi}\frac{1}{\{(D + x)^2 + y^2 + z^2\}^{\frac{1}{2}}}$$

So the total potential at P is

$$E_P = \frac{I\rho}{2\pi}\left[\frac{1}{\{(D-x)^2 + y^2 + z^2\}^{1/2}} - \frac{1}{\{(D+x)^2 + y^2 + z^2\}^{1/2}}\right] \quad (2.15)$$

Now the potential gradient perpendicular to the central plane $QRST$, i.e. in the x direction is dE_P/dx, and this is

$$\frac{dE_P}{dx} = \frac{I\rho}{2\pi}\left[\frac{(D-x)}{\{(D-x)^2 + y^2 + z^2\}^{3/2}} + \frac{(D+x)}{\{(D+x)^2 + y^2 + z^2\}^{3/2}}\right] \quad (2.16)$$

On the central plane, $x = O$, and so at any point on the plane, the potential gradient perpendicular to it is

$$\frac{dE_P}{dx}(x=0) = \frac{I\rho}{2\pi}\left[\frac{2D}{\{D^2 + y^2 + z^2\}^{3/2}}\right]$$

$$= \frac{I\rho D}{\pi}\left[\frac{1}{\{D^2 + y^2 + z^2\}^{3/2}}\right] \quad (2.17)$$

The current density is equal to the potential gradient divided by the resistivity and so the current density σ at any point on the central plane is

$$\sigma = \frac{ID}{\pi}\left[\frac{1}{(D^2 + y^2 + z^2)^{3/2}}\right] \quad (2.18)$$

At the point O on the surface midway between A and B, $y = z = O$ and the current density σ_0 at this point is

$$\sigma_0 = \frac{ID}{\pi} \cdot \frac{1}{D^3} = \frac{I}{\pi D^2} \quad (2.19)$$

From this $I/\pi = \sigma_0 D^2$ and if this is substituted in equation (2.18) then

$$\frac{\sigma}{\sigma_0} = \frac{D^3}{(D^2 + y^2 + z^2)^{3/2}} \quad (2.20)$$

It is interesting to examine the variation in the current density on the vertical line from O downwards, i.e. with $y = O$. This is given by

$$\frac{\sigma}{\sigma_0} = \frac{D^3}{(D^2 + z^2)^{3/2}} = \frac{1}{\left[1 + \left(\frac{z}{D}\right)^2\right]^{3/2}} \quad (2.21)$$

Values calculated from this equation are plotted in the curve of Fig. 2.5. Many investigators have suggested that when four electrodes are placed in a straight line at equal intervals, the effective depth of penetration of the current is equal to this interval. In Fig. 2.4 the current electrodes are at a distance $2D$, consequently the distance between adjacent electrodes is $2D/3$. So, if the suggestion that the depth of

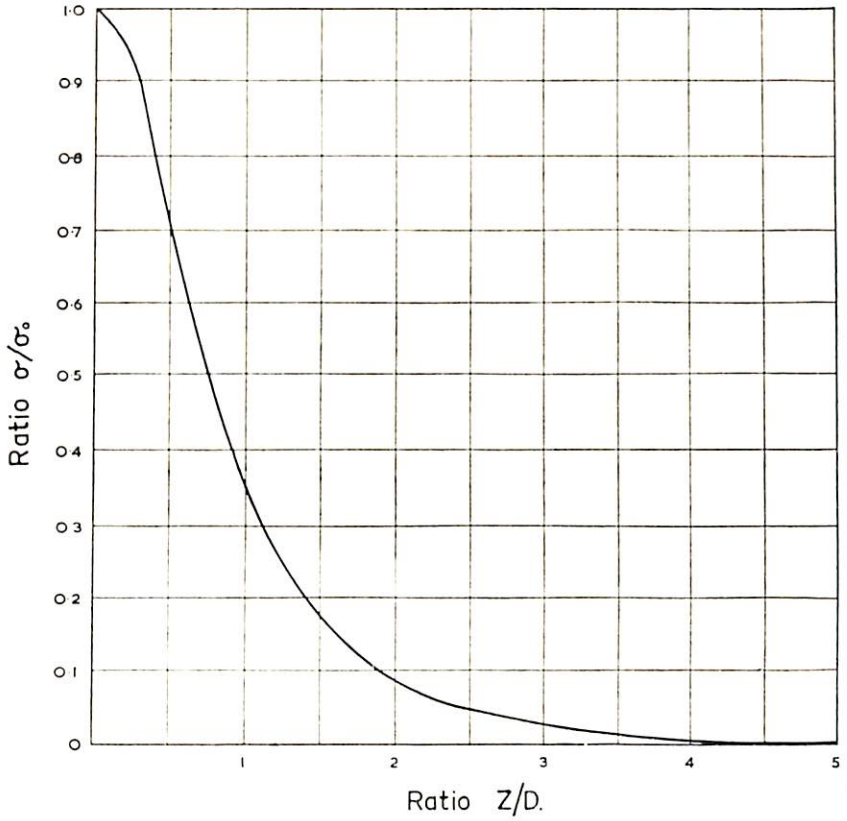

Fig. 2.5. *Variation of current density with depth.*

penetration is equal to this distance is correct, then the value of z/D corresponding to this is 2/3 or 0·667. It will be seen from Fig. 2.5, that at this value of z/D, the current density is still about 58 per cent of what it is at the surface, and consequently a considerable fraction of the total current must penetrate to a depth greater than this.

It must be borne in mind that these remarks apply to soil which is perfectly homogeneous in all directions, and that the current distribution is likely to be modified considerably by the presence of non-homogeneities.

The Wenner Configuration

The arrangement of four electrodes in a straight line at equal intervals as in Fig. 2.6 is known as the Wenner configuration. In his original paper, Wenner did not confine his theoretical investigation to this one system of electrodes but, as shown earlier, gave formulae which could be applied to any system of four electrodes. In the practical application which he describes in the latter part of his paper he does, however, use the system of Fig. 2.6 and this is probably why this arrangement bears his name.

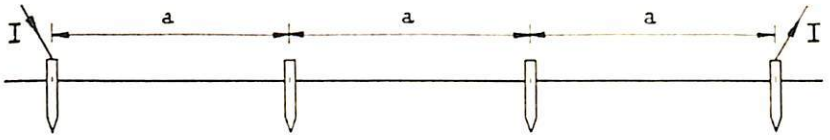

Fig. 2.6. Wenner four-electrode system.

It is also interesting to note that many subsequent investigators have used this system and, while there are other systems which have been suggested and used, this has remained on the whole the most popular. In homogeneous soil the resistivity is given by the formula;

$$\rho = 2\pi aR \tag{2.22}$$

This is a comparatively simple formula and one easily remembered. Most other systems of electrodes lead to more involved formulae.

Lee's Method of Partitioning

Although this chapter is concerned entirely with homogeneous soil, it must of course be realised that such a condition very seldom exists in practice. When considering the design of earth electrodes it may be sufficient to consider the soil as homogeneous and to make measurements accordingly.

It will be shown in a later chapter that the resistivity is an important factor in determining the resistance of any electrode. When measurements of resistivity are made in order to make deductions as to the nature of the underlying soil, then it is definitely non-homogeneities which are being sought. This subject is also dealt with fully in later chapters, but it is mentioned here because one system of electrodes has been devised with the special aim of locating changes in the soil.

Lee has suggested what he calls a method of partitioning, illustrated

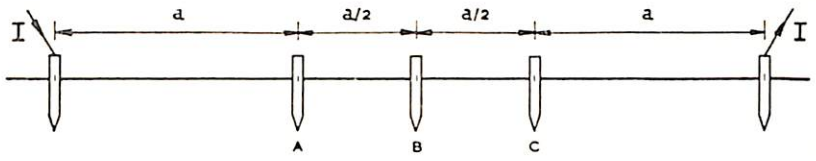

Fig. 2.7. Lee's method of partitioning.

in Fig. 2.7. In this case five electrodes are used in an arrangement as shown, four only being used in any one measurement. The outer electrodes are used as current electrodes, and two measurements are made, the first using electrodes A and B as potential electrodes and the second using electrodes B and C. It will readily be seen that from symmetry, the potential of electrode B will be zero. The potential of electrode A is

$$V_A = \frac{I\rho}{2\pi}\left(\frac{1}{a} - \frac{1}{2a}\right)$$

and this will also be the potential difference between A and B. If R_{AB} is the measured resistance between A and B equal to V_{AB}/I, this expression of V_{AB} leads to the formula

$$\rho = 4\pi a R_{AB} \tag{2.23}$$

By a similar argument if R_{BC} is the resistance measured between the electrodes B and C, then

$$\rho = 4\pi a R_{BC} \tag{2.24}$$

It is argued that two measurements can be made without moving the electrodes, and if the values of resistivity obtained from these two formulae (2.23) and (2.24) differ, then the soil is not homogeneous. With the aid of further measurements, obtained by altering the value of a, or by moving the location of the electrode system, deductions can be made as to the position and nature of the non-homogeneity in question. It is obvious that this method will only locate lateral changes and not vertical ones.

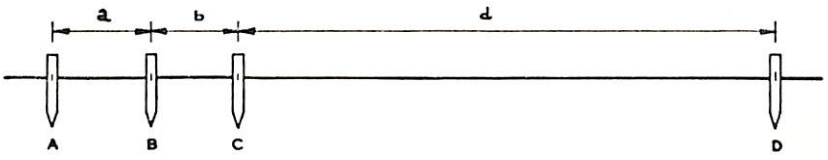

Fig. 2.8. Central electrode system.

Central Electrode System

When the Wenner four-electrode system, Fig. 2.6, is used and it is desired to change the electrode separation or the location of the test, all four electrodes and their connecting wires have to be moved. In rough country this can be very laborious, and the central electrode system has been devised, so that it is necessary to move two electrodes instead of four. This electrode system is shown in Fig. 2.8.

Here again the electrodes A and D are the current electrodes. The electrode D is placed sufficiently far away that the potential difference between the electrodes B and C due to the current I leaving at D can be neglected. Now the potential difference between B and C due to the current entering at electrode A is

$$V_{AB} = \frac{I\rho}{2\pi}\left[\frac{1}{a} - \frac{1}{(a+b)}\right] = \frac{I\rho}{2\pi}\left[\frac{b}{a(a+b)}\right] \tag{2.25}$$

If R_{AB} is the measured resistance $= V_{AB}/I$, then equation (2.25) leads to the following expression for the resistivity:

$$\rho = \frac{2\pi a(a+b)R}{b} \tag{2.26}$$

This is a slightly more complicated formula than that for the Wenner configuration.

As already mentioned it is assumed that the potential difference between B and C due to the current I leaving at D is negligible. A rough idea of the distance d between C and D for this to be so can be obtained as follows.

The potential difference between B and C due to the current I leaving at D is

$$\frac{I\rho}{2\pi} = \frac{b}{d(d+b)}$$

and if it is assumed that this must not be more than 1 per cent of that due to I entering at A, then

$$\frac{b}{a(a+b)} \Big/ \frac{b}{d(d+b)} > 100$$

or

$$\frac{d(d+b)}{a(a+b)} > 100$$

If a and d are large compared with b, then

$$\frac{d^2}{a^2} > 100 \quad \text{or} \quad \frac{d}{a} > 10$$

So if d is more than 10 times a the effect of electrode D can be neglected.

In order to ensure a uniform flow of current away from electrode A, the electrode D often takes the form of a ring of electrodes round A as centre, all connected together and forming the return electrode.

One difficulty with this method is the amount of space which is required for the electrode system compared with the Wenner four electrode configuration.

Alternative Electrode Connections

In the Wenner configuration the usual arrangement of electrodes is C, P, P, C, where C represents a current electrode and P a potential electrode. The C and P electrode can be interchanged without altering the value of the measured resistance.

Carpenter[1] has suggested that three possible measurements can be made with four electrodes, and from these three, value of the resistivity can be calculated. In completely homogeneous soil these three values are the same, but it is claimed that in non-homogeneous soil the three resistivities will not be the same and from the differences deductions can be made as to the non-homogeneities. The possible arrangements of electrodes are given in Table 2.1.

TABLE 2.1

Electrode Arrangement				*Resistivity formula*
C	P	P	C	$\rho_1 = 2\pi a R_1$
P	C	C	P	
C	C	P	P	$\rho_2 = 6\pi a R_2$
P	P	C	C	
C	P	C	P	$\rho_3 = 3\pi a R_3$
P	C	P	C	

The formulae for the resistivities are deduced in a similar manner as for the Wenner configurations. It can be shown that

$$3\rho_1 = \rho_2 + 2\rho_3$$

and

$$R_1 = R_2 + R_3$$

for homogeneous soil.

Electrical Coring[2]

An interesting application of resistivity measurements is that known as electrical coring, the principle of which is illustrated in Fig. 2.9. The

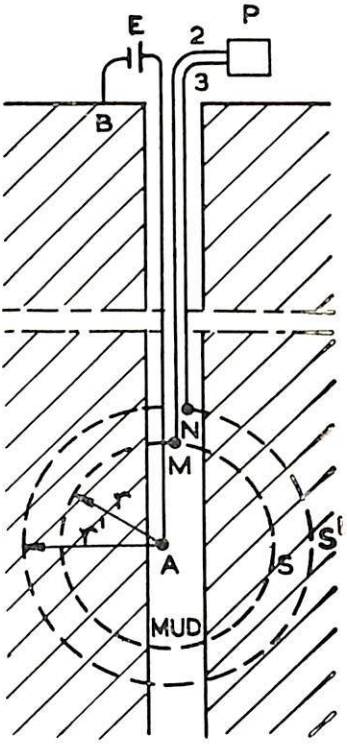

Fig. 2.9. Principle of electrical coring.

usual four electrodes, two current and two potential are used, but in this case, one current electrode is on the surface, and the other three are lowered into a bore hole containing mud or other liquid material. It can generally be considered that the effect of the electrode on the surface can be neglected, and the potential difference between the two

potential electrodes can be considered as that due to the current entering by the current electrode A in the bore hole.

In homogeneous soil, the potential at M due to the current I entering at A is $I\rho/2\pi r_1$ and that at N is $I\rho/2\pi r_2$. So the potential difference between M and N is

$$E = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

so if the measured resistance $R = V/I$ then

$$\rho = \frac{2\pi R}{\frac{1}{r_1} - \frac{1}{r_2}} \tag{2.27}$$

The main use of this method is, of course, on non-homogeneous soil.

Unequal Electrode Spacings

In the Wenner configuration the electrodes are equally spaced, but it is argued by some investigators[3] that an unequal spacing may sometimes be advantageous. If, for instance, the distance between the potential electrodes is increased, the value of the measured resistance

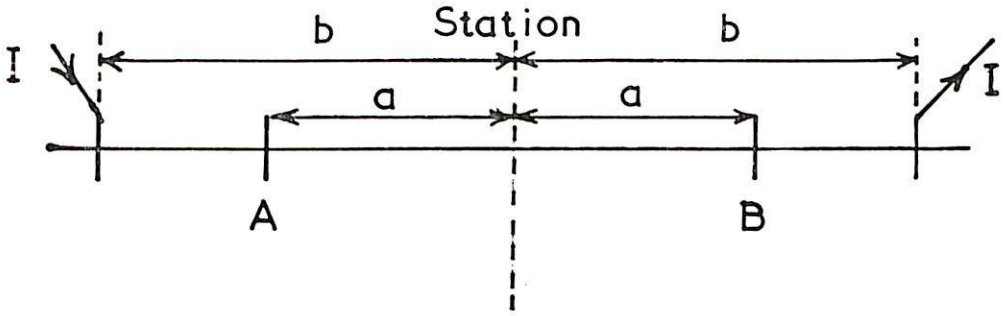

Fig. 2.10. *Unequal spacing.*

is also increased and this may be a help in making the actual measurement more easy. In Fig. 2.10 one suggested arrangement is shown, in which the system of electrodes is still symmetrical about the centre, but the distance between the potential electrodes is greater than that between adjacent current and potential electrodes.

In Fig. 2.10 the potential at A will be

$$\frac{\rho I}{2\pi}\left[\frac{1}{b-a}-\frac{1}{b+a}\right]$$

and by symmetry the potential difference between A and B will be twice this. Hence the measured potential V is

$$V=\frac{2\rho I}{2\pi}\left[\frac{1}{b-a}-\frac{1}{b+a}\right]=\frac{\rho I}{2\pi}\left[\frac{4a}{b^2-a^2}\right]$$

The measured resistance $R = V/I$ and so

$$R=\frac{\rho}{2\pi}\left[\frac{4a}{b^2-a^2}\right]=\frac{\rho}{2\pi a}\left[\frac{4}{(b/a)^2-1}\right]$$

If $b/a = \alpha$, then

$$\rho = 2\pi aR\left[\frac{\alpha^2-1}{4}\right] \tag{2.28}$$

In the Wenner uniformly spaced system $\alpha = 3$ and this equation then reduces to

$$\rho = 4\pi aR.$$

The value of a in this case, however, is only one half the value in the equation for the Wenner system, so if this adjustment is made, equation (2.28) reduces to the usual $\rho = 2\pi aR$ of the Wenner system.

It is rather doubtful whether there is much advantage to be gained by this unequal spacing, the amount of evidence available being too little to make an accurate judgement.

General Note

In this chapter the various methods and electrode arrangements have been described, and the theory has been given for the special case of homogeneous soil.

In the design of earth electrodes, as has already been stated, the resistivity is important and it may often be sufficient in some cases to assume that the soil is homogeneous. In resistivity surveying, however, it is the non-homogeneities which are important and in such cases it is usual to apply these formulae for homogeneous soil. The resistivity so obtained is known as the apparent resistivity. This will vary with electrode spacing and location of the electrode system, and it is these variations from which deductions can be made as to the nature of the sub-soil.

In some cases theoretical investigations can assist in the interpretation and can lead to methods of interpreting the results obtained. In other cases, qualitative interpretation is all that is necessary.

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3

RESISTIVITY SURVEYING IN NON-HOMOGENEOUS SOIL

As already explained in Chapter 2, the soil is nearly always non-homogeneous and these non-homogeneities can take many forms. In most cases there are several layers of soil, which may be loam, sand, gravel, clay or mixtures of these, and rocks. These layers may be approximately horizontal and parallel to the surface or may be inclined at an angle to the surface. It should be noted here that the water table acts as an additional layer, since the increase in moisture content below the water table will result in a significant change in resistivity in a number of instances. From a theoretical point of view the parallel layer problem is a comparatively simple one, although the result may not be considered as being very simple.

Lateral changes in resistivity can be produced by a vertical fault, and the soil on each side of this fault may be of different constituents, or the depths to the various layers may change. In either case, there is likely to be a significant change in resistivity and such a change can be detected and used to locate the position of the fault. Buried objects can also be located under certain favourable conditions, since the presence of such a body can produce a difference in the apparent resistivities obtained when the electrode system is over the buried object and when the electrode system is well removed from the vicinity of the object. The location of buried objects underground has many applications, for instance, archaeological features, mineral bodies, and lateral changes in the soil are examples of what can be located.

The Two-layer Problem

One of the simplest, while at the same time one of the most important, problems is that which is generally known as the two-layer problem

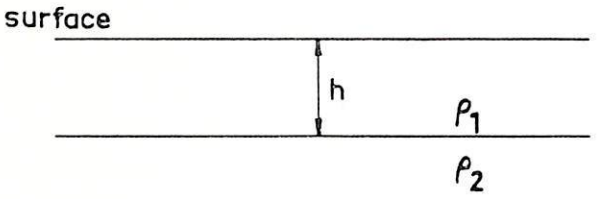

Fig. 3.1.
Two layer problem.

and is illustrated in Fig. 3.1. There is a surface layer of resistivity ρ_1 of thickness h overlaying a second layer of resistivity ρ_2, which extends to an infinite distance in the downward direction.

While this is perhaps only an approximation to many practical cases, it does serve a useful purpose as a starting point for more complicated problems, while the methods of interpretation based on it can be used successfully in a number of cases.

Statement of the Problem

If a current enters at a point A on the surface, the problem resolves itself into the calculation of the potential at all points in the soil, and any expressions for the potential must satisfy the usual conditions governing the flow of current in a semi-infinite medium.

Suppose V_1 = the potential at any point in the upper layer.
V_2 = the potential at any point in the lower layer.
then first both potentials must satisfy Laplace's equation

$$\nabla^2 V = 0.$$

It is most convenient in this case to use cylindrical co-ordinates, r, θ, z, and in this case, Laplace's equation becomes, bearing in mind that there is cylindrical symmetry and so θ is eliminated,

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}.\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{3.1}$$

Suppose that a solution may be found in the form

$$V = \Phi(r)\, e^{\pm \lambda z}$$

If this is substituted in equation (3.1), it leads to

$$\frac{\mathrm{d}^2\Phi}{dr^2} + \frac{1}{r}.\frac{\mathrm{d}\Phi}{dr} + \lambda^2\Phi = 0 \tag{3.2}$$

This is Bessel's equation of order zero, and the classical solutions are $J_0(\lambda r)$ and $Y_0(\lambda r)$, together with any linear combination of these functions with constant coefficients. In particular there are the Hankel integrals.

$$\int \theta(\lambda) J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda \quad \text{and} \quad \int \psi(\lambda) J_0(\lambda r)\, e^{+\lambda z}.\mathrm{d}\lambda.$$

taken between any fixed limits.

In addition Weber has shown that the fundamental solution $1/R$ of Laplace's equation can be written in the form

$$\frac{1}{R} = \int_0^\infty J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda$$

The potentials V_1 and V_2 can be considered as made up of two parts. The first is the potential which would exist if the soil were perfectly homogeneous of resistivity ρ_1. The second is an additional function representing the disturbing effect of the lower layer. So the potentials can be written

$$V_1 = \frac{I\rho_1}{2\pi}\left[\int_0^\infty J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda + \int_0^\infty \theta_1(\lambda) J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda + \int_0^\infty \psi_1(\lambda) J_0(\lambda r)\, e^{+\lambda z}.\mathrm{d}\lambda\right] \quad (3.3)$$

$$V_2 = \frac{I\rho_1}{2\pi}\left[\int_0^\infty J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda + \int_0^\infty \theta_2(\lambda) J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda + \int_0^\infty \psi_2(\lambda) J_0(\lambda r)\, e^{+\lambda z}.\mathrm{d}\lambda\right] \quad (3.4)$$

It is now necessary to determine the values of the functions $\theta_1(\lambda)$, $\theta_2(\lambda)$, $\psi_1(\lambda)$ and $\psi_2(\lambda)$.

The Boundary Conditions

These functions will be determined by the boundary conditions. At the surface $z = 0$, the current flow will be along the surface so,

$$\frac{\mathrm{d}V_1}{\mathrm{d}z} = 0 \quad (3.5)$$

As z goes to infinity, the value of V_2 must go to zero.

At the interface $z = h$, the potential in both layers must be the same, i.e. $V_1 = V_2$ when $z = h$. Also the flow of current must be the same on both sides of the interface so,

$$\frac{1}{\rho_1}.\frac{\mathrm{d}V_1}{\mathrm{d}z} = \frac{1}{\rho_2}\frac{\mathrm{d}V_2}{\mathrm{d}z}$$

The condition that V_2 goes to zero as z becomes infinite means that terms involving $e^{+\lambda z}$ cannot be used or $\psi_2(\lambda) = 0$. At the surface the first term in the expression for V_1 always satisfies the condition that $dV_1/dz = 0$, and applying this condition to the rest of the equation means

$$\frac{I\rho_1}{2\pi}\int_0^\infty \{-\theta_1(\lambda)\lambda + \psi_1(\lambda).\lambda\}J_0(\lambda r).d\lambda = 0$$

This is satisfied by the condition

$$\theta_1(\lambda) = \psi_1(\lambda)$$

Thus the expressions for the potentials become:

$$V_1 = \frac{I\rho_1}{2\pi}\left[\int_0^\infty J_0(\lambda r)\, e^{-\lambda z}.d\lambda + \int_0^\infty \theta_1(\lambda)J_0(\lambda r)\, e^{-\lambda z}.d\lambda + \int_0^\infty \theta_1(\lambda)J_0(\lambda r)\, e^{+\lambda z}.d\lambda\right]$$

$$V_2 = \frac{I\rho_1}{2\pi}\left[\int_0^\infty J_0(\lambda r)\, e^{-\lambda z}.d\lambda + \int_0^\infty \theta_2(\lambda)J_0(\lambda r)\, e^{-\lambda z}.d\lambda\right]$$

The condition $V_1 = V_2$ at $z = h$ leads to

$$\theta_1(\lambda)\, e^{-\lambda h} + \theta_1(\lambda)\, e^{+\lambda h} = \theta_2(\lambda)\, e^{-\lambda h}$$

or

$$\theta_1(\lambda)(e^{-\lambda h} + e^{+\lambda h}) = \theta_2(\lambda)\, e^{-\lambda h} \tag{3.6}$$

The second condition leads to

$$\frac{I}{2\pi}\left[\int_0^\infty \{-\lambda\, e^{-\lambda h} - \lambda\theta_1(\lambda)\, e^{-\lambda h} + \lambda\theta_1(\lambda)\, e^{+\lambda h}\}J_0(\lambda r).d\lambda\right] = \frac{I}{2\pi}\frac{\rho_1}{\rho_2}\left[\int_0^\infty \{-\lambda\, e^{-\lambda h} - \lambda\theta_2(\lambda)\, e^{-\lambda h}\}J_0(\lambda r).d\lambda\right]$$

or

$$-e^{-\lambda h} - \theta_1(\lambda)\{e^{-\lambda h} - e^{+\lambda h}\} = \frac{\rho_1}{\rho_2}\{-e^{-\lambda h} - e^{-\lambda h}\theta_2(\lambda)\} \tag{3.7}$$

From (3.6)

$$\theta_1(\lambda) = \frac{\theta_2(\lambda)\, e^{-\lambda h}}{e^{-\lambda h} + e^{+\lambda h}} = \frac{\theta_2(\lambda)\, e^{-2\lambda h}}{e^{-2\lambda h} + 1}$$

From (3.7)

$$-e^{-\lambda h}\left\{1-\frac{\rho_1}{\rho_2}\right\}-\theta_1(\lambda)\{e^{-\lambda h}-e^{+\lambda h}\}=-\frac{\rho_1}{\rho_2}e^{-\lambda h}\theta_2(\lambda)$$

$$-e^{-2\lambda h}\left\{1-\frac{\rho_1}{\rho_2}\right\}-\theta_1(\lambda)\{e^{-2\lambda h}-1\}=-\frac{\rho_1}{\rho_2}e^{-2\lambda h}\theta_2(\lambda)$$

Substituting for $\theta_1(\lambda)$ gives

$$-e^{-2\lambda h}\left\{1-\frac{\rho_1}{\rho_2}\right\}-\frac{\theta_2(\lambda)\,e^{-2\lambda h}\{e^{-2\lambda h}-1\}}{e^{-2\lambda h}+1}=-\frac{\rho_1}{\rho_2}e^{-2\lambda h}\theta_2(\lambda)$$

$$\theta_2(\lambda)\left[\frac{\rho_1}{\rho_2}e^{-2\lambda h}-\frac{e^{-4\lambda h}-e^{-2\lambda h}}{e^{-2\lambda h}+1}\right]=e^{-2\lambda h}\left\{1-\frac{\rho_1}{\rho_2}\right\}$$

$$\theta_2(\lambda)\left[\frac{\frac{\rho_1}{\rho_2}e^{-4\lambda h}+\frac{\rho_1}{\rho_2}e^{-2\lambda h}-e^{-4\lambda h}+e^{-2\lambda h}}{e^{-2\lambda h}+1}\right]=e^{-2\lambda h}\left\{1-\frac{\rho_1}{\rho_2}\right\}$$

$$\therefore\ \theta_2(\lambda)\left[\frac{-e^{-4\lambda h}\left(1-\frac{\rho_1}{\rho_2}\right)+e^{-2\lambda h}\left(1+\frac{\rho_1}{\rho_2}\right)}{e^{-2\lambda h}+1}\right]=e^{-2\lambda h}\left\{1-\frac{\rho_1}{\rho_2}\right\}$$

$$\therefore\ \theta_2(\lambda)[-e^{-4\lambda h}(\rho_2-\rho_1)+e^{-2\lambda h}(\rho_2+\rho_1)]$$
$$=(\rho_2-\rho_1)\{e^{-4\lambda h}+e^{-2\lambda h}\}$$

Dividing through by $e^{-2\lambda h}$ and writing $k=(\rho_2-\rho_1)/(\rho_2+\rho_1)$

$$\therefore\ \theta_2(\lambda)[-k\,e^{-2\lambda h}+1]=k[e^{-2\lambda h}+1]$$

$$\therefore\ \theta_2(\lambda)=\frac{k(1+e^{-2\lambda h})}{1-k\,e^{-2\lambda h}}$$

So

$$\theta_1(\lambda)=\frac{k(1+e^{-2\lambda h})}{1-k\,e^{-2\lambda h}}\cdot\frac{e^{-2\lambda h}}{(1+e^{-2\lambda h})}$$
$$=\frac{k\,e^{-2\lambda h}}{1-k\,e^{-2\lambda h}}$$

The important potential is V_1 and this becomes

$$V_1=\frac{I\rho_1}{2\pi}\left[\int_0^\infty J_0(\lambda r).e^{-\lambda z}\,d\lambda+\int_0^\infty\frac{k\,e^{-2\lambda h}(e^{-\lambda z}+e^{+\lambda z})}{1-k\,e^{-2\lambda h}}\cdot J_0(\lambda r)\cdot d\lambda\right]$$

The surface potential at $z = 0$ becomes

$$V_1 = \frac{I\rho_1}{2\pi}\left[\int_0^\infty J_0(\lambda r).\mathrm{d}\lambda + \int_0^\infty \frac{2k\, e^{-2\lambda h}}{1 - k\, e^{-2\lambda h}}.J_0(\lambda r).\mathrm{d}\lambda\right] \tag{3.8}$$

Solution of the Potential Equation

The solution of the first integral in equation (3.8) is, as already indicated $1/r$, and in order to evaluate the second integral, it is necessary to expand the fraction. Now

$$\frac{k\, e^{-2\lambda h}}{1 - k\, e^{-2\lambda h}} = k\, e^{-2\lambda h} + k^2\, e^{-4\lambda h} + k^3\, e^{-6\lambda h} +$$

$$= \sum_{n=1}^{n=\infty} k^n.e^{-2n\lambda h} \tag{3.9}$$

So the second integral becomes

$$2\int_0^\infty \sum_{n=1}^{n=\infty} k^n\, e^{-2n\lambda h}.J_0(\lambda r).\mathrm{d}\lambda$$

$$= 2\sum_{n=1}^{n=\infty} \frac{k^n}{[r^2 + (2nh)^2]^{1/2}}$$

and so the total solution for the potential at any point on the surface at a distance r from the current source A in Fig. 3.1 is

$$V = \frac{I\rho_1}{2\pi}\left[\frac{1}{r} + 2\sum_{n=1}^{n=\infty} \frac{k^n}{\{r^2 + (2nh)^2\}^{1/2}}\right] \tag{3.10}$$

Application to the Wenner Configuration

The Wenner configuration for two-layer the case is shown in Fig. 3.2. The current I enters at A and leaves at D, B and C being the potential electrodes. Consider first the potential at B. The current source at A is at a distance a from B so for the potential at B due to the current at A,

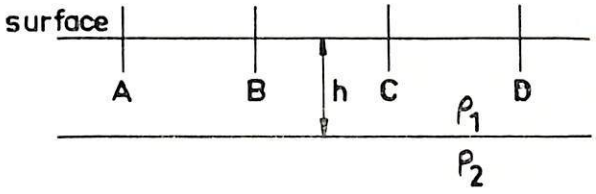

Fig. 3.2. The Wenner configuration and the two layer case.

r in equation (3.10) is equal to a. So potential at B due to current I at A is

$$\frac{I\rho_1}{2\pi}\left[\frac{1}{a}+2\sum_{n=1}^{n=\infty}\frac{k^n}{\{a^2+(2nh)^2\}^{1/2}}\right]$$

The current sink at D is at a distance $2a$ from B, so the potential at B due to D is

$$-\frac{I\rho_1}{2\pi}\left[\frac{1}{2a}+2\sum_{n=1}^{n=\infty}\frac{k^n}{\{4a^2+(2nh)^2\}^{1/2}}\right]$$

Consequently the total potential at B is

$$V_B=\frac{I\rho_1}{2\pi}\left[\frac{1}{2a}+2\sum_{n=1}^{n=\infty}\left\{\frac{k^n}{\{a^2+(2nh)^2\}^{1/2}}-\frac{k^n}{\{4a^2+(2nh)^2\}^{1/2}}\right\}\right]$$

It is obvious from the symmetry of the system that the potential at C will have the same value as but be of opposite sign to that at B and that the potential difference between B and C will be $2V$. Hence

$$V_{AB}=\frac{I\rho_1}{2\pi}\left[\frac{1}{a}+4\sum_{n=1}^{n=\infty}\left\{\frac{k^n}{\{a^2+(2nh)^2\}^{1/2}}-\frac{k^n}{\{4a^2+(2nh)^2\}^{1/2}}\right\}\right]$$

This can be written as follows:

$$V_{AB}=\frac{I\rho_1}{2\pi a}\left[1+4\sum_{n=1}^{n=\infty}\left\{\frac{k^n}{\left\{1+\left(2n\frac{h}{a}\right)^2\right\}^{1/2}}-\frac{k^n}{\left\{4+\left(2n\frac{h}{a}\right)^2\right\}^{1/2}}\right\}\right]$$

or

$$2\pi a\frac{V_{AB}}{I}=\rho_1\left[1+4\sum_{n=1}^{n=\infty}\frac{k^n}{\left\{1+\left(\frac{2nh}{a}\right)^2\right\}^{1/2}}-\frac{k^n}{\left\{4+\left(\frac{2nh}{a}\right)^2\right\}^{1/2}}\right] \qquad (3.12)$$

Now in the quantity on the left-hand side of equation (3.12), the ratio V_{AB}/I is the measured resistance R, and so the left-hand side of the equation becomes $2\pi aR$. But this is the expression which when the Wenner configuration is used in homogeneous soil gives the resistivity. Now, if this simple expression is used in the two layer problem it gives

an "apparent resistivity" which is designated by ρa. So equation (3.12) can be rewritten,

$$\frac{\rho_a}{\rho_1} = 1 + 4\sum_{n=1}^{n=\infty}\left[\frac{k^n}{\left\{1+\left(2n\frac{h}{a}\right)^2\right\}^{\frac{1}{2}}} - \frac{k^n}{\left\{4+\left(2n\frac{h}{a}\right)^2\right\}^{\frac{1}{2}}}\right]$$

$$= 1 + 4F \qquad (3.13)$$

where F is the sum of the infinite series.

This equation (3.13) is a very important one and is the basis of a number of methods which are aimed at giving satisfactory interpretations to actual field results.

The Reflection Coefficient k

The reflection coefficient k is an important quantity in expression (3.13) and indeed in all similar problems. It varies between $+1$ and -1. If the second layer is a pure insulator $\rho_2 = \infty$, then $k = +1$ and if the second layer is a perfect conductor, i.e. $\rho_2 = 0$, then $k = -1$. Now

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = \frac{\rho_2/\rho_1 - 1}{\rho_2/\rho_1 + 1}$$

and so depends on the ratio ρ_2/ρ_1 alone. The values of ρ_2/ρ_1 corresponding to various values of k are given in Table 3.1.

TABLE 3.1

Value of k	ρ_2/ρ_1	*Value of k*	ρ_2/ρ_1
+1·0		−1·0	0
+0·9	19·0	−0·9	0·0526
+0·8	9·0	−0·8	0·1111
+0·7	5·67	−0·7	0·1765
+0·6	4·0	−0·6	0·2500
+0·5	3·0	−0·5	0·3333
+0·4	2·333	−0·4	0·4286
+0·3	1·857	−0·3	0.5384
+0·2	1·50	−0·2	0·6667
+0·1	1·222	−0·1	0·8182
0	1·000		

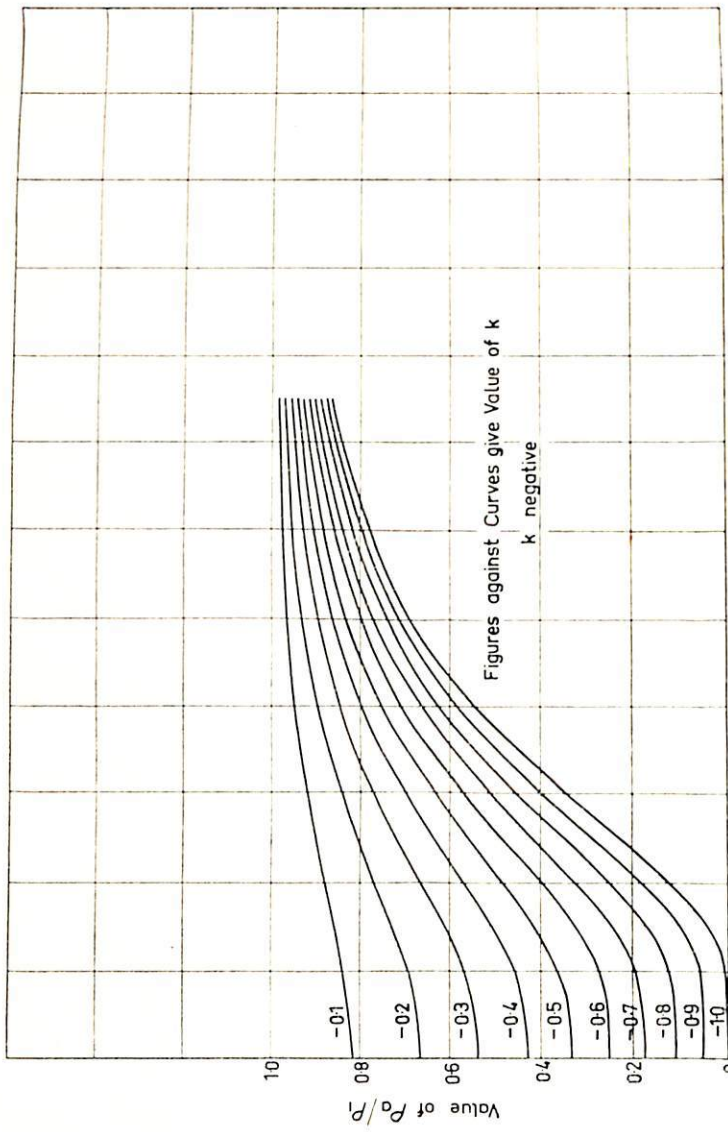

Fig. 3.3.

Fig. 3.3. Master curves—two-layer problem—lower layer more conducting.

Master Curves

For any given value of k, it is possible to calculate from equation (3.13) a curve connecting ρ_a/ρ_1 and the ratio h/a. This can be repeated for all possible values of k, and if k is negative this results in the set of curves given in Fig. 3.3.

When k is positive, it is more convenient to calculate the reciprocal of the ratio ρ_a/ρ_1 since this results in numbers less than unity. So if σ_a is the apparent conductivity $= 1/\rho_a$, and σ_1 = the conductivity of the

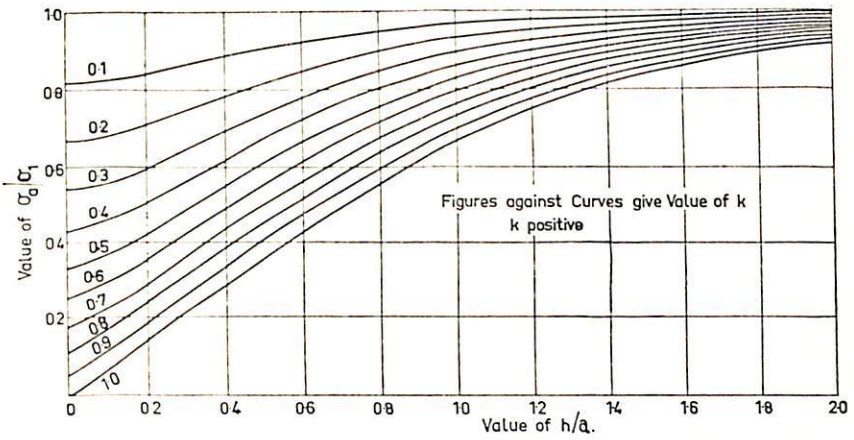

Fig. 3.4. *Master curves—two-layer—problem—lower layer less conducting.*

surface layer $= 1/\rho_1$, then the set of curves in Fig. 3.4 shows the relation between σ_a/σ_1 and h/a for all positive values of k. The figures from which these curves are drawn are given in Appendix 1.

Typical Two-layer Cases

Examples of the types of curves of apparent resistivity against electrode separation which can be expected under these idealised conditions are given in Figs. 3.5 and 3.6. In Fig. 3.5 the upper layer has a resistivity of 10,000 ohm-cms and a thickness of 50 ft, and the lower layer has a resistivity of 1,111 ohm-cms. The ratio ρ_2/ρ_1 is thus 1/9 and the reflection coefficient k is $-0{\cdot}8$. In Fig. 3.6 the upper layer is the same but the lower layer has a resistivity of 90,000 ohm-cms, making the ratio $\rho_2/\rho_1 = 9$ and the reflection coefficient $k = +0{\cdot}8$.

A point to be noted about these curves is that there is nothing significant at the value of electrode separation equal to the thickness of the upper layer. There is necessarily a point of inflection in these curves, but it is often difficult to locate it accurately by inspection and

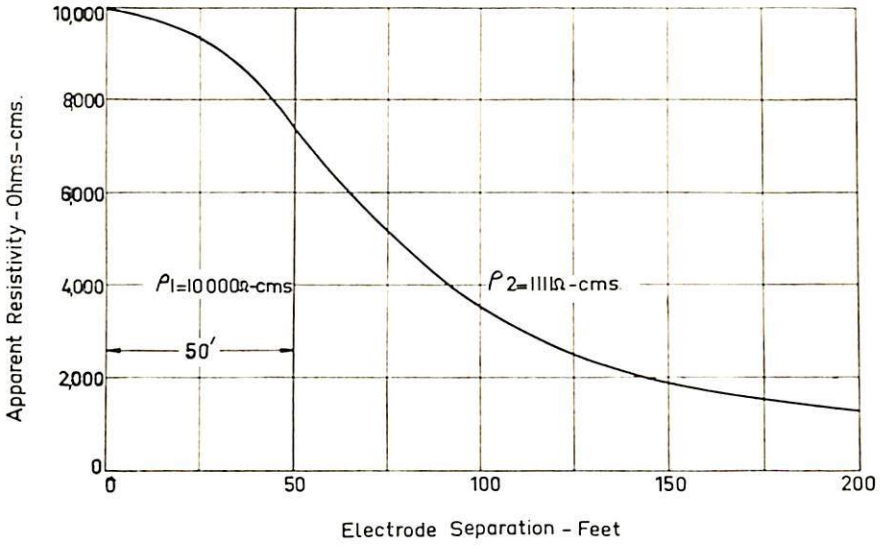

Fig. 3.5. *Typical two-layer curve—lower layer more conducting.*

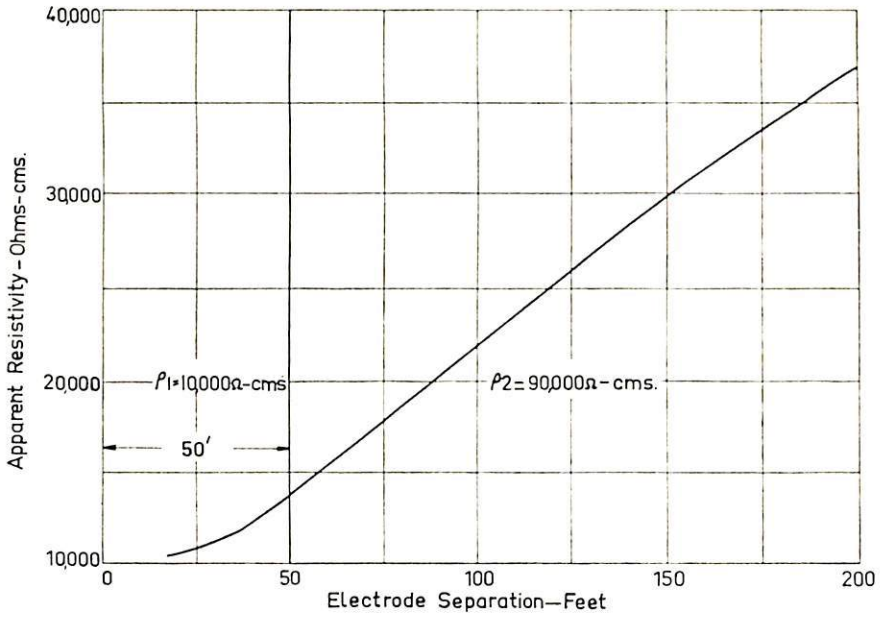

Fig. 3.6. *Typical two-layer curve—lower layer less conducting.*

it certainly does not occur at the point $h/a = 1$. These points are important and will be referred to again in the chapter dealing with the interpretation of resistivity curves.

The Multi-layer Case

Unfortunately, the idealised two-layer case does not occur very frequently in practice and, in general, it is necessary to consider the ground as consisting of a number of layers. The theory of the multi-layer problem is not particularly difficult but the calculation can be very laborious and the possible number of variables can be considerable.

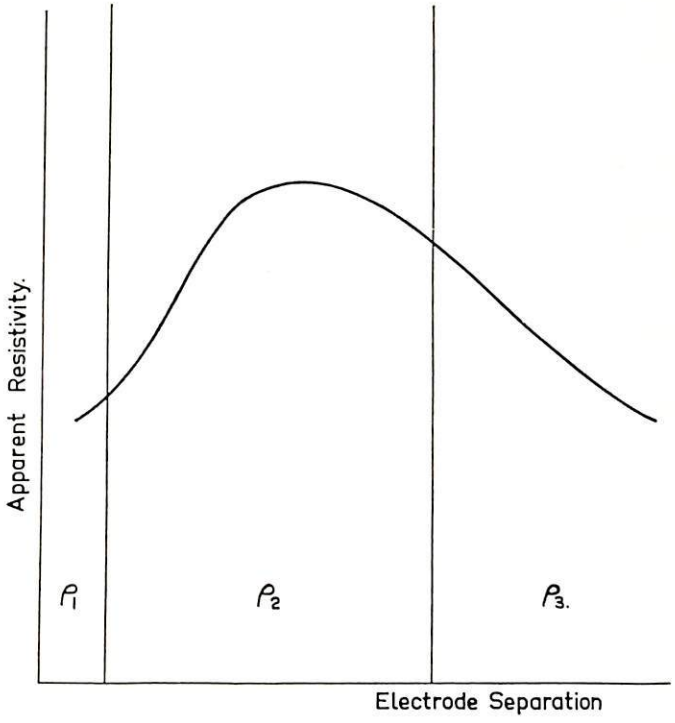

Fig. 3.7. *Typical three-layer curve.*

For example, in a four-layer problem there are four resistivities and three depths, i.e. a total of seven variables all of which are independent of each other and can have any value. With a greater number of layers the number of variables is increased and it is obviously impossible to produce a series of solutions which will cover all practical cases.

Nevertheless, a number of investigators have worked out the theory, some by extending the theory on pages 31 to 37, others by using the method of electrical images. Among these are Hummel,[1] Stefanesco

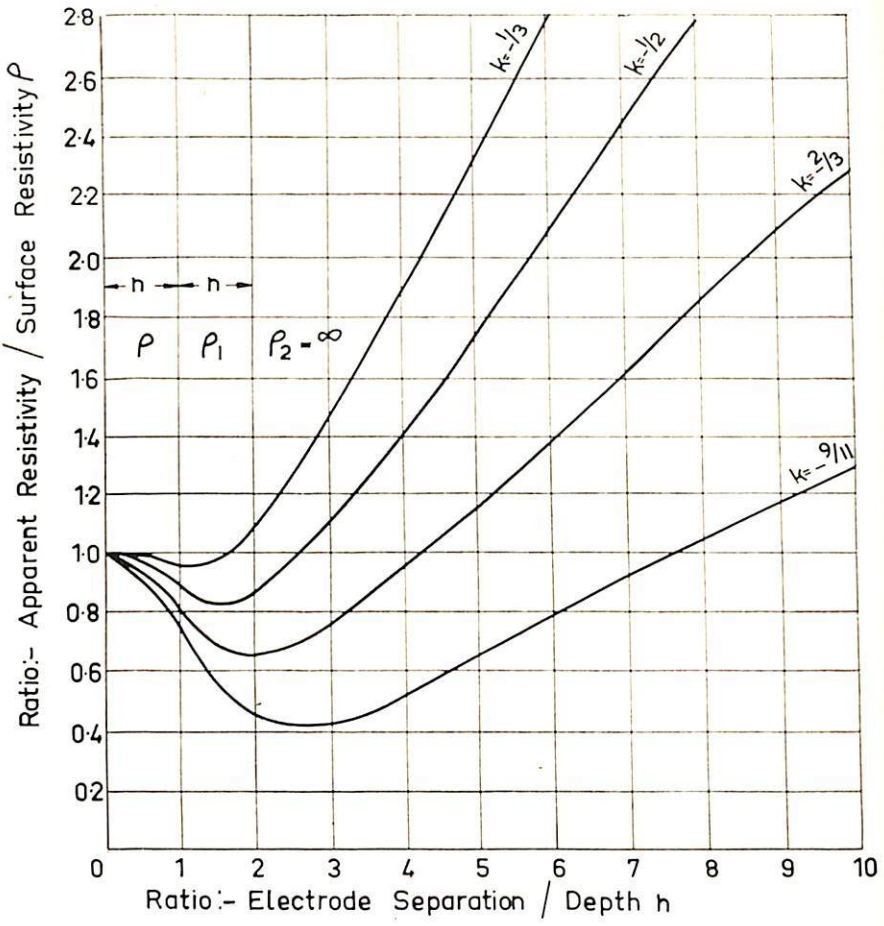

Fig. 3.8. *Typical three-layer curves.*

and Schlumberger[2] and Ehrenburg and Watson[3] and recently sets of curves for the two, three and four layer problems have been published by Mooney and Wetzel.[4]

An example of the type of curve which can be obtained in the three-layer case is given in Fig. 3.7. In this case, the middle layer has a resistivity ρ_2 which is higher than those of the first and third layers ρ_1 and ρ_3. Other typical curves are given in Fig. 3.8 in which the middle layer has the lowest resistivity of the three. The reflection coefficients against the curves refer to the upper two layers.

Inclined Layers

The above analysis and discussion has been concerned with layers which are horizontal and parallel to the surface. It often happens that the lower layers are inclined at an angle to the surface and this theoretical problem has attracted the interest of a number of investigators. It is a more difficult problem than that of the parallel layers and the works of Maeda,[5] Skalskaya[6] and Unz[7] can be quoted as typical. In both cases a rigid mathematical analysis is presented, and there are several other investigators who have put forward approximate solutions.

The idea behind all this work is again to relate the apparent resistivity as measured on the surface to the characteristics of the soil, in this case the resistivities involved and the angle of dip. Maeda, for instance,

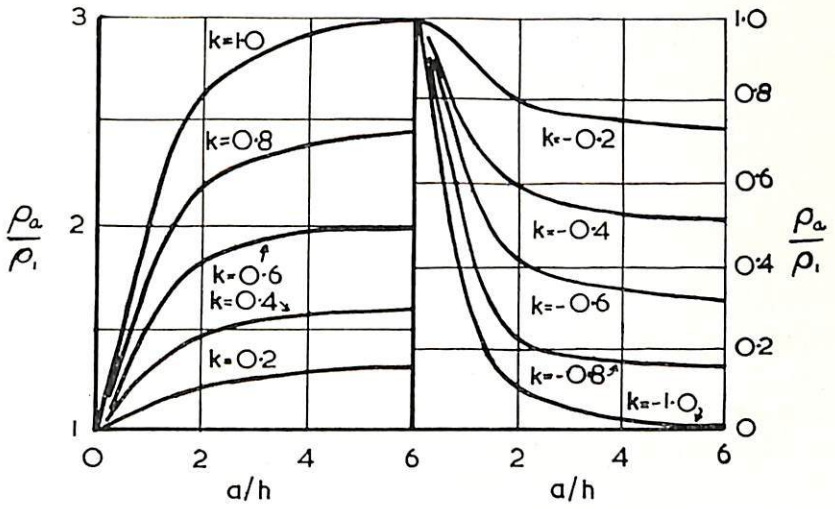

Fig. 3.9. *Maeda's curves for beds dipping at* 60°.

gives a series of curves correlating the apparent resistivity with the angle of dip, the resistivities as determined by the reflection coefficient and for the Wenner and other electrode configurations. Fig. 3.9 gives Maeda's curves for the case where the line of electrodes is parallel to the line of strike. It will be seen that the general shape of these curves is similar to those for the parallel beds, and it is not easy to determine from measurements in the field whether parallel or dipping beds are involved. If the line of electrodes crosses the line of strike, then an indication is

given of a sudden change in resistivity, and it is then apparent that some such effect as that due to an inclined layer reaching the surface has taken place.

The Vertical Fault

A limiting case of a dipping bed occurs when the angle of dip is 90°, and this becomes the case of a vertical fault. This can be used as an illustration of what happens when the line of electrodes crosses the fault. The analysis of this problem is based on the simple arrangement shown in Fig. 3.10. This shows a point source of current I at a point P embedded in a medium of resistivity ρ_1, and at a distance d from a plane separating the medium ρ_1 from a second medium of resistivity ρ_2. It is necessary to calculate the potential in both media. The analysis

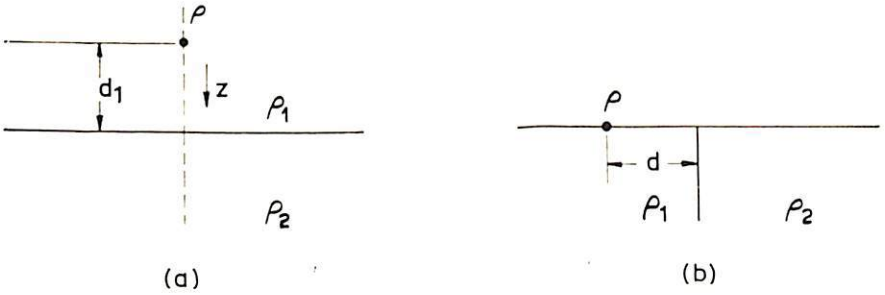

Fig. 3.10. *Simple electrode arrangement for analysis of problem of the vertical fault.*

will follow the same lines as in the "Statement of the Problem" on page 31. In this case the expressions for the potentials in the two media can be written as

$$V_1 = \frac{I\rho_1}{4\pi}\left[\int_0^\infty J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda + \int_0^\infty \theta_1(\lambda) J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda + \int_0^\infty \psi_1(\lambda) J_0(\lambda r)\, e^{+\lambda z}.\mathrm{d}\lambda\right] \quad (3.14)$$

$$V_2 = \frac{I\rho_1}{4\pi}\left[\int_0^\infty J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda + \int_0^\infty \theta_2(\lambda) J_0(\lambda r)\, e^{-\lambda z}.\mathrm{d}\lambda + \int_0^\infty \psi_2(\lambda).J_0(\lambda r)\, e^{+\lambda z}.\mathrm{d}\lambda\right] \quad (3.15)$$

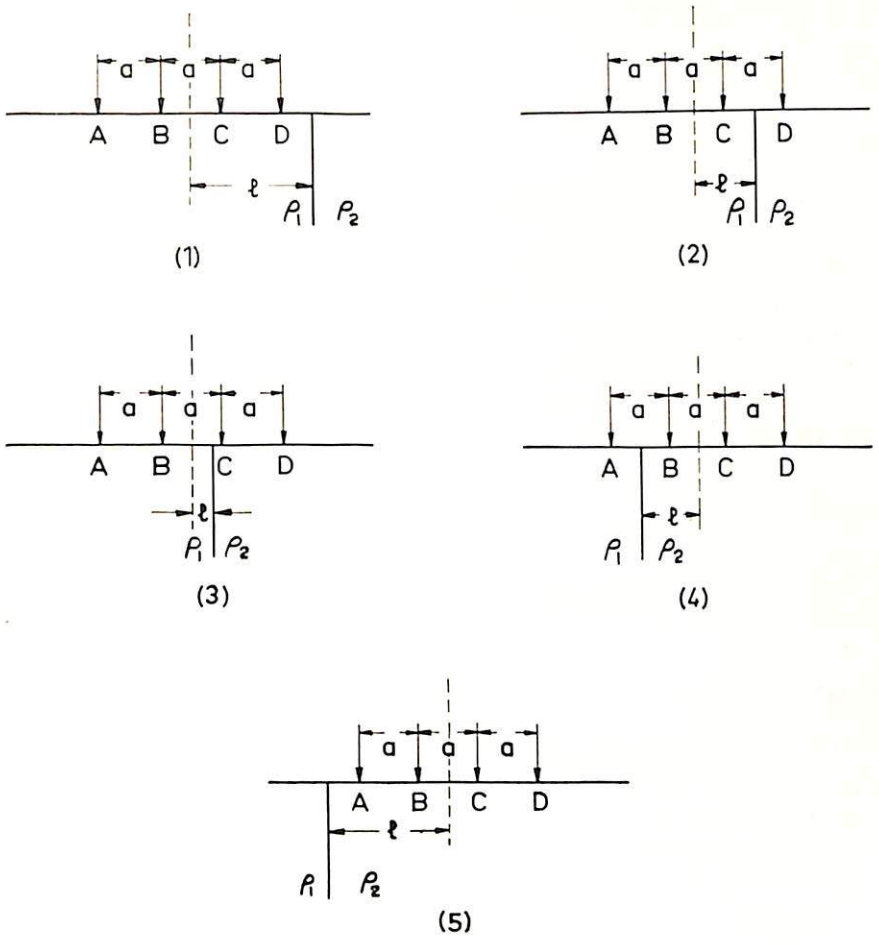

Fig. 3.11. *Effect of electrode position—showing five possible positions for the electrodes.*

The boundary conditions to be satisfied are

(a) As z goes to $-\infty$ V_1 must approach zero

(b) As z goes to $+\infty$ V_2 must approach zero

(c) At $z = d$, $V_1 = V_2$

$$\frac{1}{\rho_1} \cdot \frac{dV_1}{dz} = \frac{1}{\rho_2} \cdot \frac{dV_2}{dz}$$

Applying these conditions to equations (3.14) and (3.15) and solving

the integrals leads to the following expressions for the potentials:

$$V_1 = \frac{I\rho_1}{4\pi}\left[\frac{1}{\sqrt{\{r^2 + z^2\}}} + \frac{k}{\sqrt{\{r^2 + (2d - z)^2\}}}\right]$$

$$V_2 = \frac{I\rho_1}{4\pi}\left[\frac{1 + k}{\sqrt{\{r^2 + z^2\}}}\right]$$

It is evident that this system is symmetrical about the vertical plane passing through the dotted line in Fig. 3.10(*a*) and perpendicular to the paper. Thus, if the right-hand side is removed and replaced by air, the potential distribution is unaffected but to maintain the same values the current must be doubled. So, in the case shown in Fig. 3.10(*b*), the potential at any point is given by the equations:

$$V_1 = \frac{I\rho_1}{2\pi}\left[\frac{1}{\sqrt{\{r^2 + z^2\}}} + \frac{k}{\sqrt{\{r^2 + (2d - z)^2\}}}\right] \tag{3.16}$$

$$V_2 = \frac{I\rho_1}{2\pi}\left[\frac{1 + k}{\sqrt{\{r^2 + z^2\}}}\right] \tag{3.17}$$

Effect of Electrode Position

When the Wenner configuration is used with the line of electrodes at right-angles to the fault, there are five possible positions for the electrodes as shown in Fig. 3.11. Using equations (3.16) and (3.17) leads to the following expressions for the apparent resistivity

(1) With all four electrodes, *A*, *B*, *C* and *D* in medium ρ_1,

$$\frac{\rho_a}{\rho_1} = 1 + k\left(\frac{q}{q^2 - 1} - \frac{4q}{4q^2 - 1}\right) \tag{3.18}$$

where $q = l/a$

(2) With electrodes *A*, *B* and *C* in medium ρ_1 and electrode *D* in medium ρ_2,

$$\frac{\rho_a}{\rho_1} = 1 + k\left(\tfrac{1}{2} - \frac{1}{2q + 2} + \frac{1}{2q + 2}\right) \tag{3.19}$$

(3) With electrodes *A*, *B* in medium ρ_1 and electrodes *C*, *D* in medium ρ_2

$$\frac{\rho_a}{\rho_1} = \left\{\frac{1 + k^2}{1 - k}\right\} + \frac{k}{(2q + 2)} - \frac{k(1 + k)}{(1 - k)}\left\{\frac{1}{2 - 2q}\right\} \tag{3.20}$$

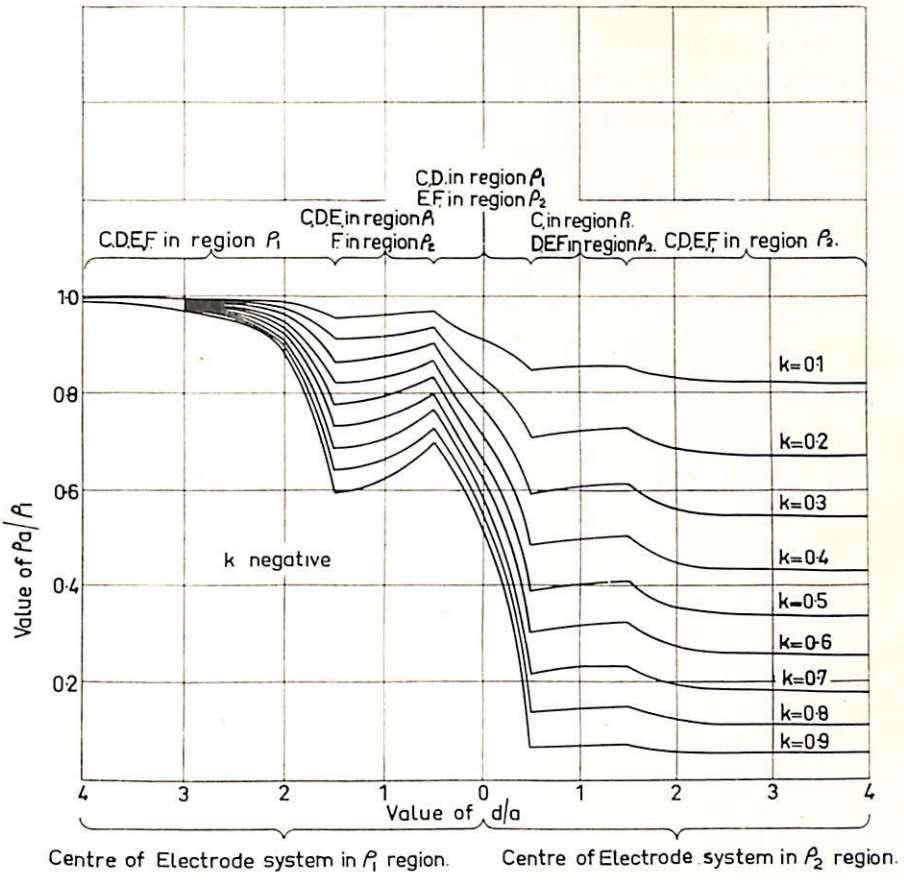

Fig. 3.12. *Vertical fault—k negative.*

(4) With electrode A in medium ρ_1, B, C, D in medium ρ_2

$$\frac{\rho_a}{\rho_1} = \left\{\frac{1+k}{1-k}\right\}\left[1 + k\left\{\frac{1}{(2q+1)} - \frac{1}{(2q+2)} - \frac{1}{2}\right\}\right] \qquad (3.21)$$

(5) With all electrodes A, B, C, D in medium ρ_2

$$\frac{\rho_a}{\rho_1} = \left\{\frac{1+k}{1-k}\right\}\left[1 + k\left\{\frac{4q}{(4q^2-1)} - \frac{q}{(q^2-1)}\right\}\right] \qquad (3.22)$$

Figures calculated from equations (3.18) to (3.22) inclusive for negative values of k ($\rho_2 < \rho_1$) are given in the curves of Fig. 3.12. For positive values of k it is again more convenient to plot the reciprocal of ρ_a/ρ_1,

i.e. σ_a/σ_1 and the resulting curves are given in Fig. 3.13. These curves all show four discontinuities, each occurring as one of the electrodes crosses the fault. In actual practice these discontinuities will not be so sharp, as they are usually somewhat obscured by surface variations, but the presence of these four discontinuities is typical of the presence of a fault.

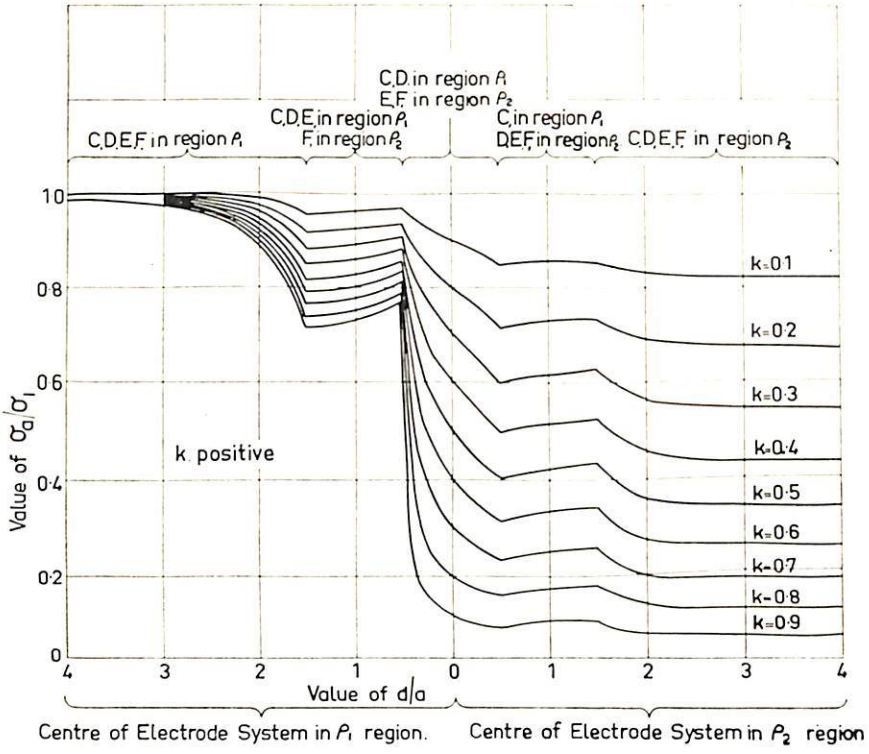

Fig. 3.13. *Vertical fault—k positive.*

Line of Electrodes at an Angle to the Fault

The case treated in the preceding sections is somewhat special as there may be no means of deciding whether the line of electrodes is perpendicular to the line of the fault. In the more general case the line of the electrodes may be inclined at an angle θ to the line of the fault as in Fig. 3.14. The quantity d in equation (3.16) then becomes $l + 3a/2 \sin\theta$, for electrode A, and $l - 3a/2 \sin\theta$ for electrode D.

The resulting expression for the apparent resistivity then becomes:

$$\frac{\rho_a}{\rho_1} = 1 + k\left[\frac{1}{\{\cos^2\theta + (2q + 2\sin\theta)^2\}^{\frac{1}{2}}} - \frac{1}{\{4\cos^2\theta + (2q + \sin\theta)^2\}^{\frac{1}{2}}}\right.$$

$$\left. - \frac{1}{\{4\cos^2\theta + (2q - \sin\theta)^2\}^{\frac{1}{2}}} + \frac{1}{\{\cos^2\theta + (2q - 2\sin\theta)^2\}^{\frac{1}{2}}}\right] \quad (3.22)$$

In the case when $\theta = 90°$, this reduces to equation (3.18)

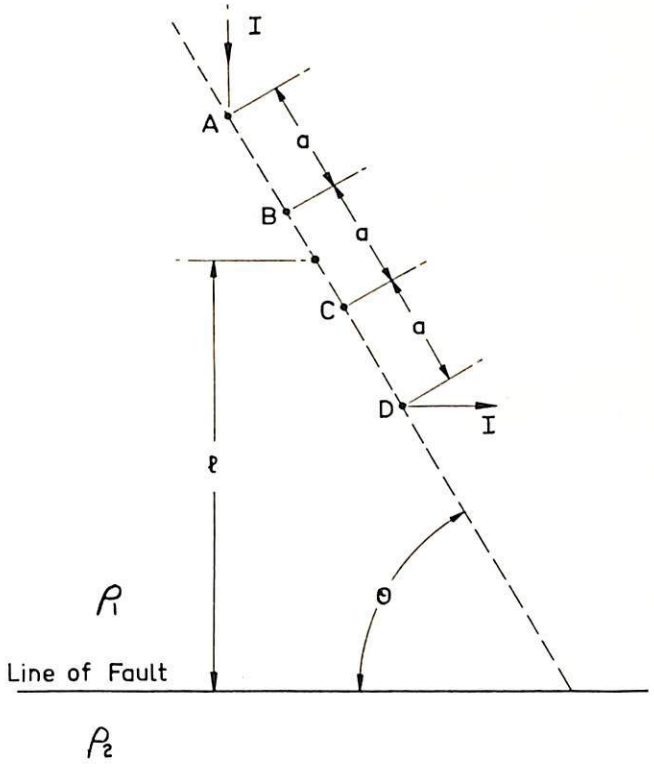

Fig. 3.14. *Line of electrodes at angle* θ *to line of fault.*

In the case when $\theta = 0$, equation (3.22) reduces to

$$\frac{\rho_a}{\rho_1} = 1 + 2k\left[\frac{1}{(4q^2 + 1)^{\frac{1}{2}}} - \frac{1}{(4q^2 + 4)^{\frac{1}{2}}}\right] \quad (3.23)$$

Curves drawn from this equation are given in Fig. 3.15.

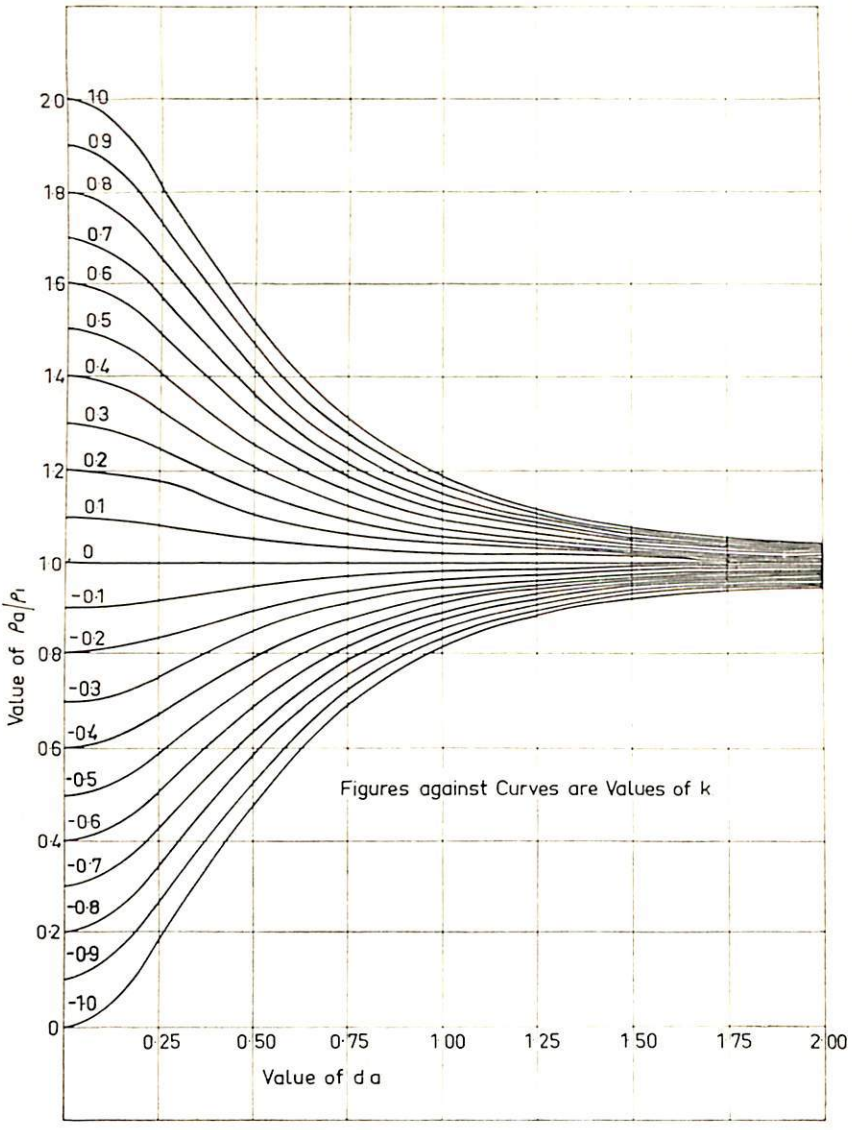

Fig. 3.15. Line of electrodes parallel to line of fault.

Again if $\theta = 45°$, equation (3.22) reduces to:

$$\frac{\rho_a}{\rho_1} = 1 + k\left[\frac{1}{[0{\cdot}5 + (2q + \sqrt{2})^2]^{1/2}} - \frac{1}{[2 + (2q + 1/\sqrt{2})^2]^{1/2}} - \frac{1}{[2 + (2q - 1/\sqrt{2})^2]^{1/2}} + \frac{1}{[0{\cdot}5 + (2q - \sqrt{2})^2]^{1/2}}\right] \quad (3.24)$$

Curves drawn from this equation are given in Fig. 3.16.

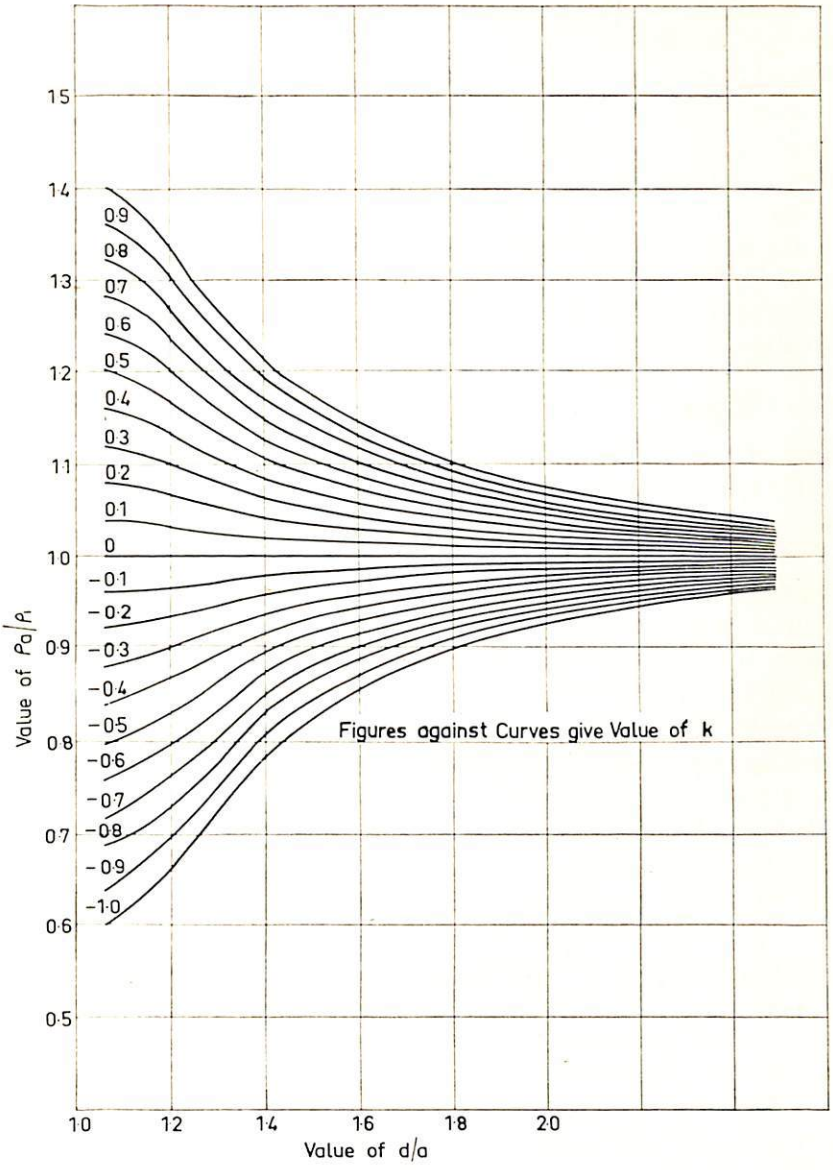

Fig. 3.16. *Line of electrodes at* 45° *to line of fault.*

General Note on Curves for Vertical Fault

It will be noted that the apparent resistivity curves for the case of a vertical fault are very similar in shape to those of the two-layer case, and it is difficult in practice, to decide from an inspection of a practical curve, whether the case being investigated is that of parallel layers or

that of a fault. An application of one of the methods of interpretation given in Chapter 4, will help to decide this point. These methods are designed to meet the case of parallel layers and, if a satisfactory interpretation cannot be obtained, this may be due to the fact that the case in point is not that of horizontal layers but one where the layers are inclined and may even be at right-angles to the surface as in the case of a vertical fault.

If the line of the electrodes does not cross the fault at right-angles, four discontinuities similar to those in Figs. 3.12 and 3.13 will occur, but will not be so marked. They may also be somewhat obscured by surface inequalities.

Buried Objects

One of the important uses of resistivity measurements is that of locating buried objects. These may be lodes of ore or tunnels and caves. Such an application does not lend itself very readily to theoretical investigation, although a number of attempts have been made. For example, Hummel[8] in 1930 obtained an expression for the apparent resistivity of a spherical cavity in a homogeneous medium. He did however treat only one special case.

Van Nostrand[9] in 1953, considered the case of a buried conducting sphere and came to the conclusion that it is very difficult to locate a conducting body buried to a depth greater than the mean of its linear dimensions. Palmer[10] does not agree that this conclusion holds for high resistivity bodies and his treatment of the problem is briefly as follows.

The problem is illustrated in Fig. 3.17. In this a spherical cavity of radius r is buried at a depth h from the surface. Palmer normally uses unequally spaced electrodes as shown. The cavity is supposed to be buried in a half space of resistivity ρ, with an upper half space consisting of air. The ratio $b/a = \alpha$ is kept constant, in order that the results can be interpreted quantitatively although the field work is rather slower than it is when the current electrodes are kept in a fixed position. The two half spaces are supposed to extend to infinity. The current electrodes C_1 and C_2, a distance $2a$ apart and the potential electrodes P_1 and P_2, a distance $2b$ apart are placed in line symmetrically above the cavity. The current passing from C_1 to C_2 is I amperes. If Maxwell's method of images is used, the equivalent system is that shown in Fig. 3.18. The cavity is replaced by an electric doublet Q with its images at Q^1. The two current electrodes C_1 and C_2 are replaced by a complete source and sink of strength $2I$, and the half space is replaced by a solid of resistivity ρ and extending to infinity in all directions.

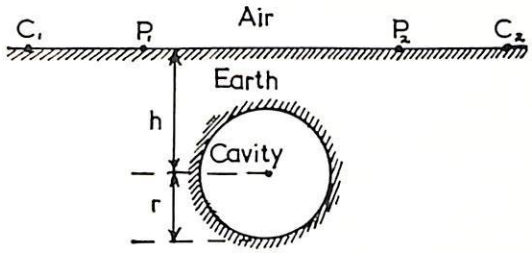

Fig. 3.17. *Spherical cavity.*

Let the radius vectors from any point P to the current source $+2I$, the current sink $-2I$, the doublet Q and its image Q^1 be d_1, d_2, d_3 and d_4 respectively, then the expression for the potential at the point P is, to a first approximation, given by

$$V_p = \frac{I\rho}{2\pi}\left[\frac{1}{d_1} - \frac{1}{d_2} + \frac{abr^3}{(a^2+h^2)^{3/2}}\left(\frac{1}{d_3{}^3} + \frac{1}{d_4{}^3}\right)\right] \tag{3.25}$$

The potential difference between P_1 and P_2 can be obtained by substituting in equation (3.25) the values for the radius vectors of P_1 and P_2. This leads to

$$V = \frac{2\alpha\rho I}{\pi(1-\alpha^2)a}\left\{1 + \frac{\beta^3(1-\alpha^2)(r/h)^3}{\alpha^3(1+\beta^2)^{3/2}[1+(\beta/\alpha)^2]^{3/2}}\right\} \tag{3.26}$$

in which $\alpha = b/a$ and $\beta = h/a$.

When there is no cavity, i.e. $r = 0$ or $h = \infty$, this reduces to

$$V = \frac{2\alpha\rho I}{\pi(1-\alpha^2)a} \quad \text{or} \quad \rho_a = \frac{V}{I}\cdot\frac{\pi(1-\alpha^2)}{2\alpha}\cdot a$$

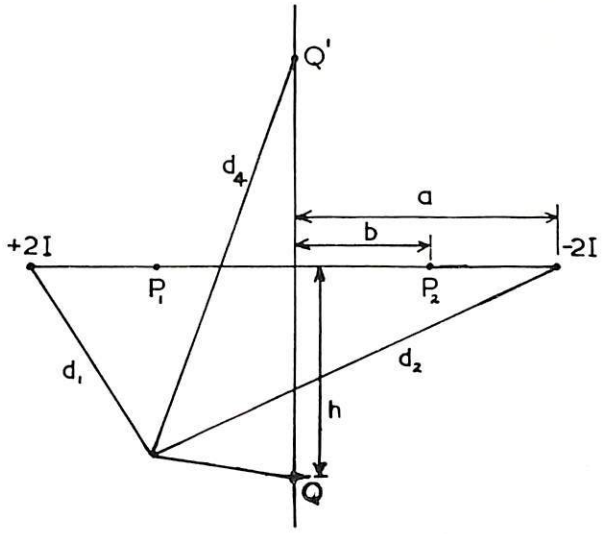

Fig. 3.18. *Electrical Equivalent of Fig.* 3.17.

But $V/I = R$, the measured resistance and so

$$\rho_a = \frac{\pi(1-\alpha^2)}{2\alpha}.aR \tag{3.27}$$

Combining equations (3.26) and (3.27) leads to

$$\frac{\rho_a}{\rho_1} = 1 + \frac{\beta^3(1-\alpha^2)(r/h)^3}{\alpha^3(1+\beta^2)^{3/2}[1+(\beta/\alpha)^2]^{3/2}} \tag{3.28}$$

Curves calculated for the Wenner configuration and for values of r/h equal to 0·5 and 0·75 are given in Fig. 3.19. The presence of the buried sphere appears as a bulge in the curve of apparent resistivity against electrode separation. The value and position of this bulge can be determined from equation (3.28) by equating the first differential to zero, α being constant.

$$\text{If} \quad \frac{\mathrm{d}(\rho_a/\rho_1)}{\mathrm{d}.a} = 0, \quad \text{then } \beta = \sqrt{\alpha} \quad \text{or} \quad h = a_0\sqrt{\alpha} \tag{3.29}$$

where a_0 is the value of a for the position of the maximum apparent resistivity. So, as α is always less than unity, the bulge on the curve will always appear lower down than the depth of the cavity. For the Wenner configuration $\alpha = 1/3$ and so

$$h = 0{\cdot}575a_0 \tag{3.30}$$

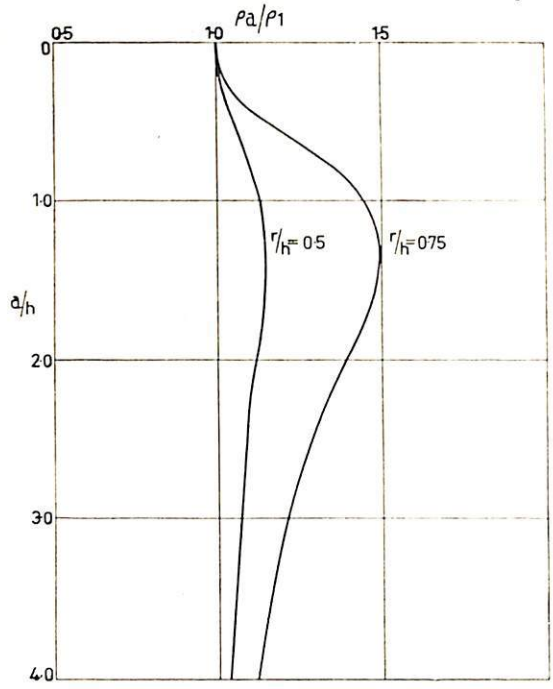

Fig. 3.19. *Effect of buried insulating sphere.*

If the value of a_0 from equation (3.29) is substituted in equation (3.28), this leads to

$$\frac{\rho_a(\max)}{\rho} = 1 + \frac{1-\alpha}{(1+\alpha)^2}(r/h)^3 \tag{3.31}$$

So the expression

$$\frac{(1-\alpha)}{(1+\alpha)^2}\left(\frac{r}{h}\right)^3$$

gives the magnitude of the bulge on the high resistance side of the curve. Again for the Wenner configuration ($\alpha = 0{\cdot}333$), the width of the bulge is $0{\cdot}375(r/h)^3$. To make the size of the anomaly large, α must be small, but then the anomaly will appear well below the depth of the cavity.

It is also clear that in order to detect the cavity it must be of dimensions of the same order of the depth h. This is generally true in the location of any buried object and is in fact obvious from the geometry of the problem. This matter is discussed further in Chapter 5.

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4

INTERPRETATION OF DEPTH CURVES

One of the important uses of resistivity measurements is to determine the depth from the surface to lower layers such as the water table, bedrock, etc. As a result, considerable attention has been paid to the interpretation of what are known as depth curves. If the Wenner configuration of four equally-spaced electrodes is used, the centre of the electrode system is kept in one place, the electrode separation varied, and the resulting resistivities are plotted against the electrode separation, the result is a depth curve. From the earliest days of resistivity surveying the interpretation of such curves has been a matter of great concern and indeed of controversy.

The methods of interpretation which have been developed can be roughly divided into two classes. The first of these is entirely empirical and is based partly on experience. The second consists of a number of methods which are based on the theories outlined in Chapter 3. There are two other methods which are a combination of the two main classes, and in any case it is necessary to emphasize that experience in such work is of great importance.

It is, of course, obvious that all the interpretation can do is to suggest that there are changes in resistivity at certain depths, and, in order to ascertain the causes of these changes, geological knowledge is necessary or access to the records of nearby drill holes. The number of methods of both classes is very large and there is a considerable literature on the subject; it will not be possible to deal with all of them here. Some indication can, however, be given of those methods which are of historic interest, and also those which are most successful and consequently are the most used.

Empirical Interpretation

The work of Gish and Rooney. Among the earliest workers in the field of resistivity measurements were Messrs. O. H. Gish and W. J. Rooney who published a number of papers[1] on their work. This was done in connection with their work on natural currents flowing in the earth for the Carnegie Institute in Washington. In their papers, the statement is made that the measurements give the average resistivity to a depth equal to the electrode separation and consequently they

deduce from a depth curve, that a change in formation, i.e. another layer is reached at a depth equal to any electrode separation at which a break or change in curvature occurs.

It has been shown in Chapter 3 that a considerable proportion of the total current penetrates in homogeneous soil to a depth greater than the electrode separation and, in non-homogeneous soil, the penetration must be further modified by the degree of non-homogeneity. Consequently, the resistivity measured cannot be considered as the average value for the soil to a depth equal to the electrode separation, and so this method has no sound basis and can lead to quite false conclusions.

Points of inflexion. An examination of the theoretical curves for the two-layer case in Figs. 3.3 and 3.4 shows that in each curve there is a point of inflexion, although the exact point is not easily determined by inspection. Nevertheless it has been suggested several times that the value of electrode separation, at which this point of inflexion occurs, bears some definite relation to the depth to the second layer. For instance, Lancaster-Jones[2] has suggested that this point of inflexion occurs near the point where $a = 3h/2$, and consequently the depth to the lower layer can be taken as 2/3 the electrode separation at which the point of inflexion occurs.

On referring back to the equation connecting ρ_a/ρ_1 and h/a in Chapter 3, (3.13); the point of inflexion will occur when

$$\frac{\mathrm{d}^2\rho_a/\rho_1}{\mathrm{d}a^2} = 0.$$

The value of h/a at which this occurs can be determined from equation (3.13), but this is a very laborious process. An approximation can however be obtained by using the figures similar to those in Appendix 1 calculated from this equation and replacing the second differential by its equivalent difference equation. The table of figures required for this purpose are those connecting ρ_a/ρ_1 with a/h, such as those given by Roman.[3]

For any value of ρ_a/ρ_1 and of k the second differential can be replaced by the difference equation.

$$w^2f''(a) = \Delta^2 f(a) - \Delta^3 f(a) + \tfrac{11}{12}\Delta^4 f(a) - \tfrac{5}{6}\Delta^5 f(a) + \qquad (4.1)$$

where w = interval between successive entries in the table
$\Delta^2 f(a)$ = second difference
$\Delta^3 f(a)$ = third difference
and so on.

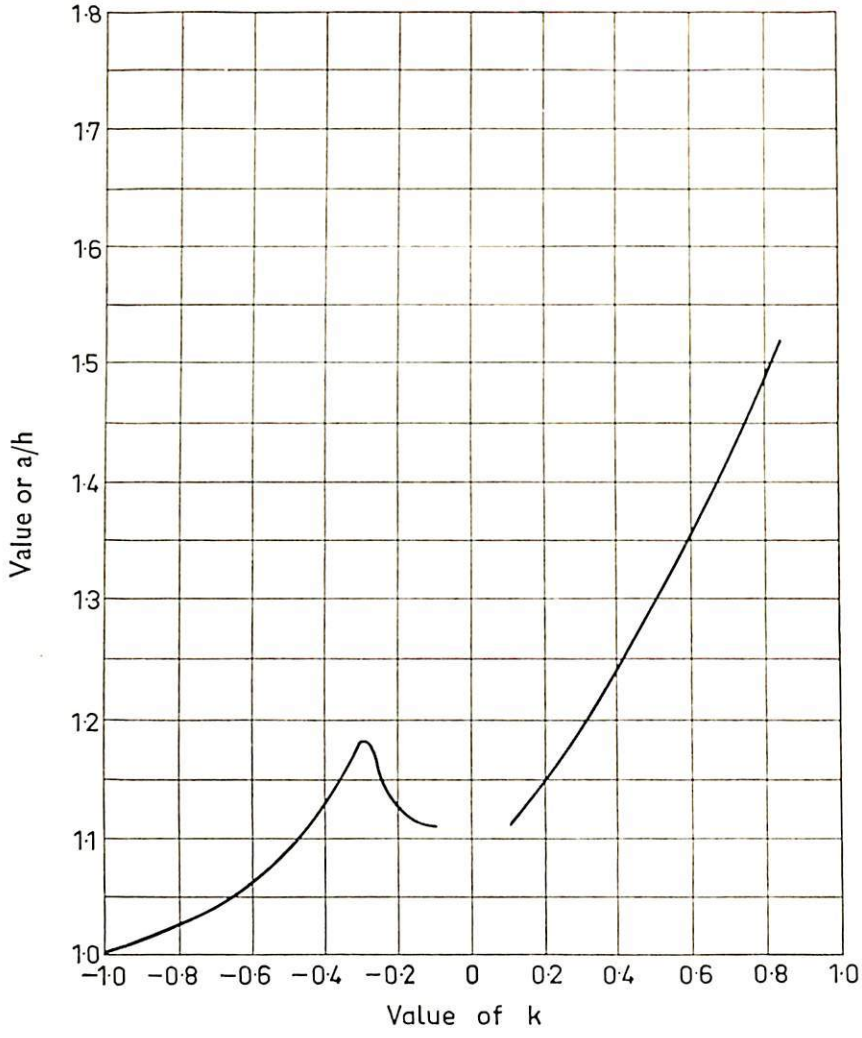

Fig. 4.1. *Points of inflexion.*

This can be calculated for each value of a/h and by plotting the results for any one value of k, it is possible to determine the value of a/h at which the second differential is zero. The results of all these calculations are plotted on the curve of Fig. 4.1 which gives the values of a/h at which the point of inflexion occurs for all values of k. It will be seen that a/h varies between 1·0 and 1·5 and only approaches the value of 1·5 suggested by Lancaster-Jones when k approaches $+1$, i.e. the lower bed has a very high resistivity. Thus it follows that the point of inflexion,

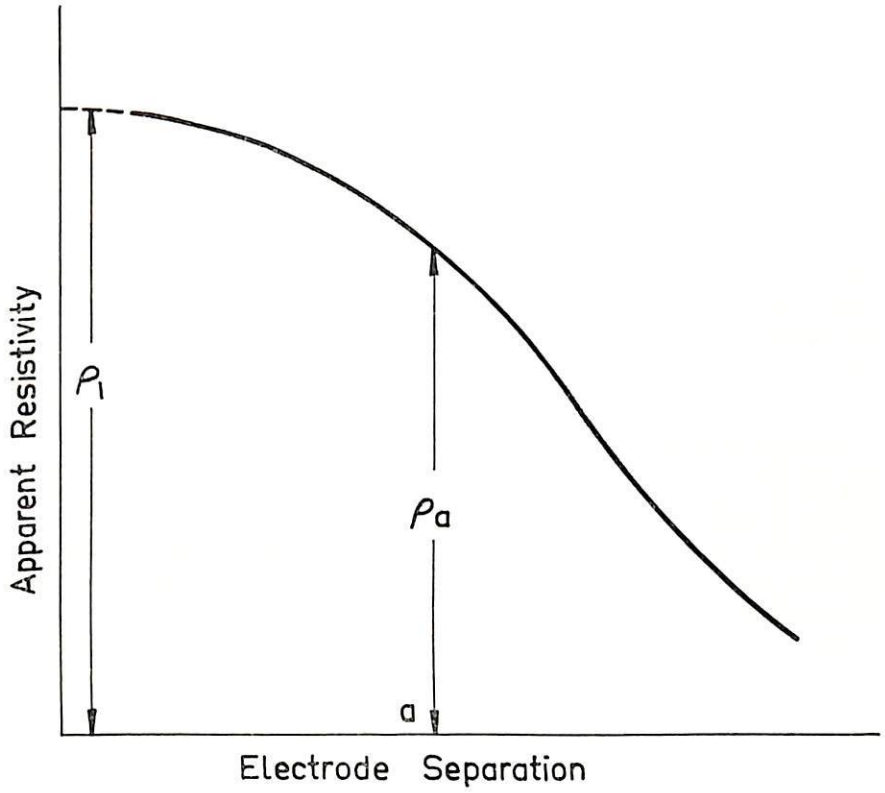

Fig. 4.2. *Typical depth curve—first method.*

if it can be located accurately, cannot be used as a means of interpretation.

Other methods which are, in essence, empirical are those due to Moore[4] and Barnes,[5] but these have been examined by other investigators and appear to be no better than the original idea of Gish and Rooney. Some other methods have been suggested which involve differentiating of the practical curve. This gives sharper indications but any accidental irregularities are emphasized. It must not be overlooked that these accidental irregularities are always likely to occur, due to surface variations, instrumental and observational errors and so on and experience is necessary to decide whether such irregularities can be ignored or must be taken seriously. In general, it may be said that empirical methods are of doubtful use, and whether they can give any useful information, except in a few special cases, is very debatable.

Direct Methods

The methods which may be given this title are those in which the theoretical analysis is used as a basis. The major difficulty here is that the theory pre-supposes ideal conditions, i.e. uniform isotropic layers with the interfaces parallel to the surface.

Unfortunately such ideal conditions are seldom encountered in practice, and so some errors and inconsistencies can appear in the deductions, but nevertheless it is often possible to obtain useful results by their use. If they fail completely to give a sensible answer, then this can be taken as an indication that the conditions differ widely from those assumed in the theory, e.g. sloping interfaces, anisotropy and so on.

Two-layer problem—the author's first method. The theoretical curves for the two-layer problem are, of course, those given in Chapter 3 (Figs. 3.3 and 3.4) and a typical depth curve obtained in practice could be such as that shown in Fig. 4.2. In this, the apparent resistivity ρ_a is plotted against the electrode separation a. It should be possible to extrapolate the curve backwards until it cuts the ρ_a axis, when the point at which it cuts will give the value of the surface resistivity ρ_1. Then, if a value of electrode separation a_1 is chosen, the corresponding value of resistivity ρ_{a1} can be read off the curve. The ratio ρ_{a1}/ρ_1 can then be calculated. In the case illustrated this will be less than unity, the dropping curve indicating that the lower bed has a smaller resistivity than the surface layer. On referring to Chapter 3 (Fig. 3.3) it will be seen that for this particular value of ρ_a/ρ_1, there is a series of possible values of h/a one for each value of the coefficient k. This series of values is read off the curves, each value of h/a multiplied by the electrode separation a, to give a value of h. The result is a set of corresponding values of h and k, which are plotted in the form of a curve. A second value of $a = a_2$ is then chosen and the process repeated, resulting in a second curve of h against k. These two curves should intersect at a point giving the true values of h and k.

It is advisable to repeat the process with at least one additional value of a, to ensure the accuracy of the interpretation. The three or more curves should all intersect in a point under ideal conditions. It is generally found that due to some irregularities in the curves which may be due to many causes, the intersection of the curves produces a small triangle, and then the centre of the triangle is taken as giving the required values of h and k.

If the practical curve is rising, i.e. the apparent resistivity is increasing

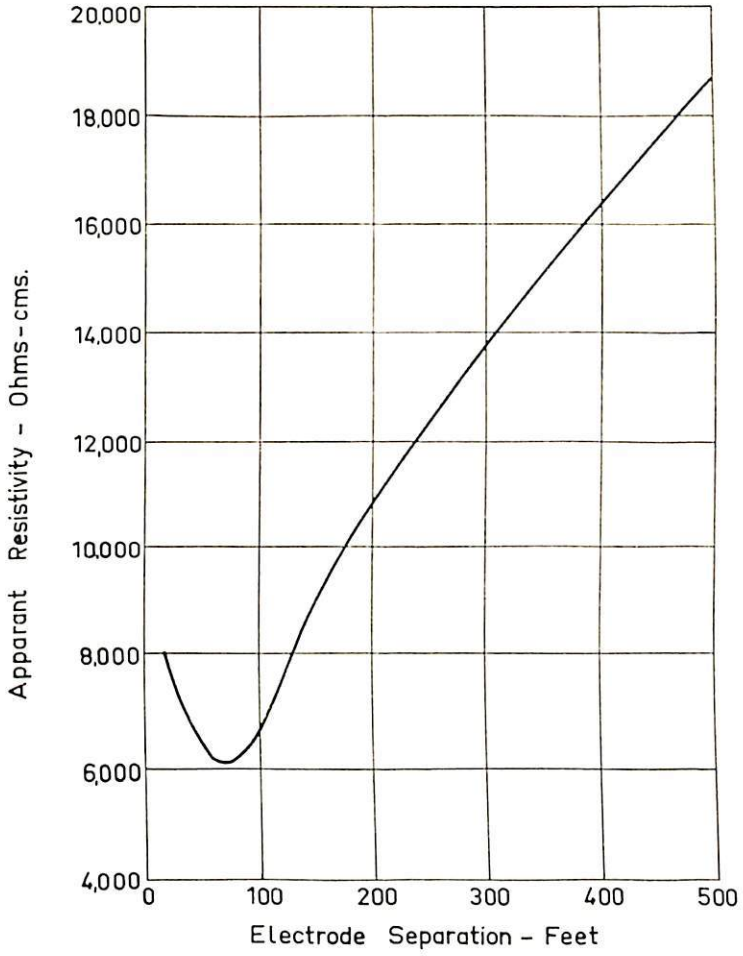

Fig. 4.3. *Practical depth curve.*

with the electrode separation, this indicates that the lower bed is of a higher resistivity than the surface and the corresponding theoretical curves are those of Fig. 3.4 The procedure is the same, except that instead of calculating the ratio ρ_a/ρ_1 for the chosen electrode separation, its reciprocal $\rho_1/\rho_a = \sigma_a/\sigma_1$ is taken and applied to the curves of Fig. 3.4.

The following is an example of the application of this method. A survey was made on a site where the surface material was limestone overlain by about six inches of loam. The depth of the limestone varied from 50 ft to 260 ft and under this was either sand or clay. The site

chosen was practically level and measurements made at electrode intervals from 20 to 500 ft. The results of these are plotted as a curve of apparent resistivity against electrode separation in Fig. 4.3, and this curve shows that the apparent resistivity is increasing with the electrode separation showing the resistance of the lower layer was higher than that of the limestone and so suggesting that it was probably sand.

The first step was to determine the surface resistivity and this presented certain difficulties due to the presence of the layer of loam on the surface which apparently had a lower resistivity than the limestone. To obtain a suitable value of ρ_1, the resistivities obtained up to an electrode separation of 70 ft were averaged and gave a value of 6,703 ohm-cms. A series of values were then read off the curve of Fig. 4.3 and the ratio ρ_a/ρ_1 determined for each. These values were all greater than unity and so the reciprocals σ_a/σ_1 were obtained. These figures are given in Table 4.1.

TABLE 4.1

Electrode Separation (ft)	*Apparent Resistivity (ohm-cms)*	*Ratio* ρ_a/ρ_1	*Ratio* σ_a/σ_1
150	8,960	1·388	0·748
200	10,740	1·601	0·625
250	12,320	1·840	0·544
300	13,860	2·068	0·483
350	15,220	2·270	0·441
400	16,480	2·460	0·407

The values of σ_a/σ_1 were then referred to the theoretical curves of Fig. 3.4 and a series of corresponding values of h/a and k read off for each value of σ_a/σ_1. These and the corresponding values of h are given in Table 4.2.

An inspection of this table shows that the values for $k = 0{\cdot}7$ agree very closely, but to obtain the exact values, the figures in Table 2 are plotted in the curves of Fig. 4.4. The six curves intersect in a small area and the centre of this area gives a value of h of 142 ft and $k = 0{\cdot}702$. This means that the thickness of the limestone at this point is 142 ft, and so far as it was possible to judge from the geological map, this was approximately correct.

Two-layer problem—the author's second method. One of the difficulties encountered in the first method is the determination of the surface

TABLE 4.2

	150 ft		*200 ft*		*250 ft*		*300 ft*		*350 ft*		*400 ft*	
	$\sigma_a/\sigma_1 = 0{\cdot}748$		$\sigma_a/\sigma_1 = 0{\cdot}625$		$\sigma_a/\sigma_1 = 0{\cdot}544$		$\sigma_a/\sigma_1 = 0{\cdot}483$		$\sigma_a/\sigma_1 = 0{\cdot}441$		$\sigma_a/\sigma_1 = 0{\cdot}407$	
k	h/a	h	h/a	h	h/a	h	h/a	h	h/a	h	h/a	h
1·0	1·19	179	0·915	183	0·770	193	0·675	202	0·610	214	0·560	224
0·9	1·12	168	0·850	170	0·705	176	0·610	183	0·545	191	0·500	200
0·8	1·045	157	0·775	155	0·640	160	0·545	163	0·485	170	0·435	174
0·7	0·96	144	0·700	140	0·565	141	0·478	143	0·410	144	0·360	144
0·6	0·87	130·5	0·620	124	0·485	121	0·390	117	0·325	114	0·280	112
0·5	0·785	118	0·525	105	0·390	97·5	0·295	74	0·226	78	0·170	68
0·4	0·66	99	0·42	84	0·270	67·5	0·16	40	0·06	21		
0·3	0·525	79	0·26	52	0·030	7·5						
0·2	0·317	47										
0·1												

resistivity ρ_1. The example just quoted is a case in point. The presence of the surface layer of loam made it impossible to extrapolate the curve backwards to obtain the value of ρ_1 and recourse had to be taken to averaging the resistivities for the first few values of electrode separation.

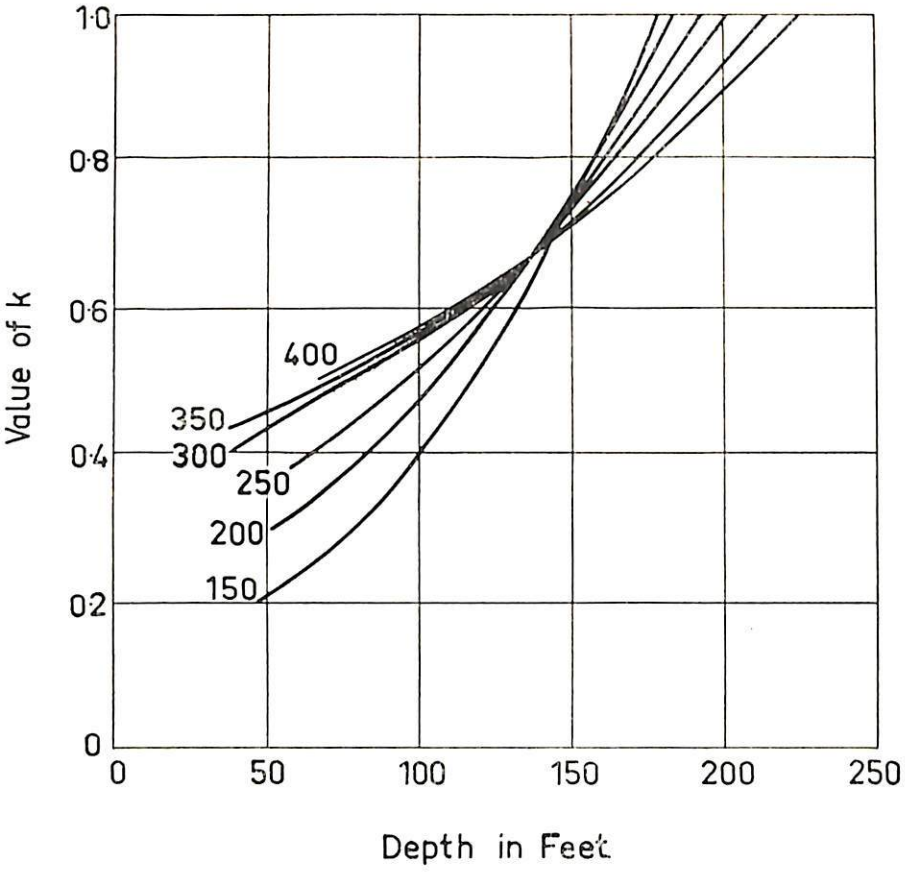

Fig. 4.4. *Curves of k against h.*

Now consider the curve of Fig. 4.5. This again is a depth curve showing the variation of apparent resistivity with electrode separation. For any chosen value of a, the resistivity is ρ_a. Next, choose a value of electrode separation equal to na where n is a suitable number. The ratio ρ_{na}/ρ_a can then be determined and will depend on the values of h/a, n and k. This ratio can be calculated from the theory and results in the sets of curves given in Figs. 4.6–4.11 inclusive. The method of using these curves is then very similar to that described for the first method, and is perhaps best described by means of an example.

In the course of an extended investigation of an area in East Anglia, it was necessary to make a number of depth tests. In this area the upper layer was very wet sand, and the lower layer was sandy clay. It was known that in certain places in this area the depth to the clay was about 50 ft, but it was suspected that in other places it was only about 15 ft from the surface.

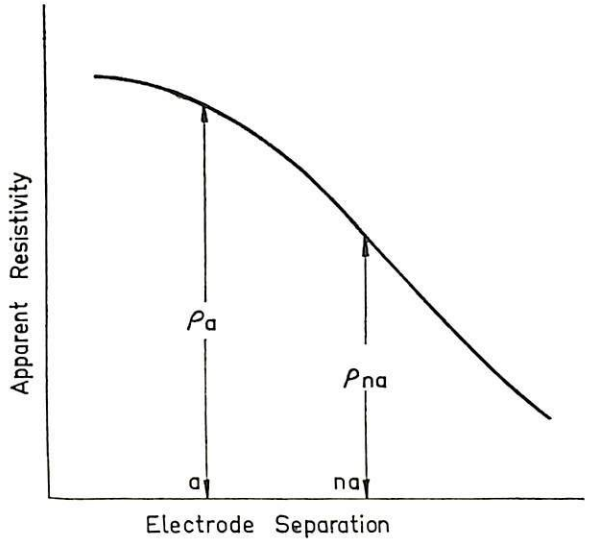

Fig. 4.5. Typical depth curve—second method

One of these latter sites was chosen for the purpose of this depth test and the curve obtained is given in Fig. 4.12. Since, in the interpretation absolute resistivities are not required but only ratios, the ordinates of this curve are the product $a \times R$, the factor 2π and scale factors being omitted. The process of interpretation is then as follows:

(1) Chose $a = 10$ ft and $n = 2$
Then $a \times R$ at 10 ft = 21·5
$a \times R$ at 20 ft = 16·4

$$\therefore \rho_{2a}/\rho_a = 0{\cdot}763.$$

(2) Choose $a = 15$ ft and $n = 2$
Then $a \times R$ at 15 ft = 18·7
$a \times R$ at 30 ft = 12·3

$$\rho_{2a}/\rho_a = 0{\cdot}657.$$

(3) Choose $a = 10$ ft and $n = 3$
Then $a \times R$ at 10 ft = 21·5
$a \times R$ at 30 ft = 12·3

$$\rho_{3a}/\rho_a = 0{\cdot}572.$$

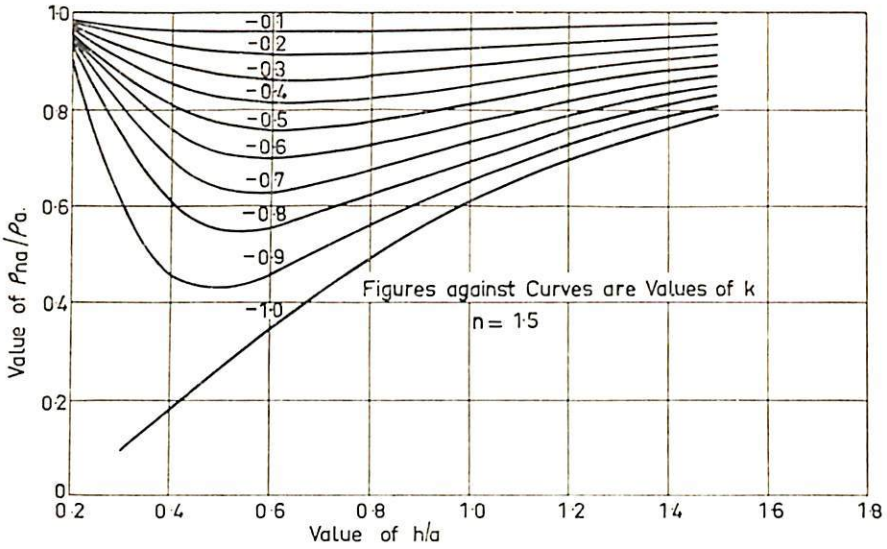

Fig. 4.6. Second method curves, k negative, n = 1·5.

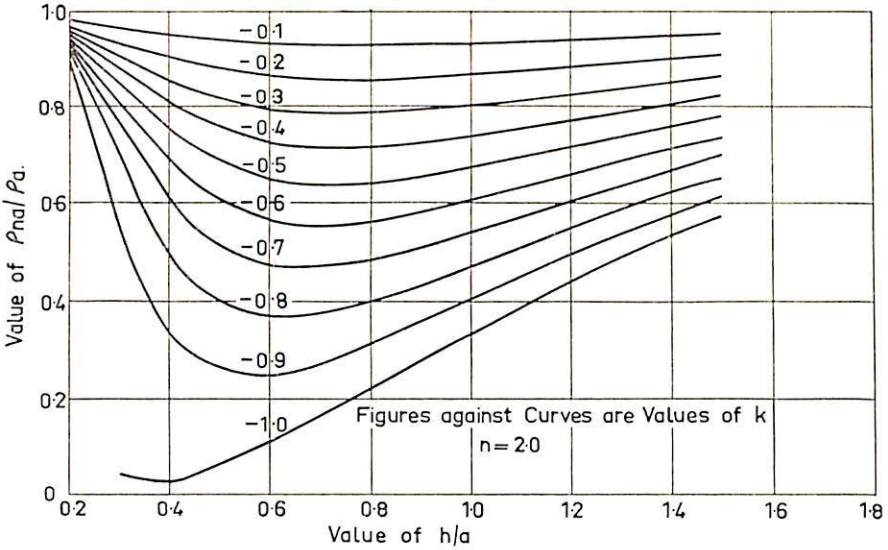

Fig. 4.7. Second method curves, k negative, n = 2·0.

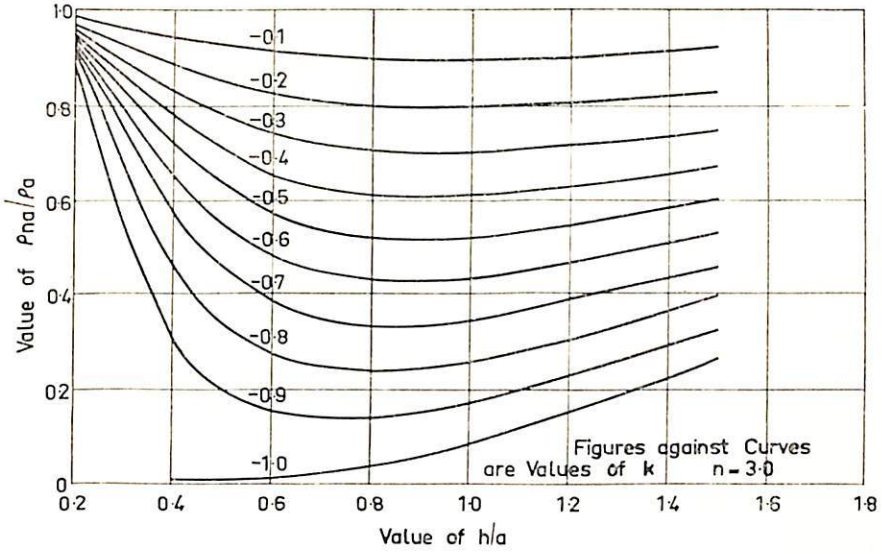

Fig. 4.8. *Second method curves, k negative, n* = 3·0.

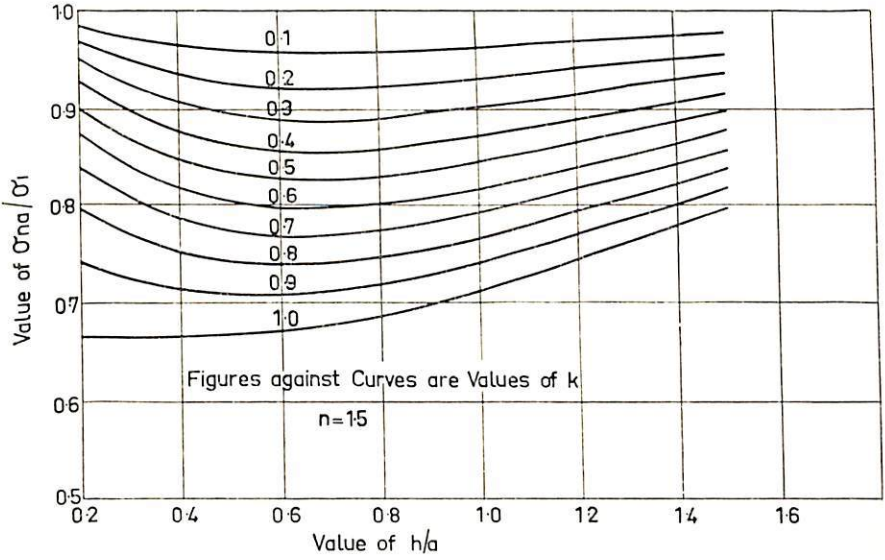

Fig. 4.9. *Second method curves, k positive, n* = 1·5.

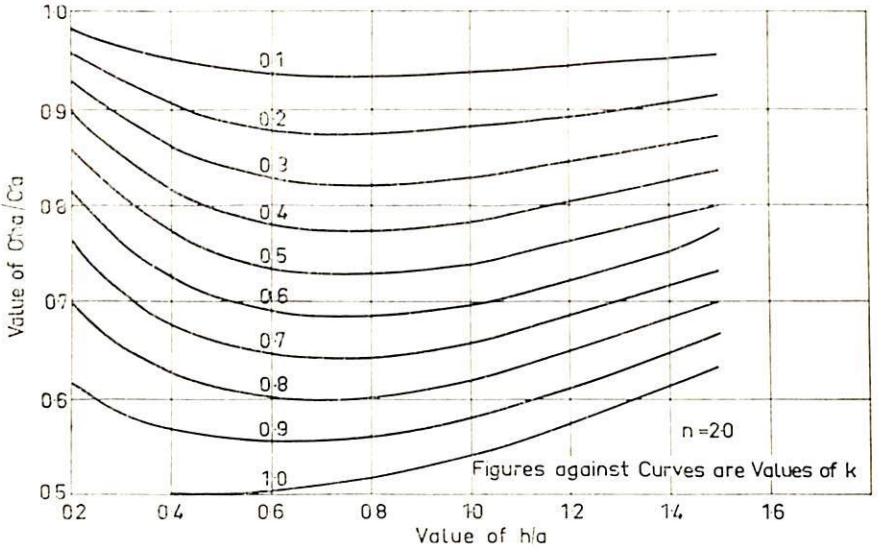

Fig. 4.10. *Second method curves, k positive,* $n = 2{\cdot}0$.

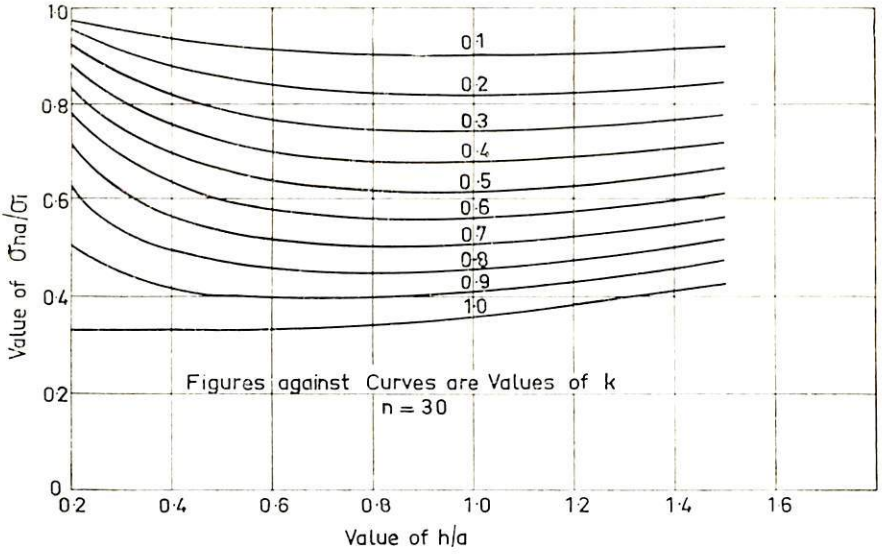

Fig. 4.11. *Second method curves, k positive,* $n = 3{\cdot}0$.

TABLE 4.3

	$a = 10$ $n = 2$ $\rho_{2a}/\rho_a = 0{\cdot}763$		$a = 15$ $n = 2$ $\rho_{2a}/\rho_a = 0{\cdot}657$		$a = 10$ $n = 3$ $\rho_{3a}/\rho_a = 0{\cdot}572$	
k	h/a	h	h/a	h	h/a	h
−1·0	—	—	—	—	—	—
−0·9	0·238	2·38	0·270	4·05	0·290	2·9
−0·8	0·270	2·70	0·318	4·77	0·347	3·47
−0·7	0·302	3·02	0·370	5·55	0·400	4·00
−0·6	0·340	3·40	0·431	6·47	0·471	4·71
−0·5	0·390	3·90	0·573	8·60	0·600	6·00
−0·4	0·484	4·84	—	—	—	—
−0·4	1·146	11·46	—	—	—	—
−0·5	1·410	14·10	0·913	13·70	1·350	13·50
−0·6	1·603	16·03	1·185	17·78	1·666	16·66
−0·7	1·766	17·66	1·347	20·21	1·912	19·12
−0·8	1·903	19·03	1·500	22·50	2·103	21·03
−0·9	2·032	20·32	1·623	24·35	2·273	22·73
−1·0	2·140	21·40	1·726	25·89	2·421	24·21

For (1) and (2) the corresponding theoretical curves are those of Fig. 4.7 while for (3) they are those of Fig. 4.8. On referring to these curves the sets of values given in Table 4.3 can be read off.

These figures are plotted in the form of curves of k against h in Fig. 4.13 and it will be seen that they give a good intersection at $h = 14{\cdot}5$ ft. From drillings done in the very close proximity to the site of this test it appears that the depth to the clay varies from 13–16 ft in this area. So this must be considered a very satisfactory interpretation.

The figures from which the curves of Figs. 4.6–4.11 can be plotted are given in Appendix 2.

Extension to the three-layer problem. The occurrence of simple two-layer cases is comparatively rare and much more frequently the investigator is presented with a depth curve which by its very shape suggests that three or more layers are present. The interpretation of such curves by direct methods is not easy but Pirson[6] has suggested a method of successive approximation by which the author's first method can be extended to the three-layer case. This method may be summarised as follows:

(1) Determine the surface resistivity by making measurements

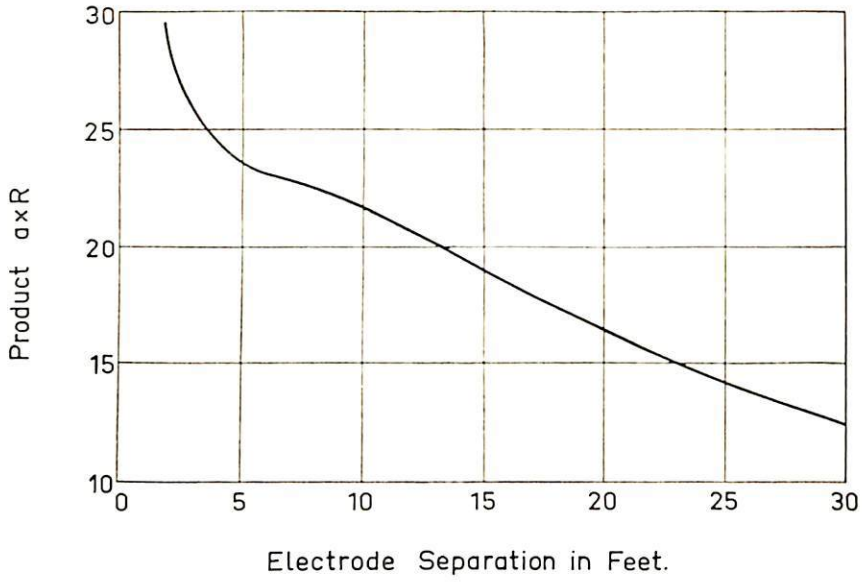

Fig. 4.12. *Depth test at a site in Norfolk.*

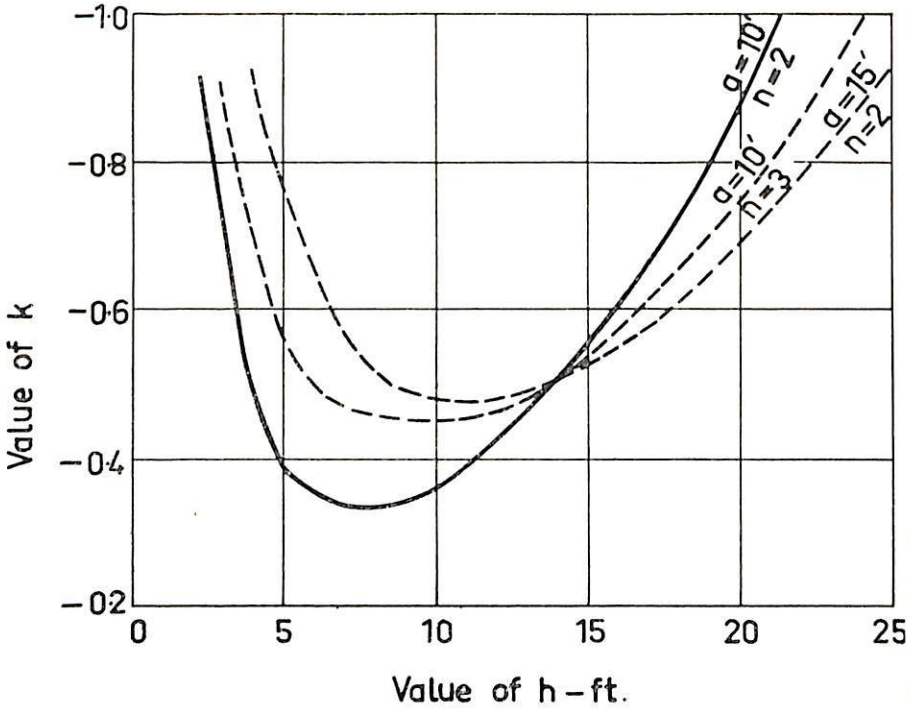

Fig. 4.13. *Curves of h against k—second method.*

with small electrode intervals, say from 5 to 20 ft, and taking the average of the results obtained. This must be done as carefully as possible as an accurate knowledge of the surface resistivity is essential if the method is to be successful.

(2) Plot ρ_1 to scale on the resistivity axis and draw a line from ρ_1 to meet tangentially the first part of the curve.

(3) Take the values of the resistivity for several electrode spacings and apply the first method for the two-layer problem (page 60). This will give the thickness h_1 of the first layer and from the resistivity factor $k_1 = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$, the resistivity ρ_2 of the second layer.

(4) Estimate the depth to the third layer by Lancaster-Jones method. Thus $h_1 + h_2 = 2/3d$ where d is the distance to the point of inflexion laying between the two lower maximum curvature points.

(5) Calculate the apparent or average resistivity $\rho_1{}^1$ of the two layers of resistivities ρ_1 and ρ_2 in parallel by the formula due to Hummel[7]

$$\frac{h_1 + h_2}{\rho_1{}^1} = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2}$$

(6) Apply the first method for the two-layer problem (page 60) to the lower part of the curve. This will give a set of curves of k against h. If they converge in a small area, they will give a value of $h_1 + h_2{}^1$ and the reflexion coefficient

$$k_2 = (\rho_3 - \rho_2)/(\rho_3 + \rho_2).$$

This latter will enable the value of the resistivity ρ_3 of the third layer to be calculated.

(7) If the area of convergence is not felt to be good enough for the accuracy required, repeat from step (5) using the value of $h_1 + h_2{}^1$ obtained from (6). This will give $h_1 + h_2{}^{11}$ which should give a more accurate result.

When the depths to the various interfaces are comparatively large, it is often possible to divide the curve up into two parts, and to apply the second method for the two-layer problem to each part separately. This will give the depths, but will not give the individual resistivities. These latter are however, in general, not so important as the depths, since resistivities are affected by many factors other than the simple composition.

Other direct methods. Several other direct methods have been suggested from time to time, but they generally tend to be more complicated than those described, and their advantages are debatable. The extension to more than three layers also presents considerable difficulties. It must be remembered that each layer introduces two unknowns; thus, while the two-layer problem can be considered as having only two, the thickness of the upper layer and the ratio of the resistivities, a four-layer problem will have eight. This makes theoretical methods of interpretation very difficult, and there is no doubt that experience can play an important part in making deductions from any given point.

Curve Matching

The matching of the experimental curve against known conditions or theoretical curves covers a number of methods which have been put forward and used. Here again experience plays an important part.

Matching against drill log. One method which has been used to a considerable extent is to make a depth test in the vicinity of a bore hole, and to compare the curve and its characteristics with the results obtained from the hole. Then, tests are made at other points in the vicinity and, by comparing the depth curves obtained at this point with that obtained at the bore-hole, it is sometimes possible to determine how the depth to a given layer varies over the area surveyed. This comparison is not always easy and it is, of course, essential that the geology of the area should not change to any marked degree over the area surveyed. The method can be used to deduce the total number of bore-holes required to delineate a particular area.

Matching against theoretical curves. A number of suggestions have been made that interpretation can be carried out by matching the experimental curve to a corresponding theoretical one. One such method has been described by Roman.[3] His method can be described as follows:

The equation for the two-layer problem is

$$\frac{\rho_a}{\rho_1} = 1 + 4F$$

where F is a function of a/h and the reflection coefficient k. Taking logarithms

$$\log \rho_a - \log \rho_1 = \log(1 + 4F)$$

Now from calculation the value of $\log (1 + 4F)$ can be calculated for each value of a/h or preferably $\log a/h$, and of the reflection coefficient k.

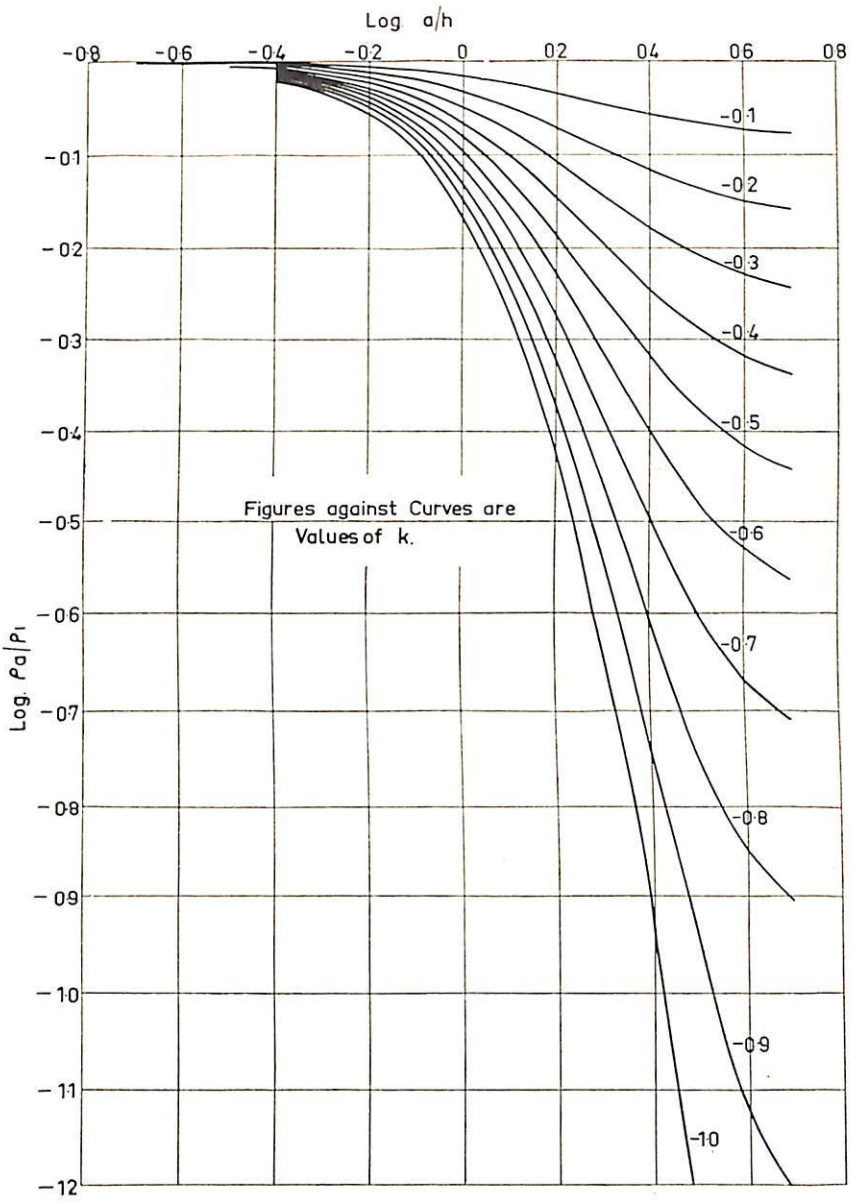

Fig. 4.14. *Curves of log* ρ_a/ρ_1 *against log a/h, k negative.*

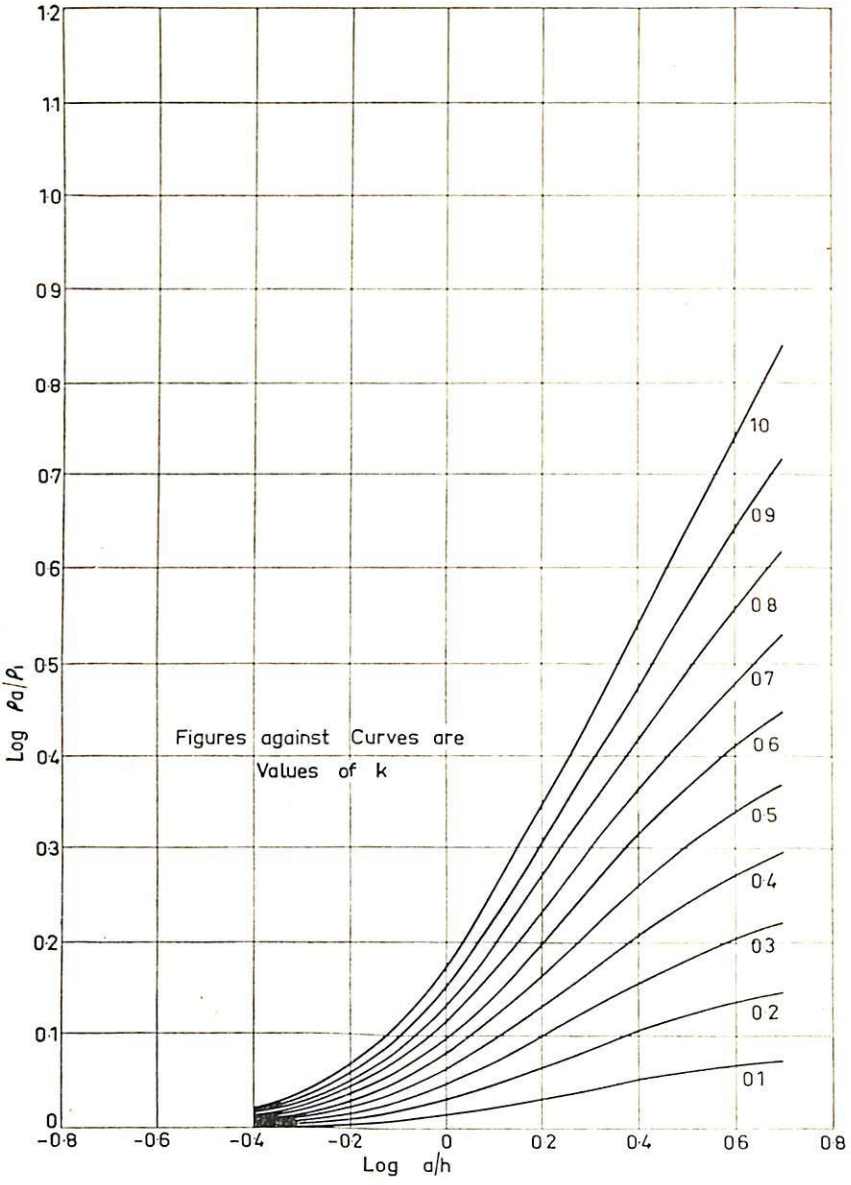

Fig. 4.15. *Curves of log* ρ_a/ρ_1 *against log a/h, k positive.*

From these it is possible to plot sets of curves of log $(1 + 4F)$ against log a/h and such curves are given in Figs. 4.14 and 4.15. If the observed values of log ρ_a are plotted against the values of log a in the same scales as in the theoretical curves, the two curves may be superimposed since

$$\log \frac{\rho_a}{\rho_0} = \log \rho_a - \log \rho_0$$

$$\log a/h = \log a - \log h$$

and since both log ρ_0 and log h are constant. If the observed curve can be made to fit one of the theoretical curves, four quantities may be determined as follows:

(1) The value of the reflection factor k is given by the theoretical curve which fits the data.

(2) For log $(1 + 4F) = 0$, log ρ_a = log ρ_0 so that the value in the observed ordinate (log ρ_0) scale corresponding to zero in the theoretical ordinate log $(1 + 4F)$ scale is the value of log ρ_0, from which the resistivity of the upper layer can be obtained.

(3) For log $a/h = 0$, $a = h$, so that the value on the observed abscissa (log a) scale corresponding to zero on the theoretical abscissa (log a/h) scale is log h, from which the depth h can be obtained.

(4) From the value of ρ_1 and k, the resistivity of the second layer is given by

$$\rho_2 = \left(\frac{1 + k}{1 - k}\right)\rho_1$$

The application is comparatively simple. The theoretical curves are plotted using suitable scales on thin graph paper. The observed curves are plotted on heavy graph paper using the same scales. The two graphs are then super-imposed preferably over a light table, and the required data can be taken directly from the curves.

Roman states that when the method was used, the fitting of the curves was in some of the cases quick and definite, the same observer obtaining the same results after intervals of days or even months. In other cases the results are more ambiguous and exact fitting is difficult. In still other cases no fitting was possible, showing that the problem was more complicated than the simple two-layer one.

It is difficult to illustrate this method as the superposition of the curves cannot be demonstrated. However, Fig. 4.16 is taken from Roman's

paper. The dots are the observed points and, by this method of superposition, the following values were obtained—reflection factor, 0·9; surface resistivity, 4,730 ohm-cms; lower bed resistivity, 90,000 ohm-cms; thickness of surface layer 33·9 ft. The full curve in Fig. 4.16 has been calculated from these figures, and it will be seen that the agreement is very good.

Curve matching in multi-layer problems. It is obviously more difficult to apply this method to multi-layer problems owing to the large number of variables involved and hence the large number of master curves which

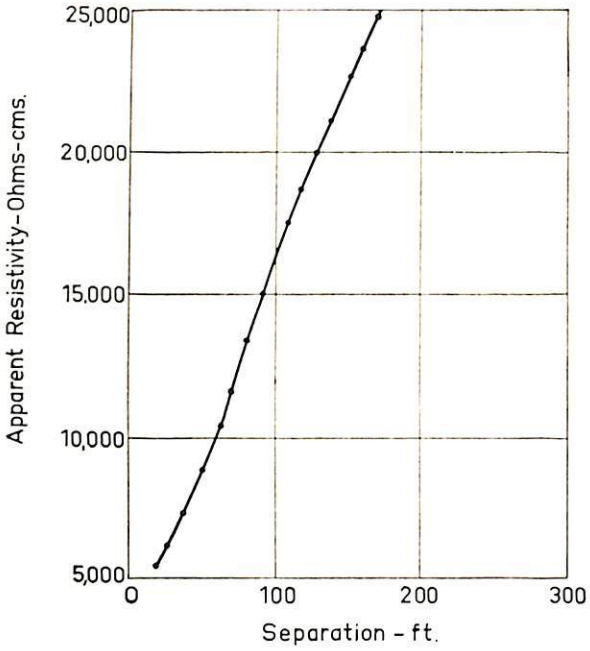

Fig. 4.16. *Result of superposition method.*

are required. Some work has been done on this but it does not appear to have been so very satisfactory, and the idea of splitting the curve up into parts may be more successful.

Summary

An attempt has been made in this Chapter to outline the most commonly used methods of interpretation. It has not been possible to deal with all the methods which have been put forward from time to time, as many of these have not been used very much. It is just as necessary to realise the limitations of the methods as it is to understand what they can do.

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5

THE LINE TRAVERSE AND THE RESISTIVITY MAP

The discussion in Chapter 4 centred round depth curves, i.e. curves of apparent resistivity against electrode separation obtained with the centre of the electrode system at the point under investigation, the object being to determine the depth to successive layers in the ground.

There is, however, another very important application of resistivity measurements which aims at locating lateral discontinuities such as buried bodies, ore bodies and the like. Also resistivity maps, i.e. maps on which lines of equi-resistivity at a constant electrode separation are drawn, can give useful information as to variations in the underlying soil. In fact the resistivity map can be a powerful technique and has had many applications including that of archaeological investigation.

The Line Traverse

The principle of the line traverse is comparatively simple. Suppose it is desired to locate a body such as that shown shaded on Fig. 5.1. Then a suitable spacing a is chosen for the four electrodes and this is kept constant. The system of four electrodes is moved bodily along a line passing over the buried body or, perhaps it is more true to say, where it is suspected that there is a buried object or vertical change in the underlying strata. As the line is moved over the body, the apparent resistivity is affected by its presence and variations in this resistivity can be used as a means of detecting the presence and position of the body. The interpretation of the results obtained is not quite so simple as perhaps this would suggest and a certain amount of care is necessary in making deductions.

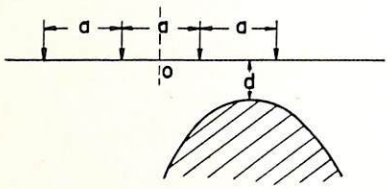


Fig. 5.1. *The line traverse.*

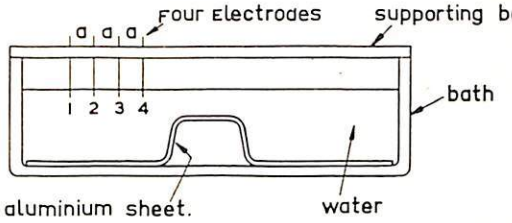


Fig. 5.2. The model.

The model. This can be best explained by examining the results obtained with a model constructed by the author for a similar purpose. A diagram of this model is given in Fig. 5.2. It consisted of a shallow bath containing water, which was intended to represent the soil generally. On the base an aluminium plate was placed and this was bumped up in one place to represent the case where an underlying layer comes nearer to the surface. The effect is similar to that of a buried object. A wooden beam spanning the bath carried the four electrodes which could be moved along any desired line across the bath. A typical result is such as that shown in Fig. 5.3.

When the four electrodes are clear of the bulge in the aluminium plate, the measured resistivity remains practically constant as shown by the section *AB*. As the first two electrodes pass over the edge of the bulge, there is an increase in apparent resistivity as shown by the section *BC*. However, when all four electrodes pass over the edge of the bulge, the resistivity falls as shown by the section *CD*. While all four electrodes are over the bulge, the apparent resistivity remains approximately constant as shown by the section *DE*. As the first two electrodes pass over the further edge of the bulge the apparent resistivity will rise

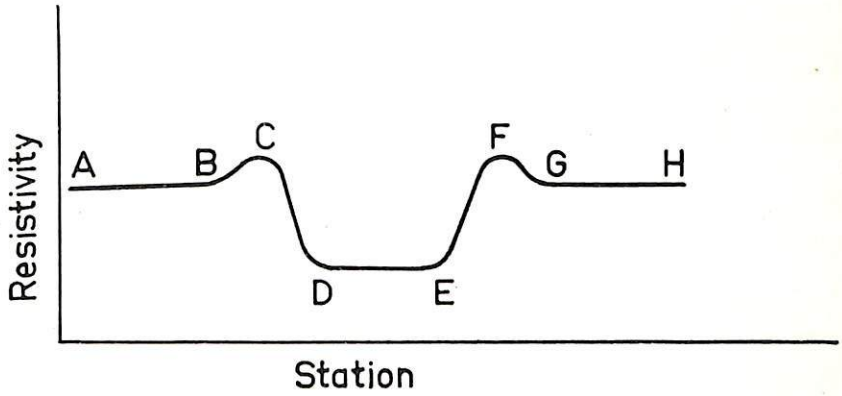


Fig. 5.3. Typical traverse from model.

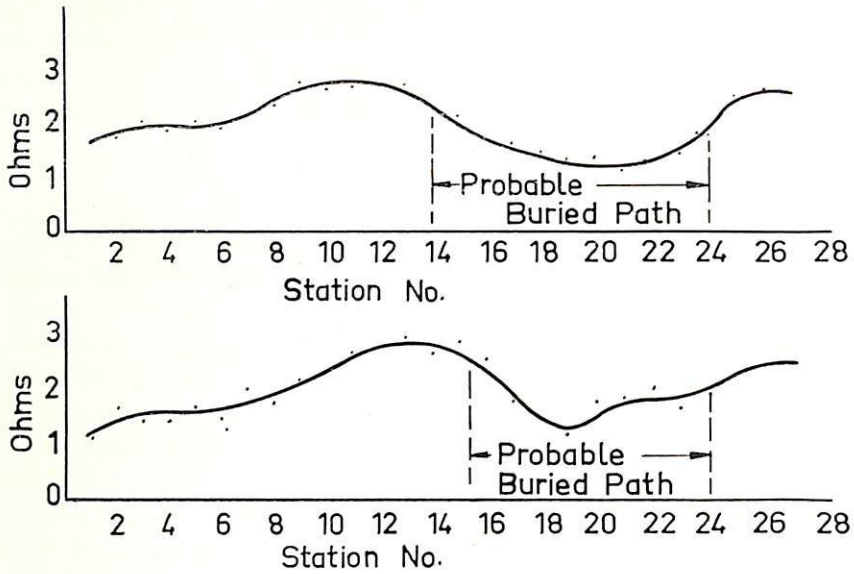


Fig. 5.4. Typical traverses across field in Lincolnshire.

as shown by the section *EF*, and when the last two electrodes are passing over the edge of the bulge, the resistivity drops as shown on the section *FG*. As all four electrodes become clear of the bulge the resistivity becomes constant again as shown in section *GH*.

The typical traverse of Fig. 5.3 thus shows three characteristic changes. If the underlying body had been of a higher resistance than the water, the direction of the changes would be reversed. Thus, in examining the results of line traverses this treble change in resistivity has to be sought. The exact reason for these three changes is not entirely clear. It would normally be expected in the conditions of Fig. 5.2 that there would be a drop in resistivity as the electrode system passes over the bulge, and the rises at *C* and *F* in Fig. 5.3 are probably due to a constriction in the current flow when one current electrode is over the bulge and the other is off it.

Examples of line traverses. In connection with a large scale investigation recently carried out by the author to locate a hidden pathway, a large number of line traverses were run across the area concerned. One important point to be decided in such a matter is the electrode separation to be used. It is obvious that, if this is too small, the effect of the

buried object will be small, since the fraction of the testing current which will reach it is small. On the other hand if it is too large, the buried object becomes small in comparison with the amount of ground included in the test and so again its effect is small. There is obviously an optimum value for any given depth of object, and this is generally found to be rather greater than the depth, usually about 50 per cent more. In the case in point it was suspected that the pathway in question was about 15 to 20 ft below the surface and so an electrode separation of 30 ft was chosen.

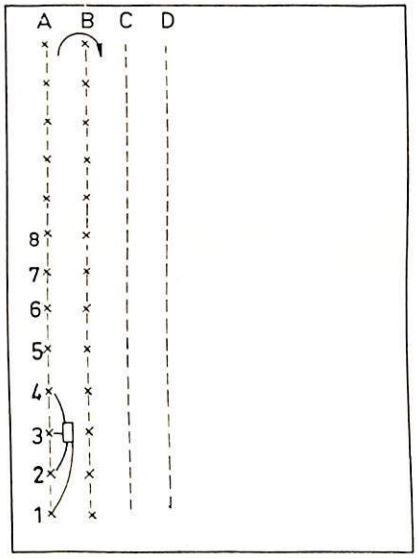


Fig. 5.5. Plan for obtaining resistivity map.

Some typical traverses are given in Fig. 5.4. The three changes in resistivity can be seen, indicating the presence of the hidden path, although they are partly obscured by surface irregularities. By an examination of all the traverses it was found possible to trace the position of the pathway in question over a distance of about three miles. This was afterwards checked by drilling.

Resistivity Map

A development from the line traverse is the resistivity map. One manner in which such a map may be made is as follows. Suppose the area to be surveyed is such as that shown by the rectangle in Fig. 5.5. The first step is to decide on the electrode spacing which is to be used. This is decided in the manner mentioned above, i.e. it is made about

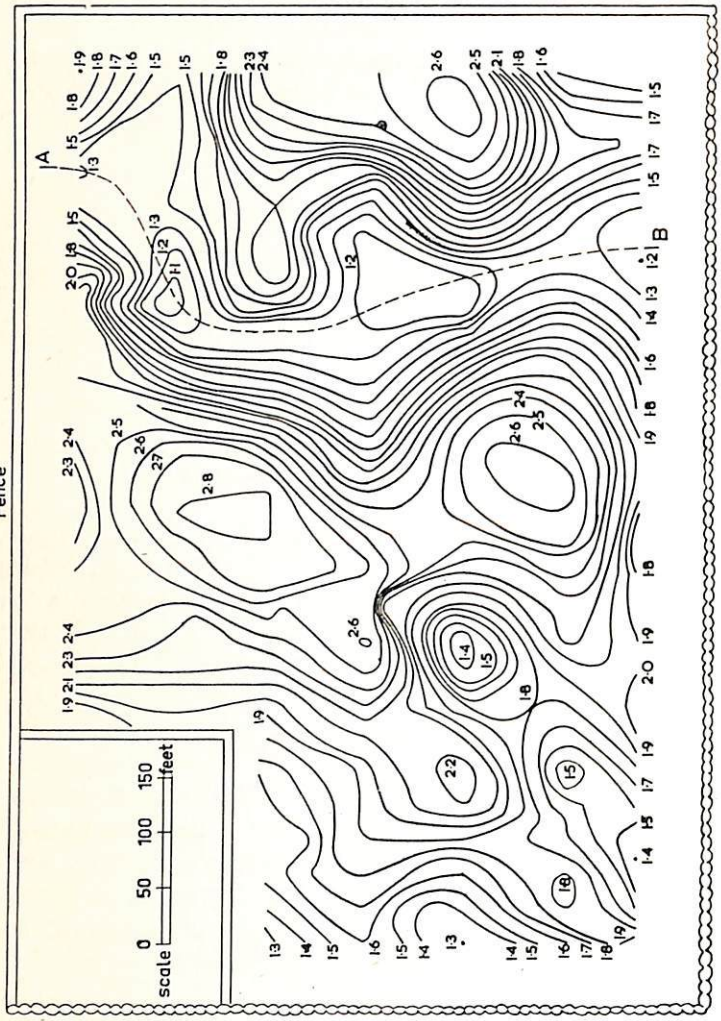


Fig. 5.6. Resistivity map of a field on a farm in Lincolnshire.

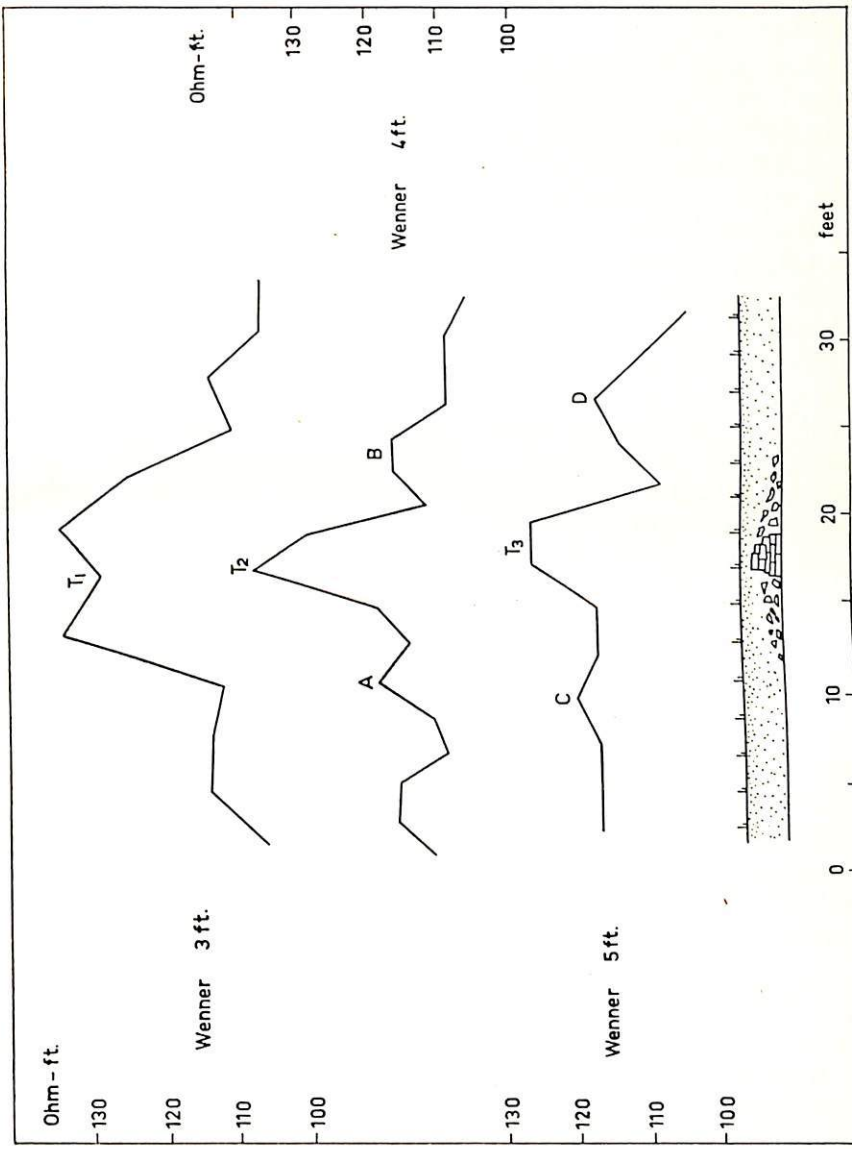


Fig. 5.7. Traverses across buried Roman wall.

50 per cent more than the depth to the features it is desired to map. A series of parallel lines such as *A*, *B*, *C*, *D*, etc. is laid out across the area, the distance between successive lines being equal to the electrode separation chosen.

It is not necessary and, indeed it is often not convenient, to lay out all the lines at once. A series of electrodes 1, 2, 3, 4, 5, etc. is set out along the first line and the equipment is placed opposite electrode 3. The first four electrodes 1, 2, 3 and 4 are connected to the apparatus and a measurement made. The apparatus is then moved forward to electrode 4 and all the connections moved up one electrode spacing, so that the next measurement is made using electrodes 2, 3, 4, 5. This process is repeated until the line *A* has been completed. The equipment is then moved to line *B*, which is traversed in the opposite direction.

In this way a series of parallel traverses is made across the area. From these traverses it is possible to pick off points of equal resistivity, to plot them on a map and to join them up as lines of equi-resistivity. The result is a resistivity map.

An example of such a map is given in Fig. 5.6. This gives the results obtained in a field in Lincolnshire when tracing the pathway mentioned in connection with the line traverse. It was known that this pathway had a lower resistivity than the surrounding soil, and in Fig. 5.6 there is clear evidence of a trough of low resistivity running across the field from *A* to *B*. There are a few pockets of low resistivity but the line *A-B* shows a continuous area of low resistance. This area lined up well with results obtained in the areas on either side of this field.

The resistivity map can be obtained comparatively quickly. It is not always necessary to have as much detail as in that of Fig. 5.6, and if it can be combined with a few bore holes in carefully chosen places, it is possible to obtain a considerable amount of useful information from it. This method has been used for tracing the variations in the depth of the water table, and the depth to bedrock in civil engineering problems.

Archaeological Investigations

A very interesting and successful application of resistivity surveying is to be found in its application to archaeological investigations. These are usually on a somewhat smaller scale than those already described, but the detection of changes in resistivity are usually produced by the presence of buried walls and filled-in ditches and pits.

An example of the detection of a buried wall is given in Fig. 5.7. The wall in question was part of the annexe to the Roman Villa at Northleigh, Oxon. The line of the wall was already known approximately

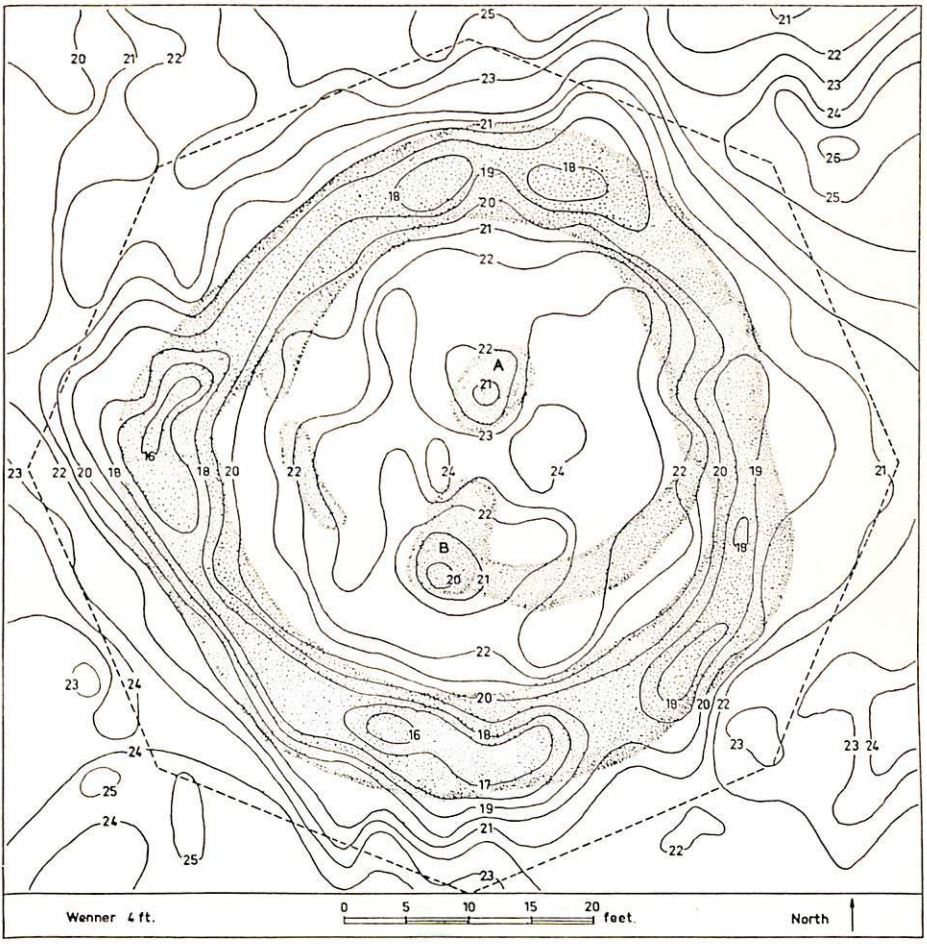


Fig. 5.8. *Resistivity map of neolithic burial site at Dorchester, Oxon.*

from air photographs. Three traverses were run along the same line, using different electrode spacings, and the high resistivity values at T_1, T_2 and T_3 were subsequently found by probing to agree with the position of the wall. The top of the wall was about 18 in below the surface and the wall was surrounded by collapsed masonry extended to about 5 ft on either side.

The use of the resistivity map is shown in Fig. 5.8. This was obtained on a neolithic burial site at Dorchester, Oxon. The area has been under cultivation for a long time and there were no traces visible on the

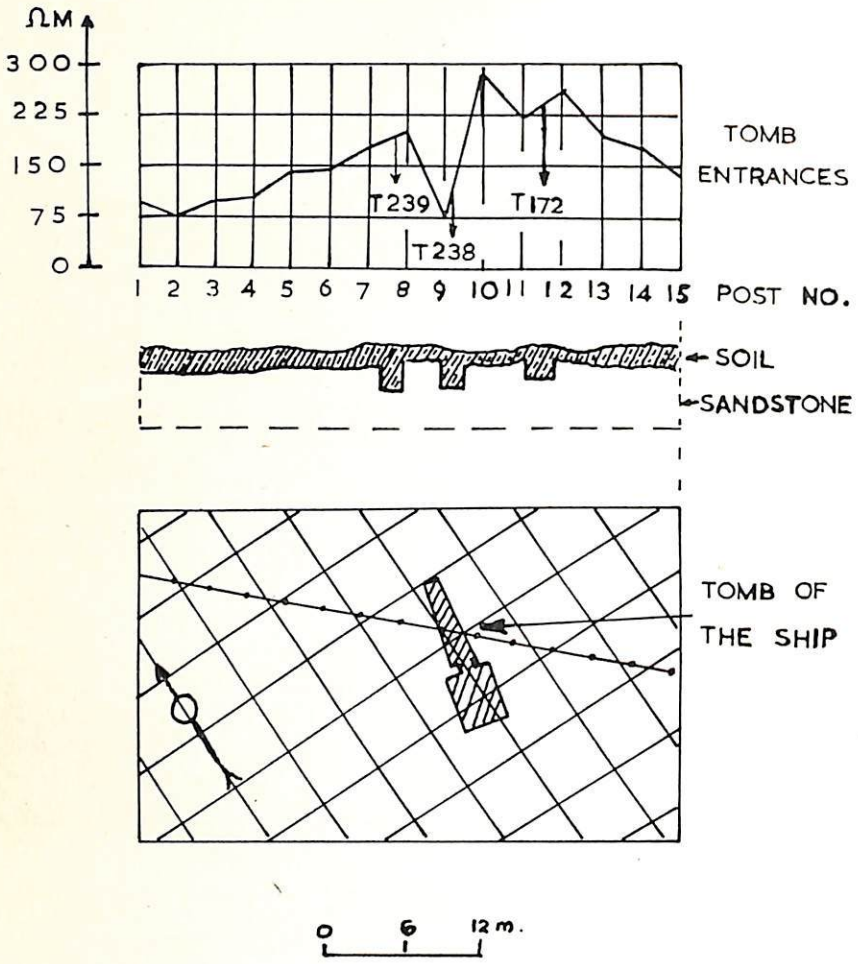


Fig. 5.9. Discovery of the tomb of the ship.
(By courtesy of the Fondazione Lerici.)

surface, but air photographs suggested that the site consisted of a roughly circular ditch surrounding a pair of pits or graves. A series of parallel traverses was run across this area, the lines being 4 ft apart, and the distance between stations and the electrode spacing both being 4 ft also. From these traverse results, lines of equal resistivity were obtained at intervals of 1 ohm. The area to be excavated was determined from this map; it was octagonal in shape and is shown by the broken line. The plan of the ditches and pits is shown by stippling and

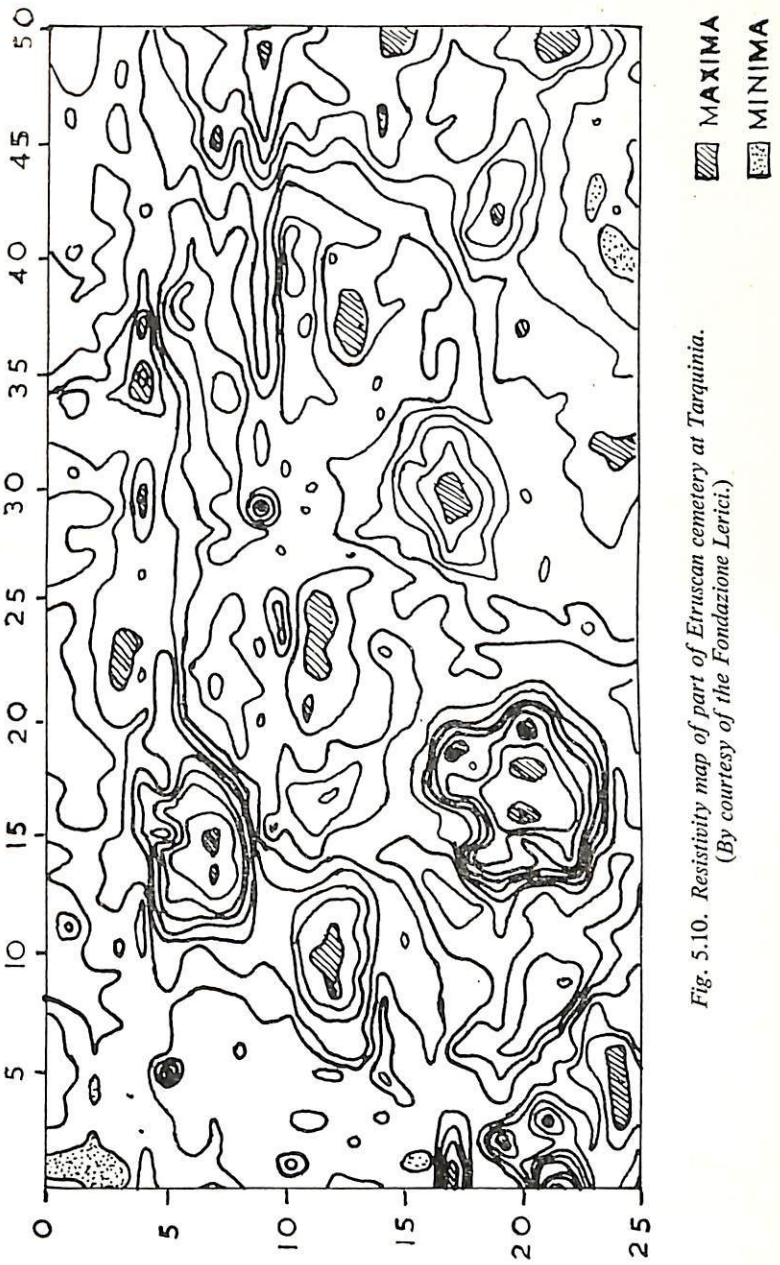


Fig. 5.10. *Resistivity map of part of Etruscan cemetery at Tarquinia.*
(By courtesy of the Fondazione Lerici.)

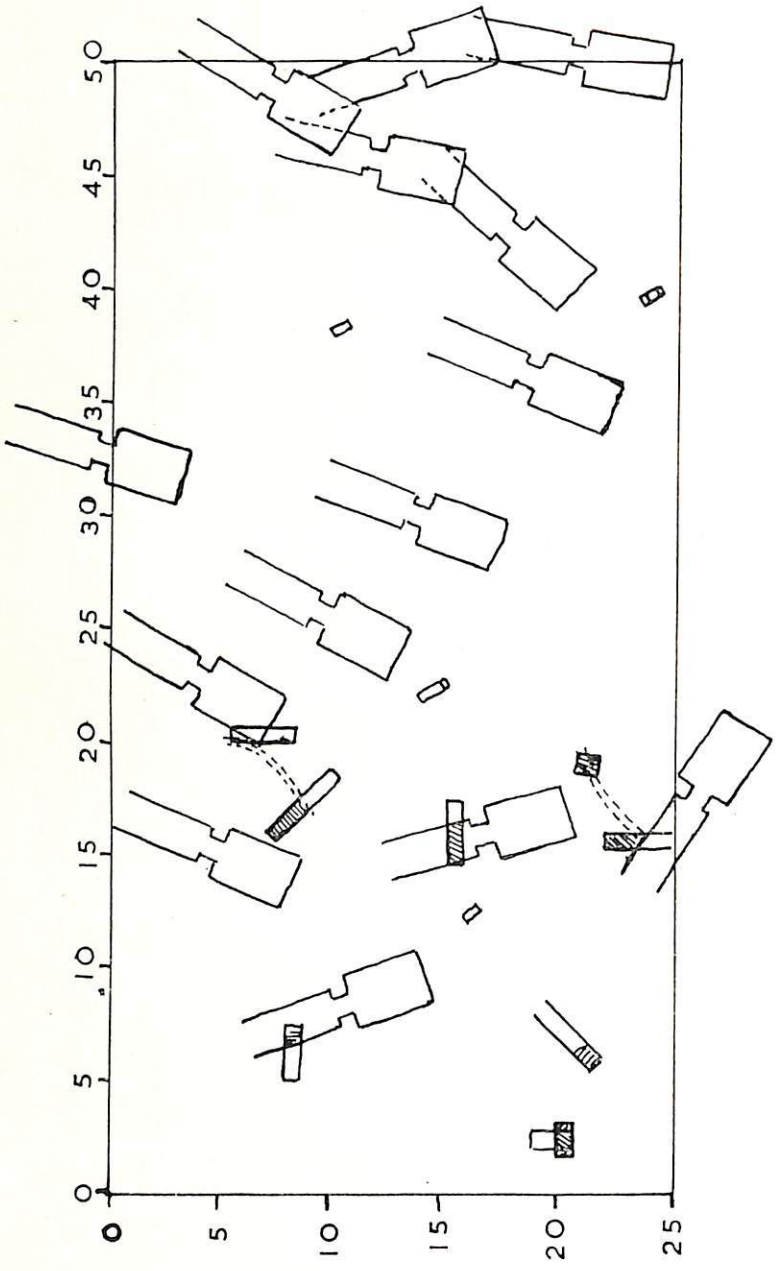


Fig. 5.11. Result of interpretation of map in Fig. 5.10.

it will be seen that the resistivity map shows the position of the surrounding ditch and the pits.

An outstanding use of resistivity surveying in archaeological prospecting is that made by the Lerici Foundation of the Milan Polytechnic in the location of Etruscan tombs. The success of this work can be judged by the results which have been obtained in a period of five years. These included the discovery of a necropolis at Fabriano, six hundred tomb chambers at Cerreteri, numerous structural remains of the ancient city at Tarquinia.

The methods used included both the line traverse and the resistivity map, and an example of the line traverse is given in Fig. 5.9, which shows the discovery of the tomb known as the Tomb of the Ship. The lower part of this diagram shows the line of the survey and the position of the tomb as subsequently confirmed. Above this is a cross-section of the line of the survey, which shows that it passed over the entrances to three tombs. The top diagram gives the results of the resistivity survey and a particularly clear indication of the tomb by the big fall in resistivity over the entrance.

Another example is given in Fig. 5.10. This is a resistivity map of part of the Etruscan cemetery at Tarquinia, and Fig. 5.11 gives the positions of the chamber tombs discovered after the interpretation of Fig. 5.10.

Well Logging

In any account of resistivity investigations it is necessary to make some reference to well logging. This has already been mentioned in Chapter 1 and is an important subject. It is, however, very specialised and a tremendous amount of work has been done, resulting in new and effective techniques for determining the nature and composition of the rocks through which a well has been bored.

The major application is to oil prospecting. A full description of the various techniques would be out of place here, but they can be found in journals dealing with petroleum technology.

6

THE GENERAL EARTH ELECTRODE AND THE SINGLE-DRIVEN ROD

The previous chapters have been concerned with the problems involved in the measurement of the resistivity of the soil, and the uses to which these measurements can be put. In making these measurements electrodes were used, but little consideration had to be paid to them, and any effect they may have on the measurements is small or negligible and is discussed in the chapter on measuring equipment.

Permanent earth electrodes are used in electricity supply systems and telecommunications systems, and in such cases the resistance is of importance. The electrode may consist of driven rods, buried strips or plates and often includes the sheaths of buried cables, fences round power stations and sub-stations. A practical earth electrode is often quite complicated, but a forecast of the resistance is very important. The resistivity of the soil is one factor which contributes to the resistance of any electrode and the size and shape of the electrodes are also important.

The Nature of an Earth Electrode Resistance

In order to examine the nature of an earth electrode resistance consider the simple form in the shape of a hemisphere as in Fig. 6.1. The total resistance can be divided into three parts:

(1) The resistance of the conductor connecting the earth electrode to the system.

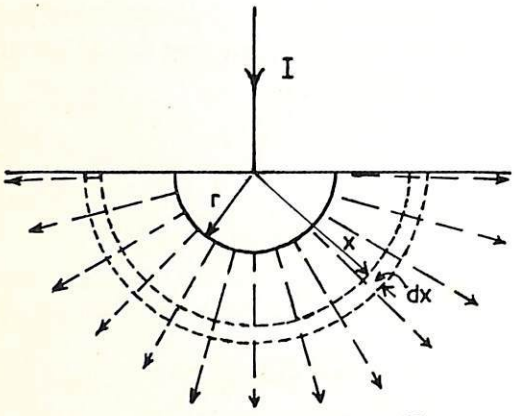


Fig. 6.1. *Hemispherical electrode.*

(2) The contact resistance between the surface of the electrode and the main body of the earth.
(3) The resistance of the body of earth immediately surrounding the electrode.

The resistance of the conductor is a normal conductor resistance and can be dealt with separately. The contact resistance has been found to be negligibly small, and the main part of any electrode resistance is that of the body of earth surrounding the electrode.

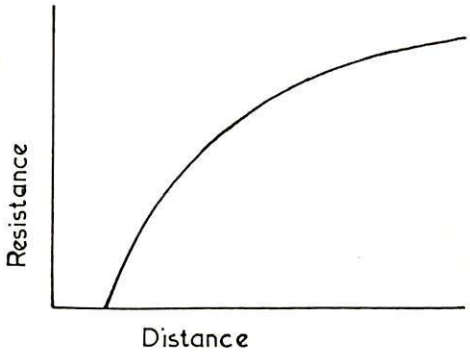


Fig. 6.2. Resistance variation.

In Fig. 6.1, suppose a current I is flowing into the earth through the hemi-spherical electrode. It will flow away in all directions and, if the return electrode or electrodes is a long way away, it will flow uniformly in all directions. So it can be considered that the current is flowing through a series of concentric spherical shells of which one is shown in the figure. These shells are all in series and, as the distance from the electrode is increased, the total resistance is increased, but the cross-section of the shells is increasing, so that the elements of increase in resistance become gradually less. So a curve of resistance against distance will take the form shown in Fig. 6.2.

If an individual shell at a radius x, has a thickness of $\mathrm{d}x$, then the resistance $\mathrm{d}R$ of the element is

$$\mathrm{d}R = \frac{\rho\,\mathrm{d}x}{2\pi x^2}$$

and the total resistance up to a distance r_1 is

$$R = \int_r^{r_1} \frac{\rho\,\mathrm{d}x}{2\pi x^2} = \frac{\rho}{2\pi}\left(\frac{1}{r} - \frac{1}{r_1}\right) \qquad (6.1)$$

The total resistance, i.e. the resistance up to $r_1 = \infty$ is

$$R_\infty = \frac{\rho}{2\pi r} \tag{6.2}$$

The General Equation for Electrode Resistance

The equation (6.2) is actually a special case of a more general expression. Consider two electrodes buried in a homogeneous conducting medium, and suppose there is a current flow from one electrode to the other.

Let V_1 = the potential of one electrode
V_2 = the potential of the second electrode.
V = the potential at any point in the medium.

Then the conditions to be satisfied by the potential are similar to those discussed in connection with the resistivity problems, namely

V must reduce to V_1 at the first electrode
V must reduce to V_2 at the second electrode
V must vanish at an infinite distance
$\nabla^2 V = 0$ at all points in the medium.

At the same time consider the analogeous problem in electrostatics. The homogeneous medium will be replaced by air, the electrodes remaining conductors. Let the two electrodes receive equal and opposite changes of electricity of such a magnitude that the potential difference between the electrodes is $V_1 - V_2$. Now let ψ be the electrostatic potential at any point in the field, and ψ_1, ψ_2 be the values of ψ at the electrodes, so that $\psi_1 - \psi_2 = V_1 - V_2$. There will be a constant C such that $\psi + C$ assumes the values V_1 and V_2 over the electrodes. It is necessary that $\nabla^2\psi = 0$ throughout the field, and so $\nabla^2(\psi + C) = 0$ and $\psi = 0$ at infinity.

Consequently $\psi + C$ satisfies the conditions which have to be satisfied by the potential V in the current case, and these are sufficient to determine V uniquely. It follows that V and $\psi + C$ must be equal. Thus the lines of flow in the current problem are identical with the lines of force when the two electrodes are changed to different potentials in air.

The current flow normal to the surface on any point of the electrode surface is

$$\frac{1}{\rho} \cdot \frac{\partial V}{\partial n}$$

so that the total flow outwards from the electrode is

$$-\frac{1}{\rho}\iint \frac{\partial V}{\partial n}.\mathrm{d}S = -\frac{1}{\rho}\iint \frac{\partial \psi}{\partial n}.\mathrm{d}S$$

where $\mathrm{d}S$ is an element of the surface of the electrodes.

If Q is the change on this electrode in the analagous electrostatic problem then, by Gauss' Theorem

$$-\iint \frac{\partial \psi}{\partial n}\,\mathrm{d}S = 4\pi Q$$

So that the total flow of current is

$$I = \frac{4\pi Q}{\rho}$$

Again, if the capacity between the electrodes in air in the electrostatic case is C, then

$$\psi_1 - \psi_2 = V_1 - V_2 = \frac{Q}{C}$$

If R is the resistance between the electrodes, then

$$R = \frac{V_1 - V_2}{I} = \frac{Q}{C}.\frac{\rho}{4\pi Q} = \frac{\rho}{4\pi C} \tag{6.3}$$

In the case of a single electrode, i.e. the return electrode being at a great distance the values of R and C, then apply to the single electrode.

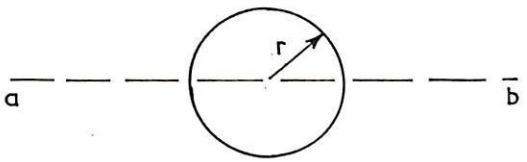


Fig. 6.3. Sphere.

Suppose in Fig. 6.3 the electrode is a sphere, then the capacity of a sphere in air is equal to its radius. So the resistance of a spherical electrode in an infinite conducting medium is

$$R = \frac{\rho}{4\pi r} \tag{6.4}$$

In the practical case the electrode is a hemisphere buried in the surface *ab* of Fig. 3 and it is evident that in this case the resistance will be doubled, i.e.

$$R = \frac{\rho}{2\pi r}$$

which is the same as equation (6.2).

This leads to the general equation applicable to any form of electrode

$$R = \frac{\rho}{2\pi C} \tag{6.5}$$

where C is the electrostatic capacity of the electrode combined with its image above the surface of the earth, the combined electrode being considered as being in air.

The Driven Rod

One of the simplest forms of electrode is the driven rod as in Fig. 6.4. There is, unfortunately, no formula for the capacity of a cylinder in air.

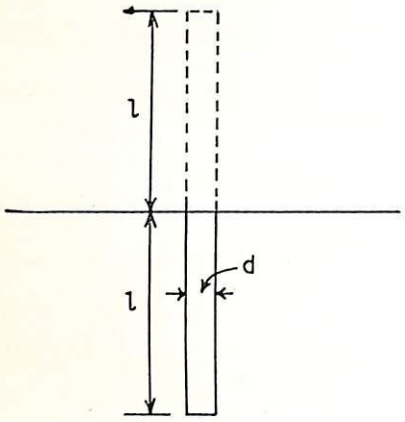


Fig. 6.4. Driven rod with image.

A good approximation can, however, be made by considering the electrode as half an ellipsoid of revolution in which the major axis is very large compared with the minor. The capacity of such an ellipsoid is given by

$$C = \frac{a}{2 \log_e \frac{2a}{b}}$$

where a is the length of the major axis, and b that of the minor axis of the ellipsoid. If this is applied to the driven rod plus its image above the surface as in Fig. 4, then

$$C = \frac{2l}{2 \log_e \frac{4l}{d}} = \frac{l}{\log_e \frac{4l}{d}}$$

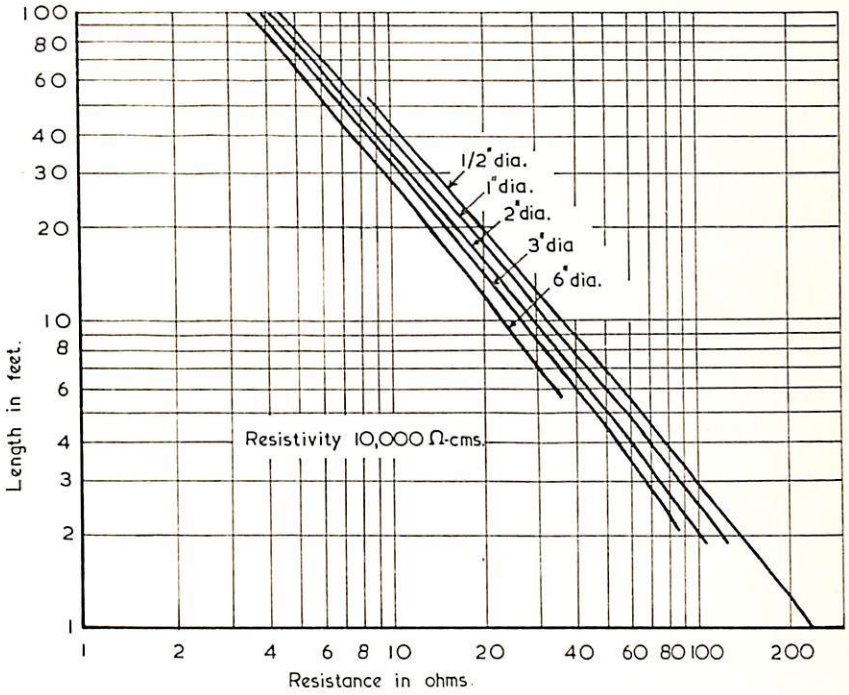


Fig. 6.5. Resistance of driven rod calculated from ellipsoid formula.

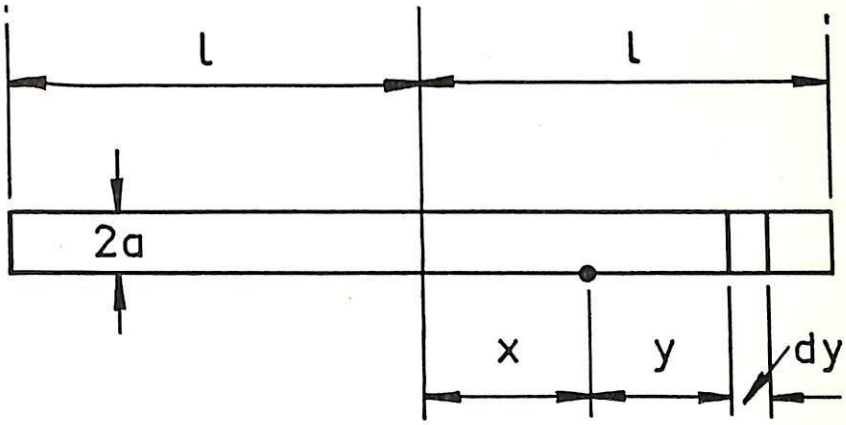


Fig. 6.6. Driven rod—uniform charge.

Substituting this in equation (6.5) gives the resistance of a driven rod as

$$R = \frac{\rho}{2\pi l} \log_e \frac{4l}{d} \tag{6.6}$$

A series of curves calculated from this formula are given in Fig. 6.5. These show the variation in resistance with length for a number of different diameters and are calculated for a value of $\rho = 10{,}000$ ohm-cms. For any other value of ρ, the values given by these curves must be

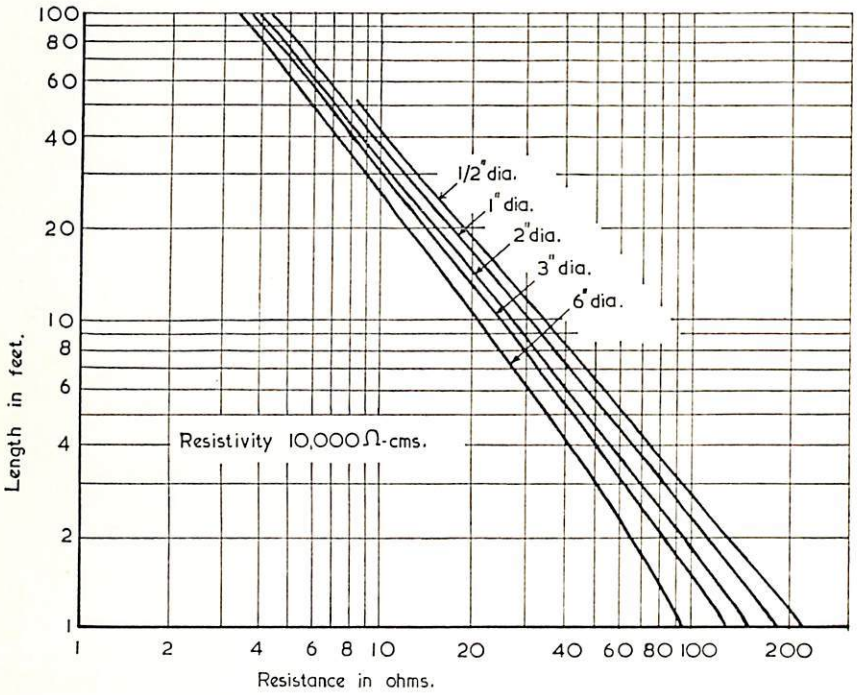


Fig. 6.7. Resistance of driven rod calculated from uniform charge formula.

multiplied by the factor $\rho/10{,}000$. The diameter of the rod has only a small effect on the resistance as is only to be expected, since the diameter appears only in the logarithmic term. An increase in length, however, always produces a lower resistance, if the soil remains homogeneous.

If for convenience l is measured in feet and d in inches, the equation (6.6) becomes

$$R = \frac{\rho}{191{\cdot}5l} \log_e \frac{48l}{d} \tag{6.7}$$

An alternative method of calculation which becomes very useful in more complicated earthing systems has been used by Dwight and others and can be described simply as the uniform charge method. Consider a driven rod as a cylindrical electrode of length L, diameter $2a$ together with its image above the surface of the earth, as shown in Fig. 6.6. The electrostatic problem is again that of calculating the capacity, and if a uniform charge over the surface is assumed, this means calculating the average potential over the electrode. Suppose the charge to be assumed to be q per cm length, then the charge on the ring $\mathrm{d}y$ is $q\,\mathrm{d}y$. The potential at any point P due to this charge can be shown to be

$$\frac{q\,\mathrm{d}y}{\sqrt{(a^2+y^2)}}$$

plus terms involving powers of the ratio $a/\sqrt{(a^2+y^2)}$.

To obtain the total potential, this must be integrated from $y=0$ to $y=l-x$ and also from $y=0$ to $y=l+x$. If this is then multiplied by $\mathrm{d}x/l$ and integrated from $x=0$ to $x=l$, the average potential will be obtained. This is

$$\frac{V}{2q}=\log_e\frac{4l}{a}-1 \text{ plus terms involving powers of } a/l.$$

Since in all practical rod electrodes, this ratio is very small, these terms may be neglected. The capacity thus becomes

$$\frac{1}{C}=\frac{E}{2ql}=\frac{1}{l}\left(\log_e\frac{4l}{a}-1\right)$$

and the resistance

$$R=\frac{\rho}{2\pi l}\left(\log_e\frac{4l}{a}-1\right)$$

if $d=$ the diameter of the rod as before, then $d=2a$, and

$$R=\frac{\rho}{2\pi L}\left(\log_e\frac{8l}{d}-1\right) \tag{6.8}$$

and this is the alternative formula.

If l is again measured in feet and d is in inches, this equation becomes

$$R=\frac{\rho}{191{\cdot}5l}\left(\log_e\frac{96l}{d}-1\right) \tag{6.9}$$

A set of curves calculated from this formula are given in Fig. 6.7 again for a value of $\rho = 10{,}000$ ohm-cms. In general formula (6.9) gives values which are slightly lower than those given by formula (6.7). The actual difference is

$$\frac{\rho}{191{\cdot}5l}\left(\log_e \frac{48l}{d} - \log_e \frac{96l}{d} + 1\right)$$

$$= \frac{\rho}{191{\cdot}5l}(\log_e 48 - \log_e 96 + 1) = \frac{0{\cdot}3069\rho}{191{\cdot}5l}$$

and if ρ is taken as 10,000 ohm-cms this becomes $1602/l$. So for a 1 foot rod the difference is 16·02 ohms or about 6·7 per cent, while for a

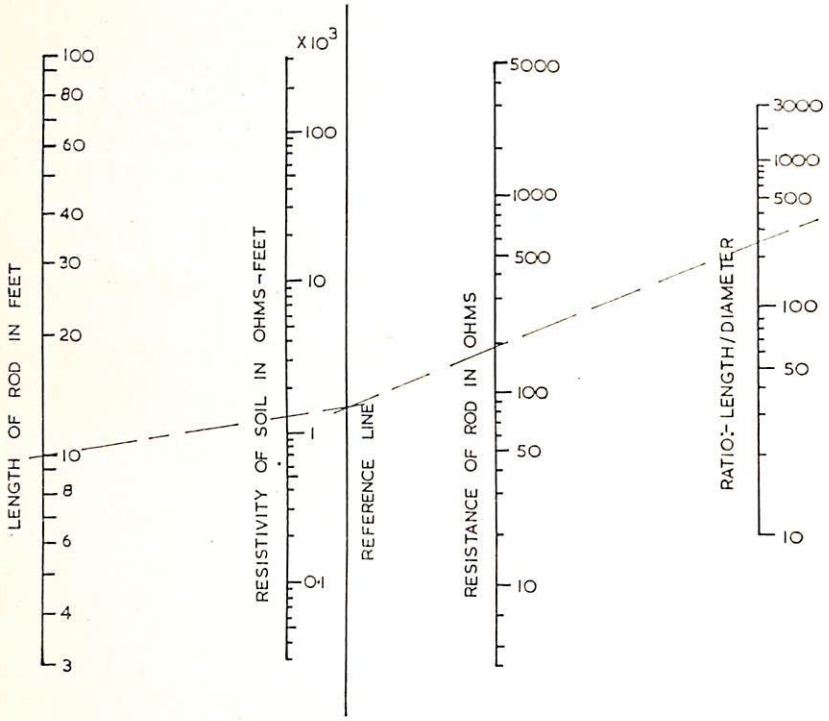


Fig. 6.8. Nomogram to give resistance of driven rod.

100 feet rod the difference is 0·1602 or about 4 per cent. Such differences are not very important since it is doubtful whether the value of ρ can be known to this accuracy, and other factors can also cause errors.

A nomogram is given in Fig. 6.8 from which the resistance of any rod electrode can be obtained for any value of soil resistivity. One scale has the values of soil resistivity in ohm-cms, a second the length of the rod

in feet, and a third the ratio, length of rod in feet/diameter of rod in inches. If a straight edge is placed across the correct values on these scales, using the reference line, the intercept on the fourth scale gives the resistance of the rod.

The Equivalent Hemisphere

A useful device for more complex calculations is to replace the rod electrode by a hemisphere having the same resistance. The resistance of a hemisphere buried in the surface is

$$R_1 = \frac{\rho}{2\pi r}$$

and if this is equated to the resistance of the driven rod, then

$$\frac{\rho}{2\pi r} = \frac{\rho}{2\pi l}\left(\log_e \frac{8l}{d} - 1\right), \text{ from which}$$

$$r = \frac{l}{\log_e \frac{8l}{d} - 1}$$

If l is in feet and r in feet, while d is in inches, then

$$r = \frac{l}{\log_e \frac{96l}{d} - 1} \tag{6.10}$$

The values of this radius for various lengths and diameters of rods or pipes are given in Table 6.1.

The Resistance or Potential Gradient Area

On referring again to Fig. 6.2 it is evident, as already explained, that, as the distance from the earth electrode is increased, the fraction of the total resistance included within this distance is also increased, although at a diminishing rate. Also from a purely theoretical point of view, the total resistance is included only when the distance becomes infinite.

From a practical point of view, however, there is a volume of earth surrounding the electrode which contributes the major part of the resistance, and the size of this volume is dependent on what is meant by this major part. The surface of this volume, i.e. on the surface of the earth is known as the resistance or potential gradient area. If a constant current is passed from the electrode into the earth, the curve of Fig. 6.2 will also represent the potential difference between the

TABLE 6.1

Length of Rod or Pipe in feet	*Radius in feet of equivalent hemisphere when diameter of rod or pipe is*				
	½ in	*1 in*	*2 in*	*3 in*	*6 in*
1	0·241	0·302	0·348	0·406	0·564
2	0·404	0·470	0·561	0·623	0·811
3	0·561	0·644	0·758	0·841	1·044
4	0·708	0·807	0·940	1·038	1·266
5	0·851	0·966	1·117	1·227	1·478
6	0·993	1·121	1·288	1·410	1·680
7	1·128	1·270	1·452	1·588	1·881
8	1·262	1·418	1·616	1·799	2·075
9	1·406	1·562	1·773	1·929	2·265
10	1·524	1·702	1·931	2·095	2·450
20	2·755	3·060	3·405	3·660	4·190
30	3·920	4·310	4·780	5·110	5·800
40	5·030	5·510	6·110	6·500	7·330
50	6·120	6·690	7·380	7·850	8·910
60	—	7·840	8·620	9·160	10·21
70	—	8·970	9·820	10·42	11·62
80	—	10·04	11·03	11·68	13·00
90	—	11·18	12·20	12·93	14·36
100	—	12·20	13·38	14·12	15·68

electrode and a point at any distance from it; so the slope of the curve can represent the potential gradient at any point.

The size of the practical resistance area can be determined in various ways. When measuring the resistance, it is governed by the accuracy required. When several electrodes are connected in parallel, the extent to which their resistance areas overlap governs the amount of the reduction in resistance obtained by the parallel connection.

In order to reduce these considerations to actual figures, consider again the hemispherical electrode of Fig. 6.1. It has been shown that the resistance included in a distance r_1 from the electrode (equation 6.1) is

$$R = \frac{\rho}{2\pi}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

so that the fraction of the total resistance included is

$$\frac{\frac{1}{r} - \frac{1}{r_1}}{\frac{1}{r}} = 1 - \frac{r}{r_1}$$

If this now is to be x per cent then

$$1 - \frac{r}{r_1} = \frac{x}{100} \text{ and so } r_1 = \frac{100r}{100 - x} \tag{6.11}$$

If this is applied to the driven rods of Table 6.1, for values of x of 95, 99 and 99·5 per cent, the results obtained are those in Table 6.2.

It will be seen from this table that the distance involved, if the fraction to be included in high, can be very large. For example, with a rod 8 ft long, 1 inch diameter, 99 per cent of the total resistance is involved in a distance of 141·8 ft, but to include 99·5 per cent it is necessary to increase this distance to 283·6 ft. This is a matter of considerable importance and will be considered again later in the chapter dealing with the measurement of the resistance.

General Note on Driven Rod

The driven rod or pipe is a very convenient form of earth electrode and its installation in the shorter lengths is comparatively simple and cheap. It is seldom, however, that a single rod, say 8 ft long will give a sufficiently low resistance, and quite often the longer rods driven in by some form of mechanical hammer are used. These sometimes have the additional advantage of reaching layers in the soil of lower resistivity and thus gaining a greater decrease in resistance than that due to the increased length alone.

In other cases, a number of rods are connected in parallel and this is dealt with in the next chapter. It must be remembered that all the calculations in this chapter are based on homogeneous soil, and the modifications necessary when this is not so, are considered in a later chapter.

TABLE 6.2

Length in feet	*Distance r_1 in feet*														
	½ *in dia.*			1 *in dia.*			2 *in dia.*			3 *in dia.*			6 *in dia.*		
	95%	99%	99·5%	95%	99%	99·5%	95%	99%	99·5%	95%	99%	99·5%	95%	99%	99·5%
1	4·82	24·1	48·2	6·04	30·2	60·4	6·96	34·8	69·6	8·12	40·6	81·2	11·28	56·4	112·8
2	8·08	40·4	80·8	9·40	47·0	94·0	11·22	56·1	112·2	12·66	63·3	126·6	16·22	81·1	162·2
3	11·22	56·1	112·2	12·88	64·4	128·8	15·16	75·8	151·6	16·82	84·1	168·2	20·88	104·4	208·8
4	14·16	70·8	141·6	16·14	80·7	161·4	18·80	94·0	188·0	20·76	103·8	207·6	25·32	126·6	253·2
5	17·02	85·1	170·2	19·32	96·6	193·2	22·34	111·7	223·4	24·54	122·7	245·4	29·56	147·8	295·6
6	19·86	99·3	198·6	22·42	112·1	224·2	25·76	128·8	257·6	28·20	141·0	282·0	33·60	168·0	336·0
7	23·56	112·8	235·6	25·40	127·0	254·0	29·04	145·2	290·4	31·76	158·8	317·6	37·62	188·1	376·2
8	25·24	126·2	252·4	28·36	141·8	283·6	32·32	161·6	323·2	35·98	179·9	359·8	41·50	207·5	415
9	28·12	140·6	281·2	31·22	156·2	312·2	35·46	177·3	354·6	38·58	192·9	385·8	45·30	226·5	453
10	30·45	152·4	304·8	34·04	170·2	340·4	38·62	193·1	386·2	41·90	209·5	419·0	49·00	245	490
20	55·10	275·5	551	61·20	306	612	68·10	340·5	681	73·20	366	732	83·80	419	838
30	79·40	392	794	86·20	431	862	95·60	478	956	102·20	511	1022	116·0	580	1160
40	100·6	503	1006	110·20	551	1102	122·20	611	1222	130·0	650	1300	146·6	733	1466
50	132·4	612	1324	133·80	669	1338	157·6	738	1576	157·0	785	1570	178·2	891	1782
60	—	—	—	156·80	784	1568	172·4	862	1724	183·2	916	1832	204·2	1021	2042
70	—	—	—	179·40	897	1794	196·4	982	1964	208·4	1042	2084	232·4	1162	2324
80	—	—	—	200·8	1004	2008	220·6	1103	2206	233·6	1168	2336	260·0	1300	2600
90	—	—	—	223·6	1118	2236	244·0	1220	2440	258·6	1293	2586	297·2	1436	2972
100	—	—	—	244·0	1220	2440	267·6	1338	2676	282·4	1412	2824	313·6	1568	3136

7

THE MULTIPLE-DRIVEN ROD ELECTRODE

The driven rod is an economical and simple means of making an earth connection but, except sometimes in the case of a deep driven rod, the resistance of a single rod is in general not sufficiently low and it is necessary to use a number connected in parallel. Considerable care has to be taken when this is done as rods so connected do not necessarily comply with the usual law of resistances in parallel.

This is because the resistance of any one rod, as already explained, is contained in a body of earth surrounding it and, if the full effect of the resistance in parallel is to be attained, the rods must be at such a distance apart, that these bodies of earth are independent of one another. To put it another way, the resistance areas of the rods must not overlap. In practice, this is very difficult and so it becomes necessary to determine what reduction in the total resistance can be obtained by connecting rods in parallel.

The calculation of the combined resistance, especially when a large number of rods is involved, is difficult and can be very lengthy and laborious at the least. Again very exact calculations are not necessarily of any greater value than approximate ones, as any calculation assumes ideal conditions, including homogeneity of soil, and such conditions are never met in practical cases. As an example, consider the simple case of the two rods in parallel.

In order to calculate the combined resistance, it is necessary first to calculate the capacity of the analogous electrostatic case. This can be done approximately by the uniform charge method already used in calculating the resistance of a single rod (see Chapter 6, page 97). In this method it is assumed that uniform charges are distributed over both rods and the potential due to these charges is calculated. The capacity is then calculated from the value of the potential and the total charge on the two rods and thence the resistance. This method can be extended to more than two rods but, as already stated, the calculations can become very laborious.

There is another approximate method which can be used. As explained in Chapter 6 (see page 99), a rod may be replaced by a hemi-

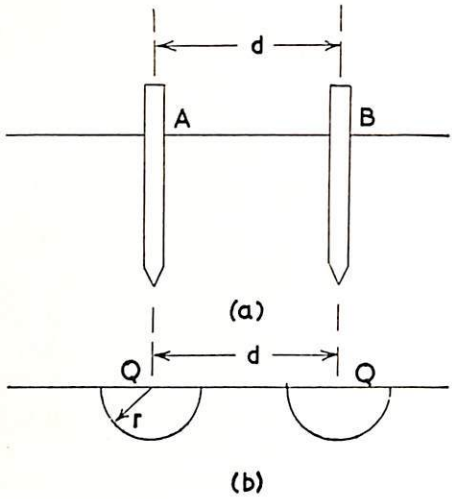


Fig. 7.1. Two rods in parallel.

spherical electrode having the same resistance, and this substitution simplifies the calculations for large numbers of rods. The method generally consists of assuming that each equivalent hemisphere carries the same charge, and to calculate the average potential of the group of rods. From this and the total charge, the capacity and the resistance can be calculated.

It is known that the capacity of a sphere is equal to its radius in centimetres and so the potential of a sphere of radius r is

$$V = \frac{Q}{r} \tag{7.1}$$

where Q is the charge on the sphere. Again the potential at a distance d from the sphere is

$$V_d = \frac{Q}{d} \tag{7.2}$$

The rods are actually replaced by hemispheres but, as the calculations are all concerned with the ratio of the combined capacity to that of a single rod, the factor 2 which would be introduced by the hemisphere can be ignored and the rods considered as replaced by spheres.

This method obviously cannot be used if the spacing between the rods is equal to or less than twice the radius of the equivalent hemisphere but, as this would be too small a spacing to be of any practical value, this limitation is not of very great importance, and the larger

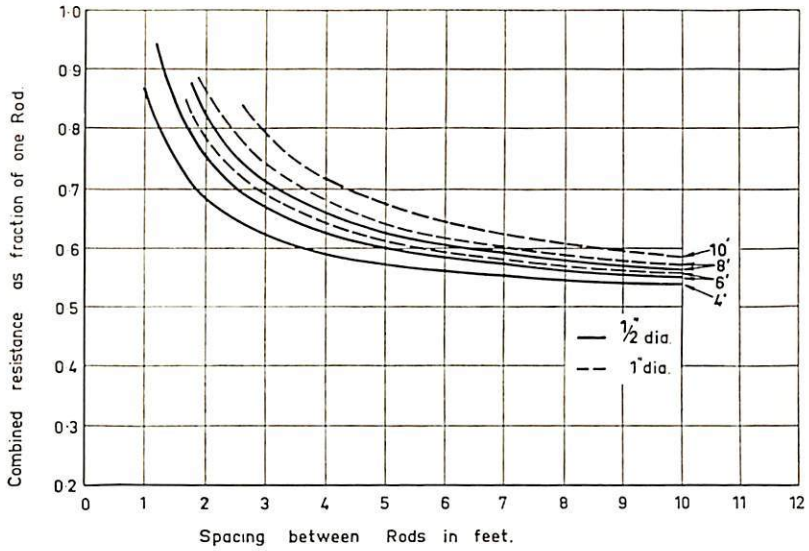


Fig. 7.2. Combined resistance of two rods in parallel.

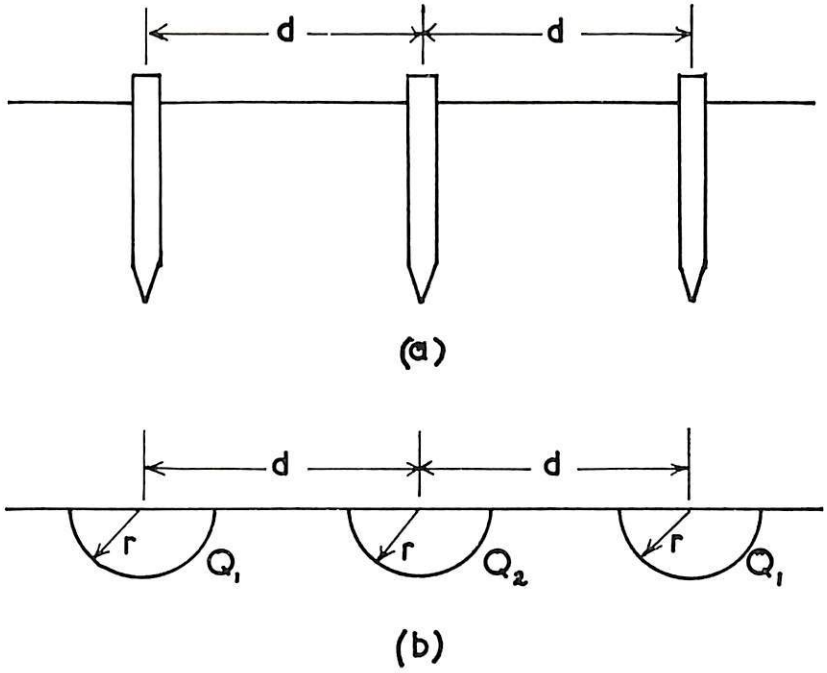


Fig. 7.3. Three rods in parallel in a straight line.

the spacing, the less is the error caused by the substitution of the hemisphere for the actual rod.

Two Rods in Parallel

Suppose there are two rods connected in parallel at a distance apart d as in Fig. 1(a). Each rod can be replaced by its equivalent hemisphere of radius r carrying a charge Q. Then the potential of either rod will be:

$$V = \frac{Q}{r} + \frac{Q}{d} = \frac{Q}{r}\left\{1 + \frac{r}{d}\right\} = \frac{Q}{r}(1 + \alpha)$$

where $\alpha = r/d$.

The total charge $= 2Q$, and so the capacity C is given by

$$\frac{1}{C} = \frac{V}{2Q} = \frac{1}{2r}(1 + \alpha)$$

So the combined resistance of the two rods in parallel will be

$$R = \frac{\rho}{2\pi C} = \frac{\rho}{4\pi r}(1 + \alpha) \tag{7.3}$$

The resistance of one rod is $\rho/2\pi r$, and so the ratio

$$\frac{\text{Resistance of two rods in parallel}}{\text{Resistance of one rod}} = \frac{\dfrac{\rho}{4\pi r}(1 + \alpha)}{\dfrac{\rho}{2\pi r}} = \frac{1 + \alpha}{2} \tag{7.4}$$

This resistance ratio has been calculated for rods of various lengths and diameters and the results are plotted in the curves of Fig. 7.2. The ideal value for this ratio is 0·5, and these curves do show the difficulty in obtaining this ideal. For instance, two 10 ft rods placed 10 ft apart produce a resistance ratio of 0·585, and the distance d would have to be increased very considerably to produce any substantial reduction in this ratio.

Three Rods in Parallel, in a Straight Line

Three rods in parallel are shown in Fig. 7.3(a), and the equivalent hemispheres in Fig. 7.3(b). In this case, as only a few rods are involved, it is not necessary to assume equal charges on all three. It is obvious that if the rods are at the same potential, as they must be, the charge

on the centre rod will be different from those on the outer rods. Let the charge on each outer rod be Q_1 and that charge on the centre rod be Q_2.

Then the potential of either of the outer rods will be

$$\frac{Q_1}{r} + \frac{Q_2}{d} + \frac{Q_1}{2d} = \frac{1}{r}\left[Q_1\left(1 + \frac{\alpha}{2}\right) + Q_2.\alpha\right]$$

The potential of the centre rod will be

$$\frac{Q_2}{r} + \frac{2Q_1}{d} = \frac{1}{r}(Q_1.2\alpha + Q_2)$$

These two potentials must be equal and so

$$Q_1\left(1 + \frac{\alpha}{2}\right) + Q_2\alpha = Q_1 2\alpha + Q_2$$

$$\text{i.e. } Q_2 = \frac{Q_1(1 - 3\alpha/2)}{(1 - \alpha)} = kQ_1 \text{ say}$$

$$\text{Then the potential} = \frac{1}{r}(Q_1.2\alpha + kQ_1) = \frac{Q_1}{r}(2\alpha + k)$$

The total charge $= Q_1(2 + k)$ and so

$$\frac{1}{C} = \frac{1}{r}.\frac{2\alpha + k}{2 + k}$$

$$\text{Therefore } R = \frac{\rho}{2\pi r}.\frac{2\alpha + k}{2 + k}$$

$$\text{Ratio: } \frac{\text{Resistance of three rods in parallel}}{\text{Resistance of one rod}} = \frac{2\alpha + k}{2 + k}$$

$$= \frac{2\alpha + \dfrac{1 - 3\alpha/2}{1 - \alpha}}{2 + \dfrac{1 - 3\alpha/2}{1 - \alpha}} = \frac{2 + \alpha - 4\alpha^2}{6 - 7\alpha} \tag{7.5}$$

Values of this ratio for various rods are given in Fig. 7.4.

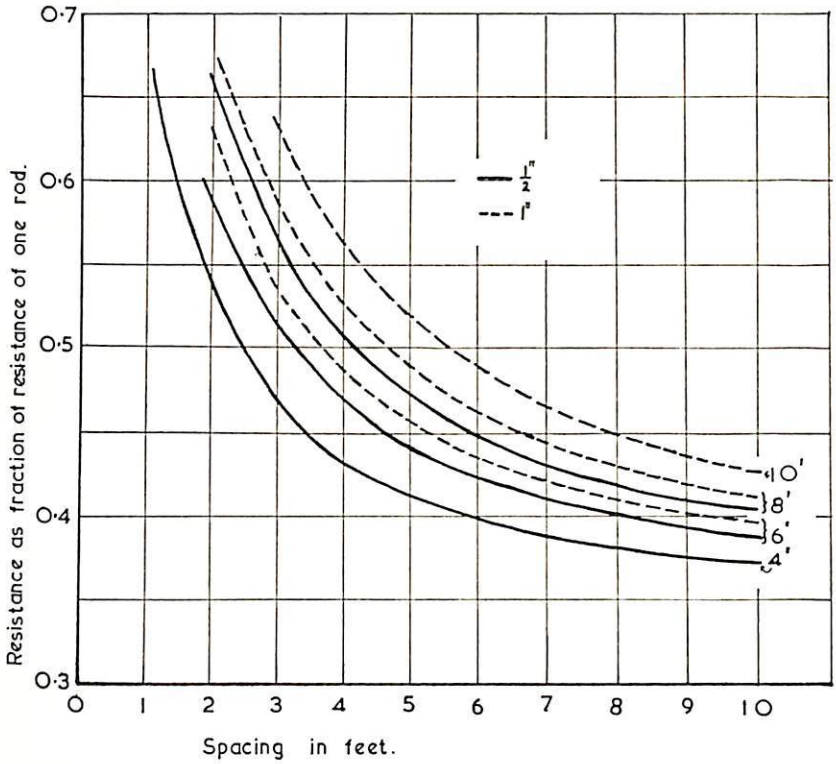


Fig. 7.4. Combined resistance of three rods in a straight line in parallel.

Three Rods in Parallel, arranged in a Triangle

A more compact arrangement of three rods consists in placing them at the corners of an equilateral triangle of side d. It can be shown by a similar calculation to those above that

$$\frac{\text{Resistance of three rods in parallel}}{\text{Resistance of one rod}} = \frac{1 + 2\alpha}{3} \qquad (7.6)$$

The values of this ratio for various rods are given in the curves of Fig. 7.5.

Four Rods in Parallel, in a Straight Line

If four rods are placed in a straight line at a distance d apart, an

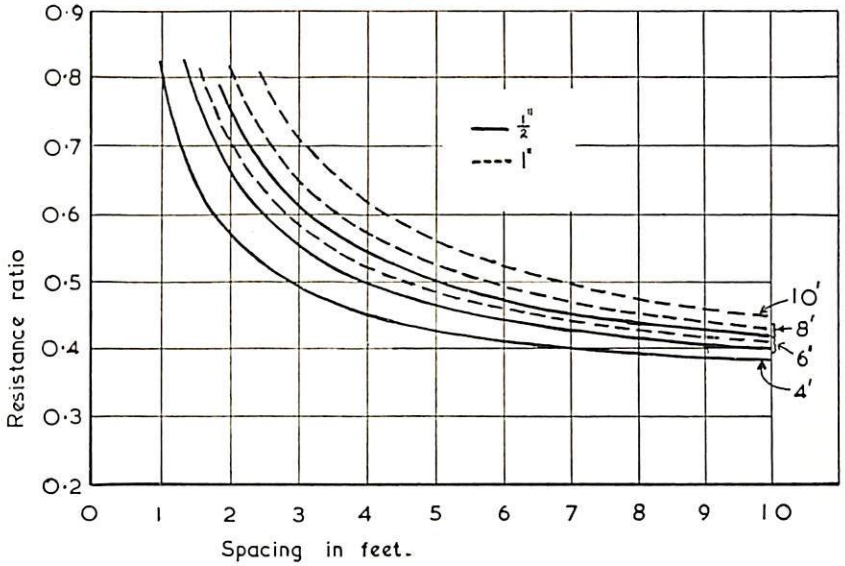
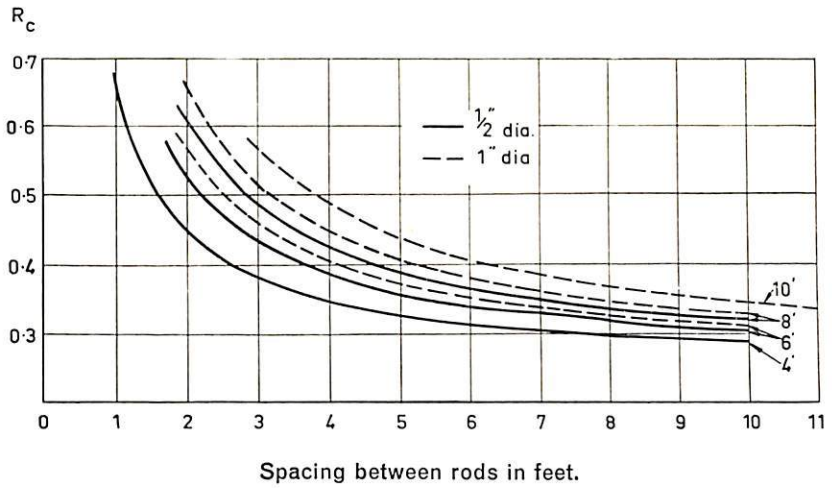


Fig. 7.5. Combined resistance of three rods in a triangle, in parallel.



Rc Combined resistance as fraction of one rod.

Fig. 7.6. Combined resistance of four rods in parallel in a straight line.

analysis similar to that of three rods in parallel, in a straight line leads to the following:

$$\frac{\text{Resistance of four rods in parallel}}{\text{Resistance of one rod}} = \frac{12 + 16\alpha - 21\alpha^2}{48 - 40\alpha} \qquad (7.7)$$

Curves calculated from this are given in Fig. 7.6.

Rods Arranged in a Hollow Square

When it is necessary to use a larger number of rods, it will usually become impracticable to place these in a straight line, and it becomes necessary to consider other arrangements. One of these is the hollow square. An example of this is illustrated in Fig. 7.7, showing eight rods arranged in a square. If there are n rods in one side of the square, the total number of rods is $4n - 4$. The method employed in calculating the combined resistance is as follows.

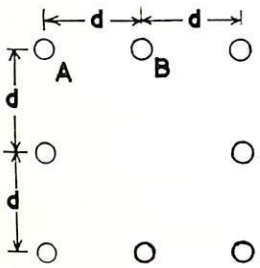


Fig. 7.7. Rods in parallel in hollow square.

It is assumed that each rod carries the same charge, and the potential of one corner rod such as A is calculated, as is the potential of a centre rod such as B. The potential of the whole system is taken as the average of these two values. This is a method which has been used successfully in other connections. From this average potential and the total charge, the capacity and thus the resistance of the array can be calculated.

If N is the total number of rods, this calculation leads to the following expression:

$$\frac{\text{Resistance of } N \text{ rods in parallel}}{\text{Resistance of one rod}} = \frac{1 + k\alpha}{N} \qquad (7.8)$$

where k is dependent on the number of rods. The values of k are given in Table 7.1.

TABLE 7.1

Number of rods in side of square	*Total number of rods*	*Value of k*
2	4	2·7071
3	8	4·2583
4	12	5·3939
5	16	6·0072
6	20	6·4633
7	24	6·8363
8	28	7·1479
9	32	7·4195
10	36	7·6551

Curves calculated from equation (7.8) are given in the curves in Figs. 7.8–7.15. The curves for four rods will be given later when dealing with a solid square.

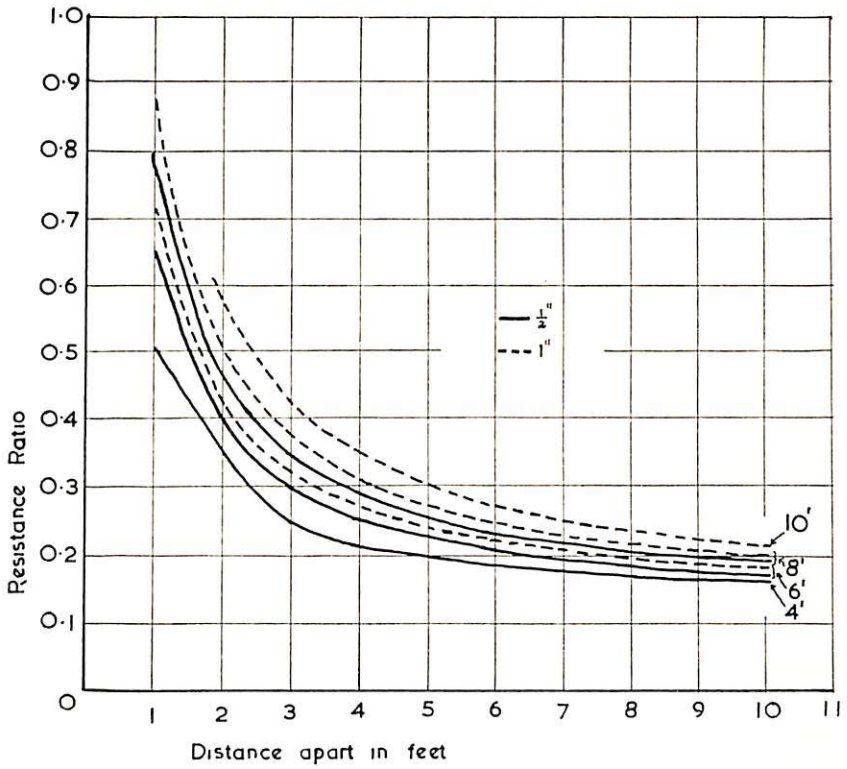


Fig. 7.8. *Eight rods in hollow square.*

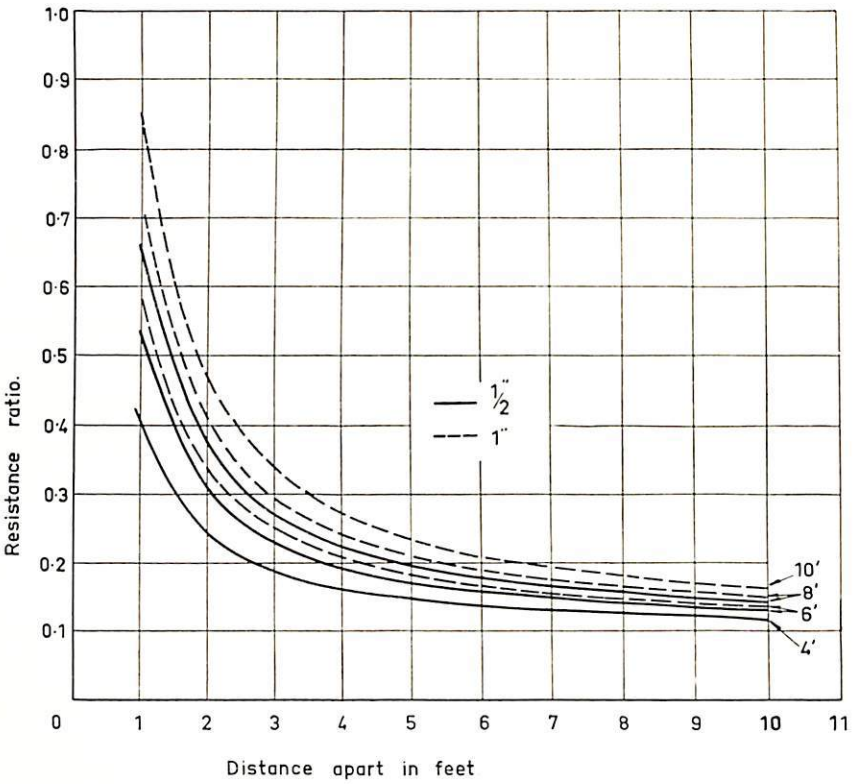


Fig. 7.9. Twelve rods in hollow square.

At first sight the curves might suggest that a lower resistance is obtained with a 4 ft rod, than with a 10 ft rod. The resistance ratio for a given spacing is certainly less, but it must be remembered that the actual resistance of a single 4 ft rod is much higher than that of a single 10 ft rod. The curves have been redrawn in terms of actual resistance on the assumption that the resistivity of the soil is 10,000 ohm-cms. These are given in Figs. 7.16–7.23.

From these curves, it is interesting to see what resistance can be obtained in a given area, and what is the most economical number of rods. The curves in Fig. 7.24 illustrate the connection between area, number of rods and resistance for rods, 8 ft long and 1 in diameter. There is obviously an economical maximum number of rods, and a minimum obtainable resistance for a given area. For instance, with an

area of 625 sq. ft, there is no advantage in increasing the number of rods beyond 28 and that a resistance of less than 5 ohms cannot be obtained under these conditions. To obtain a lower resistance it is evidently necessary to increase the area.

Rods Arranged in a Circle

An alternative to the hollow square, is to arrange the rods in a circle. This is illustrated in Fig. 7.25. If there are n rods in a circle of radius R feet, it can be shown by an analysis similar to those above that

$$\frac{\text{Resistance of circle of rods}}{\text{Resistance of one rod}} = \frac{1 + 0{\cdot}5\alpha + \alpha \sum_{s=1}^{s=(n/2-1)} \operatorname{cosec} \frac{S\pi}{n}}{n} \qquad (7.9)$$

where $\alpha = r/R$.

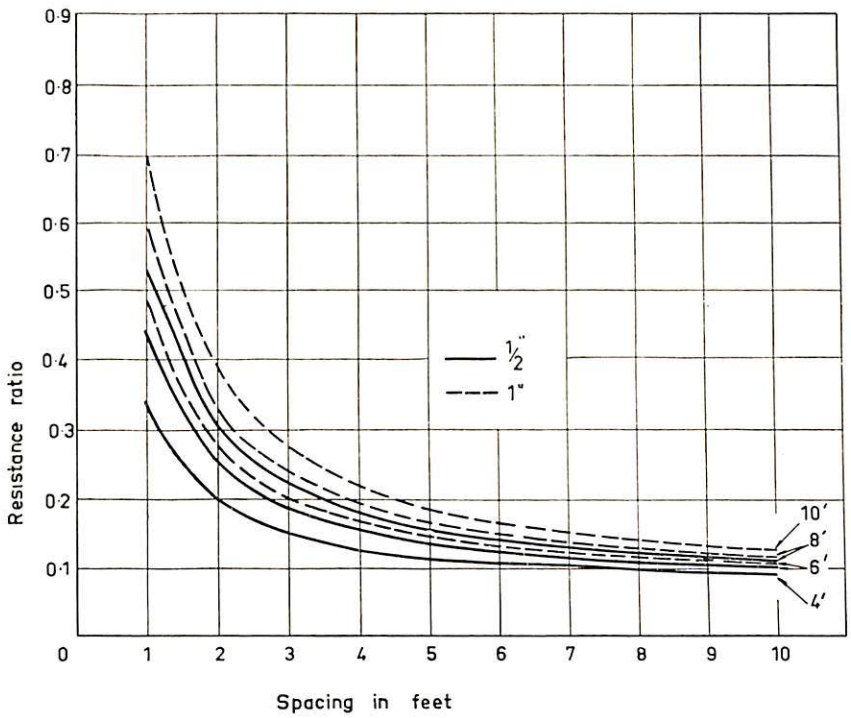


Fig. 7.10. *Sixteen rods in hollow square.*

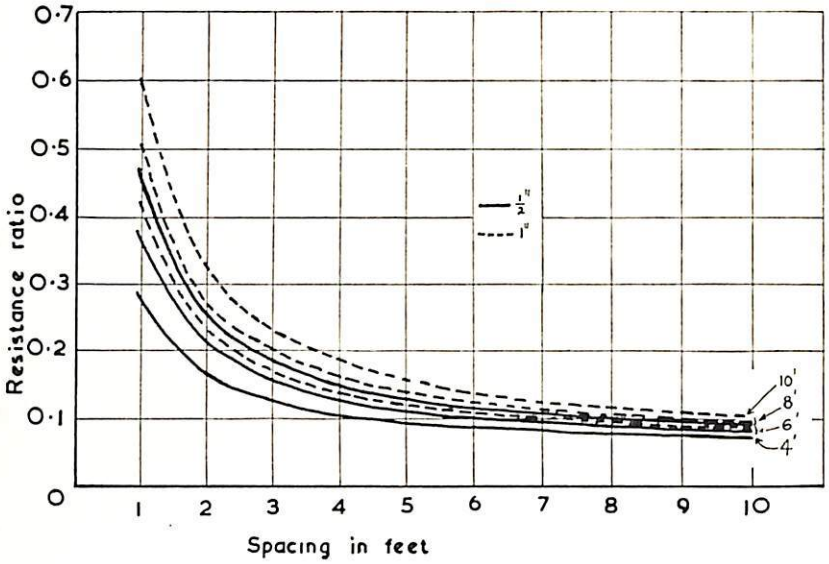


Fig. 7.11. Twenty rods in hollow square.

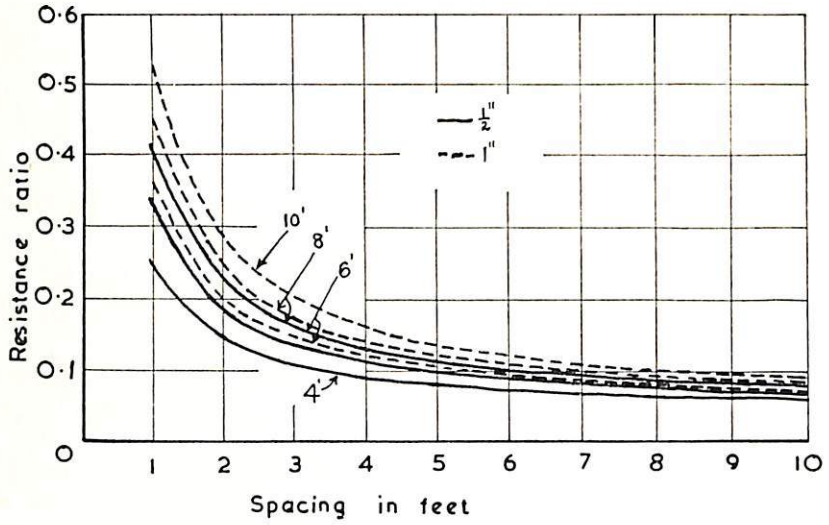


Fig. 7.12. Twenty-four rods in hollow square.

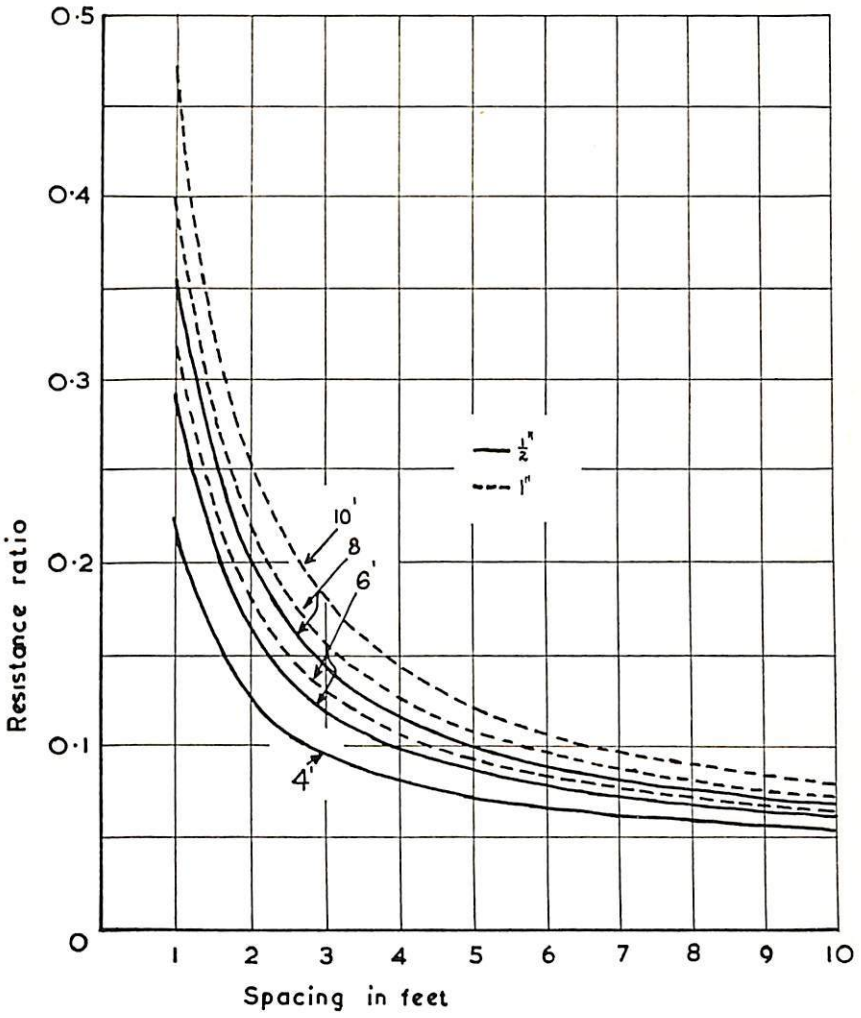


Fig. 7.13. Twenty-eight rods in hollow square.

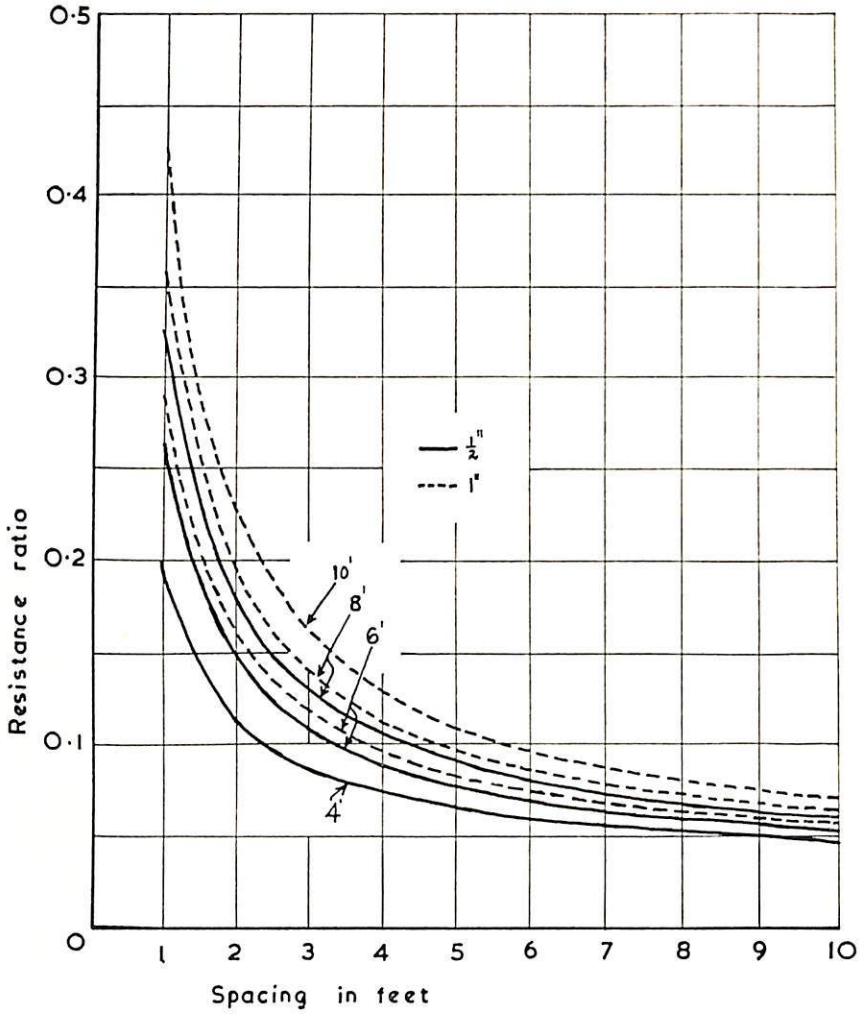


Fig. 7.14. *Thirty-two rods in hollow square.*

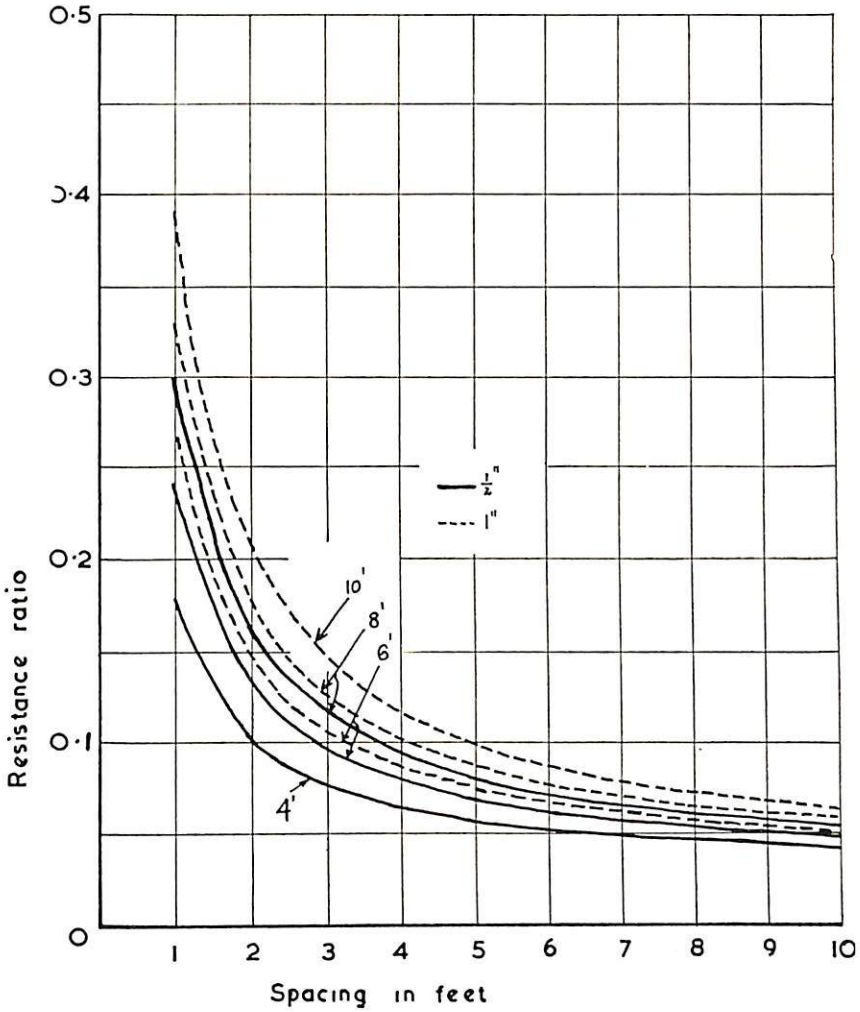


Fig. 7.15. Thirty-six rods in hollow square.

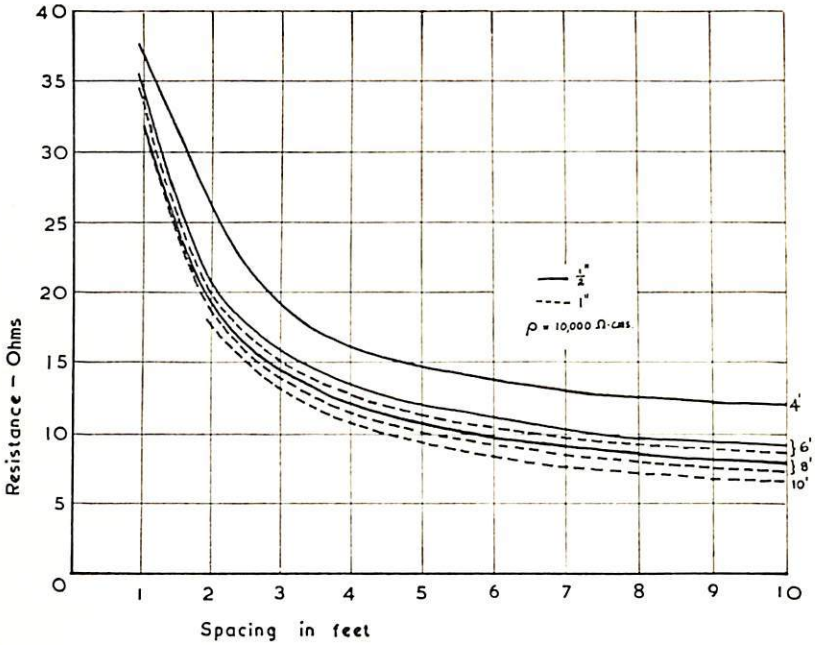


Fig. 7.16. Eight rods in hollow square.

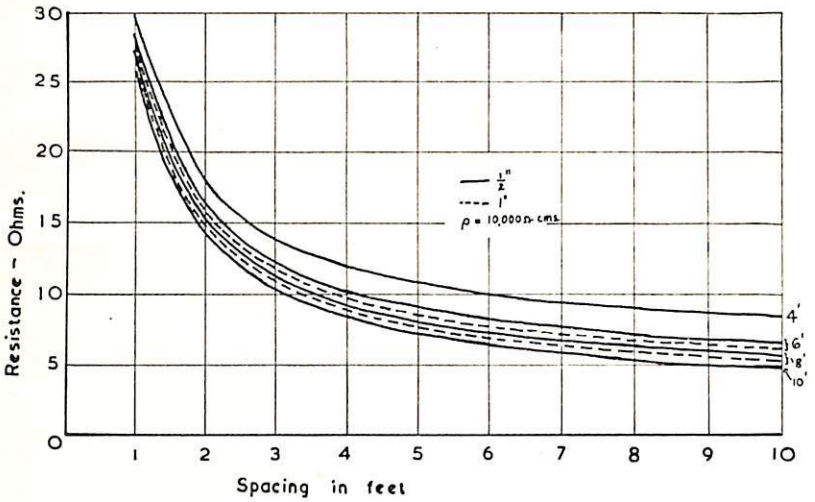


Fig. 7.17. Twelve rods in hollow square.

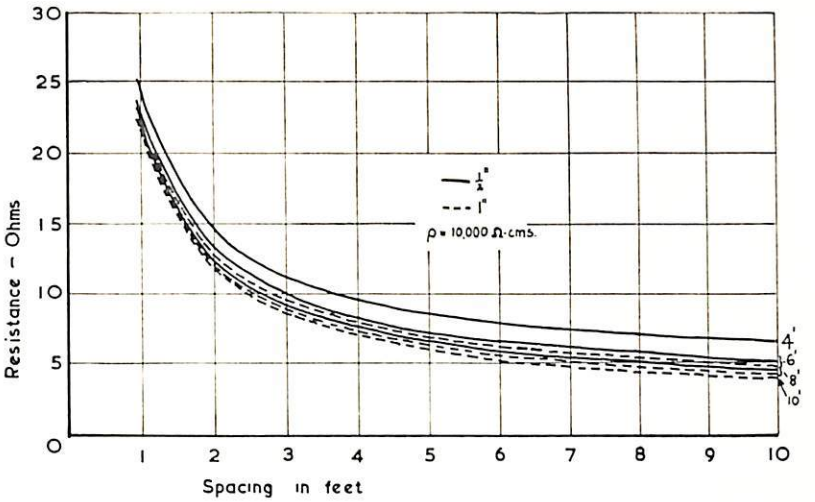


Fig. 7.18. *Sixteen rods in hollow square.*

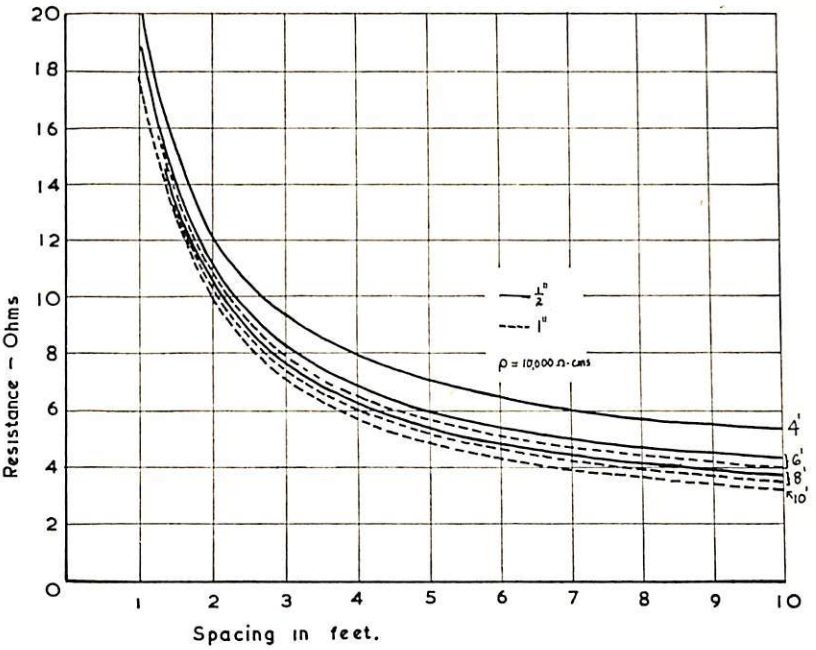


Fig. 7.19. *Twenty rods in hollow square.*

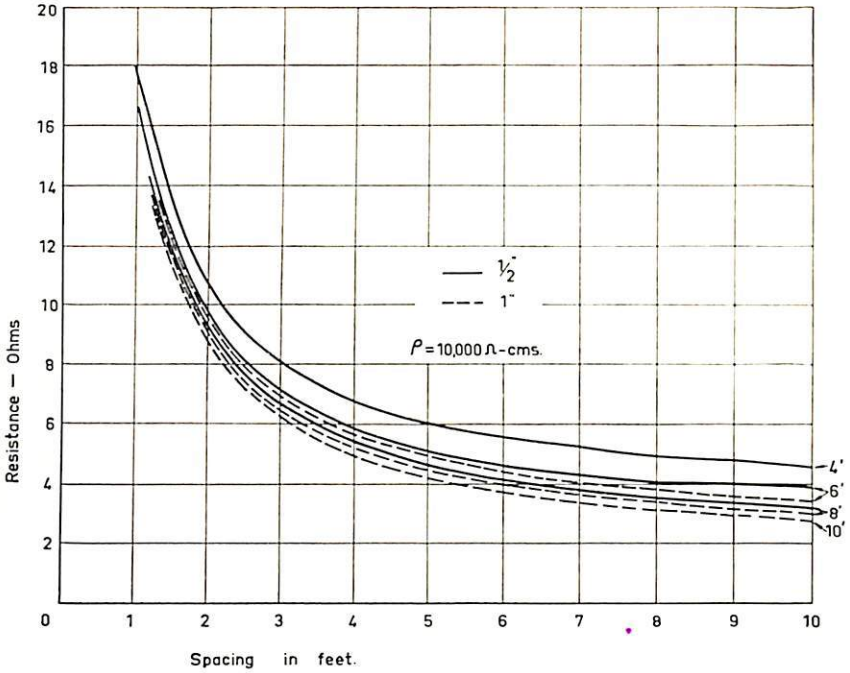


Fig. 7.20. Twenty-four rods in hollow square.

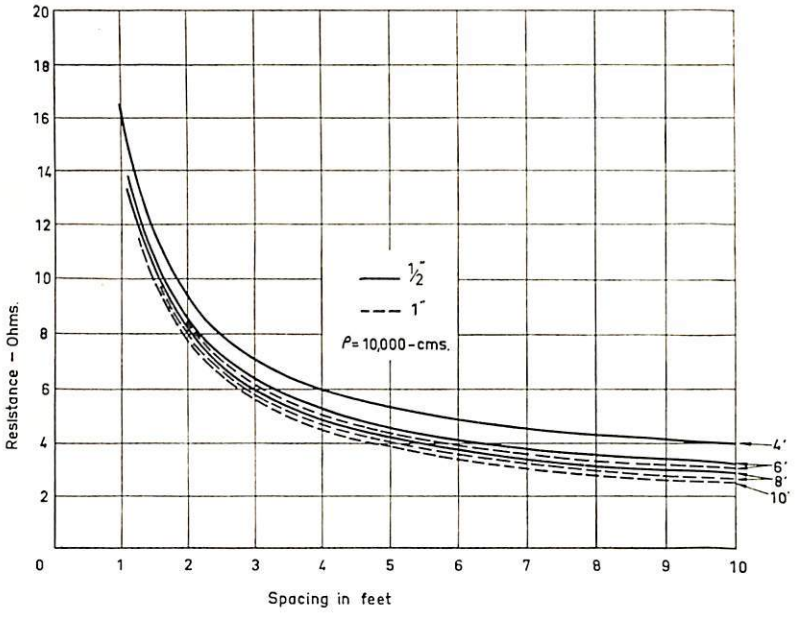


Fig. 7.21. Twenty-eight rods in hollow square.

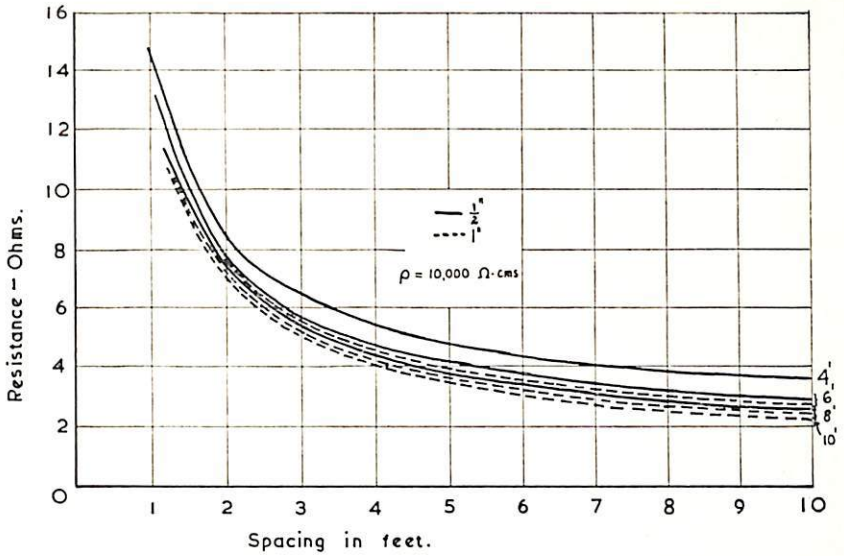


Fig. 7.22. Thirty-two rods in a hollow square.

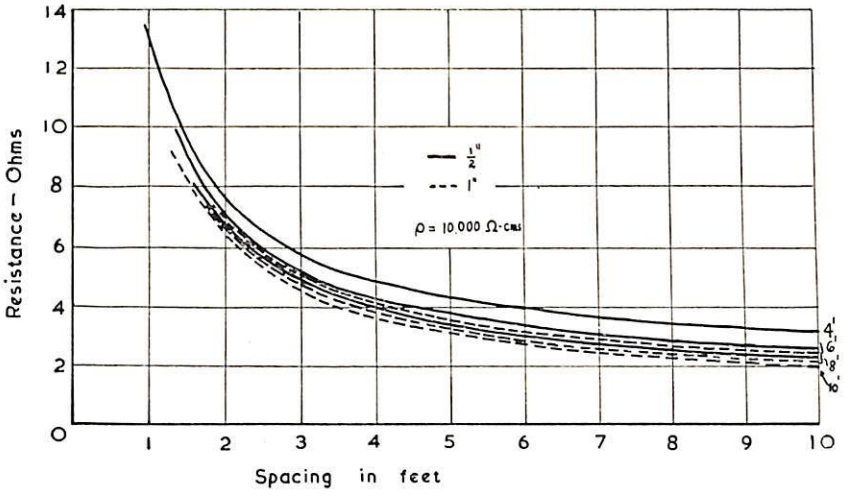


Fig. 7.23. Thirty-six rods in a hollow square.

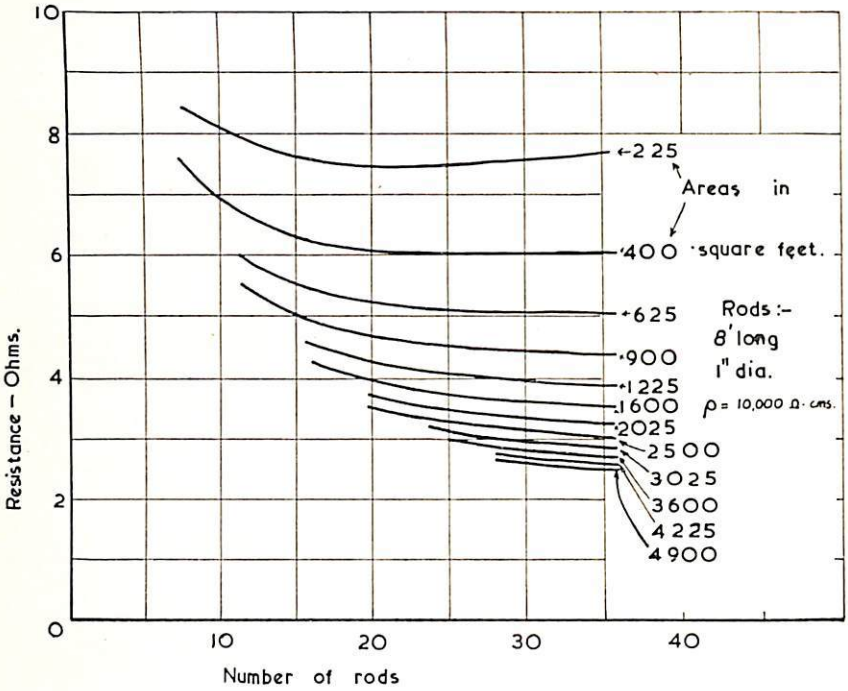


Fig. 7.24. Resistance of rods in parallel in a hollow square of given area.

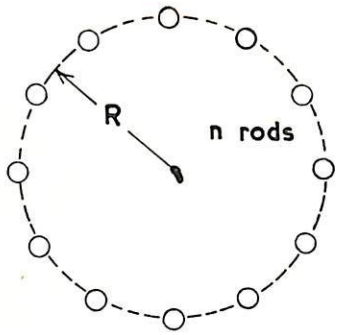


Fig. 7.25. Rods in a circle.

Curves calculated from equation (7.9) are given in Figs. 7.26–7.29, for various radii of circle and sizes of rods. These curves are redrawn in Figs. 7.30–7.33 in terms of actual resistance.

Rods Arranged in a Solid Square

It might be considered that a further substantial reduction in resistance could be obtained by putting additional rods inside the square of Fig. 7.7. A solid square array would be such as that shown in Fig. 7.34. In this case, if there are n rods in each line of the square the total number $N = n^2$. An analysis of this case gives the following relationship:

$$\frac{\text{Resistance of rods in parallel}}{\text{Resistance of single rod}} = \frac{1 + k\alpha}{N} \qquad (7.10)$$

where k has the values given in Table 7.2.

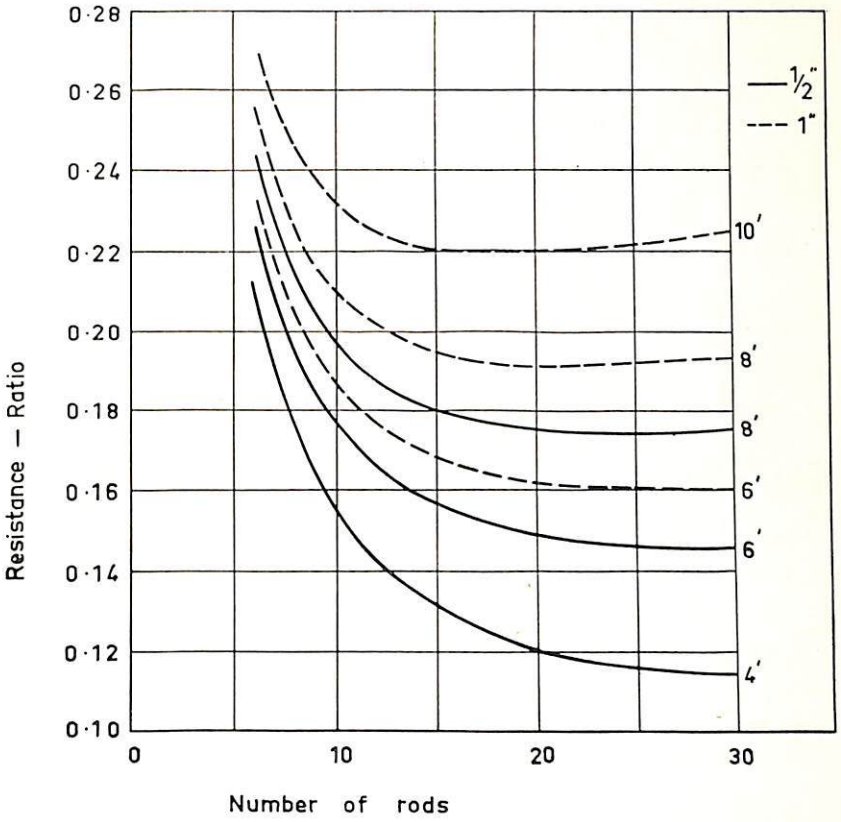


Fig. 7.26. Rods in a circle 10 *ft radius.*

TABLE 7.2

Number of rods in side of square	*Total number of rods*	*Value of k*
2	4	2·7071
3	9	5·8917
4	16	8·5545
5	25	11·4371
6	36	14·0650
7	49	16·8933
8	64	19·5003
9	81	22·3069
10	100	24·9587

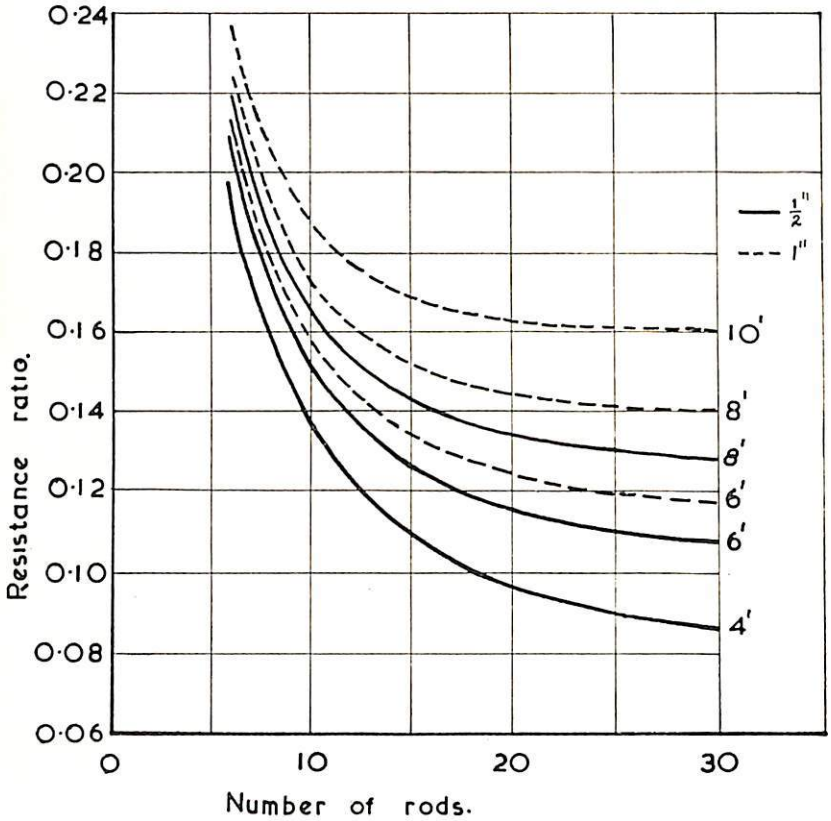


Fig. 7.27. Rods in a circle 15 ft radius.

Curves calculated from equation (7.10) are given in Figs. 7.35–7.43. In terms of actual resistance, assuming a resistivity of 10,000 ohm-cms, these curves are redrawn in Figs. 7.44–7.52. Again it is possible to see what resistance can be obtained in a given area with the minimum economical number of rods. The curves in Fig. 7.53 give the connection between area, resistance and number of rods. It is again evident that, for a given area, there is a minimum practical resistance, with a maximum

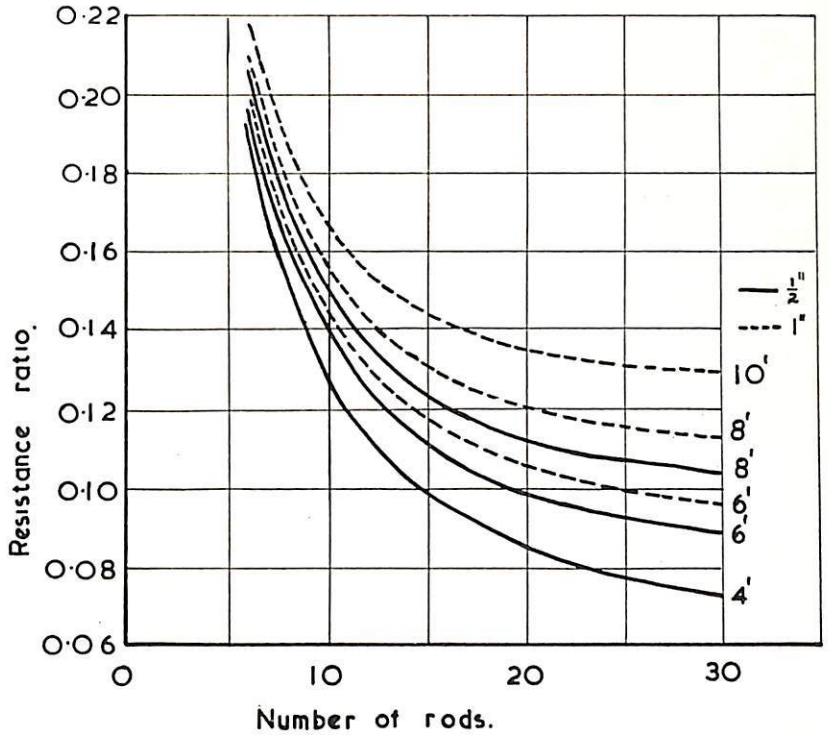


Fig. 7.28. *Rods in a circle* 20 *ft radius.*

economical number of rods. The approximate minimum resistance is plotted against area in Fig. 7.54. As a matter of interest, the figures for a hollow square are also plotted on the same diagram, and it is evident that only a comparatively small reduction in resistance results from filling in the square. This is confirmed by the experimental results described later.

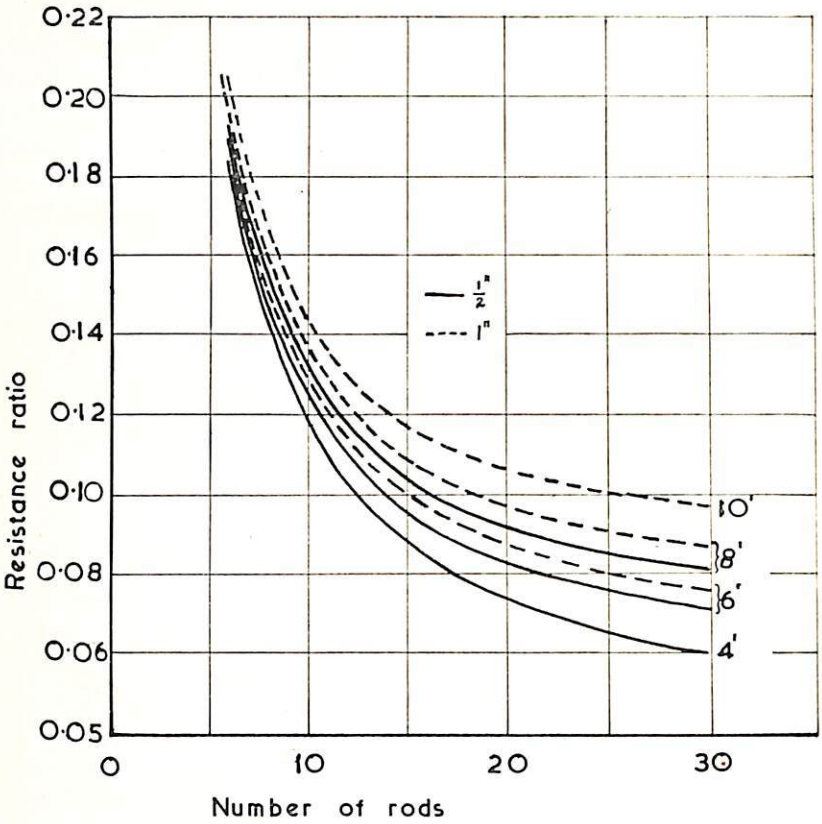


Fig. 7.29. Rods in a circle 30 ft radius.

Experimental Check on Formulae

It was thought that an experimental check on the formulae in this Chapter would be of interest. It is difficult to carry out satisfactory experiments because of the difficulty of obtaining the homogeneous conditions assumed in the theory. By using short rods it was hoped that the amount of non-homogeneity would be lessened, and so all tests were made using rods 12 in long and $\frac{1}{2}$ in diameter.

The first check was of the result of connecting two spikes in parallel as in Fig. 7.1. The measured resistance of rod A was 377 ohms. The results of this experiment are given in Table 7.3.

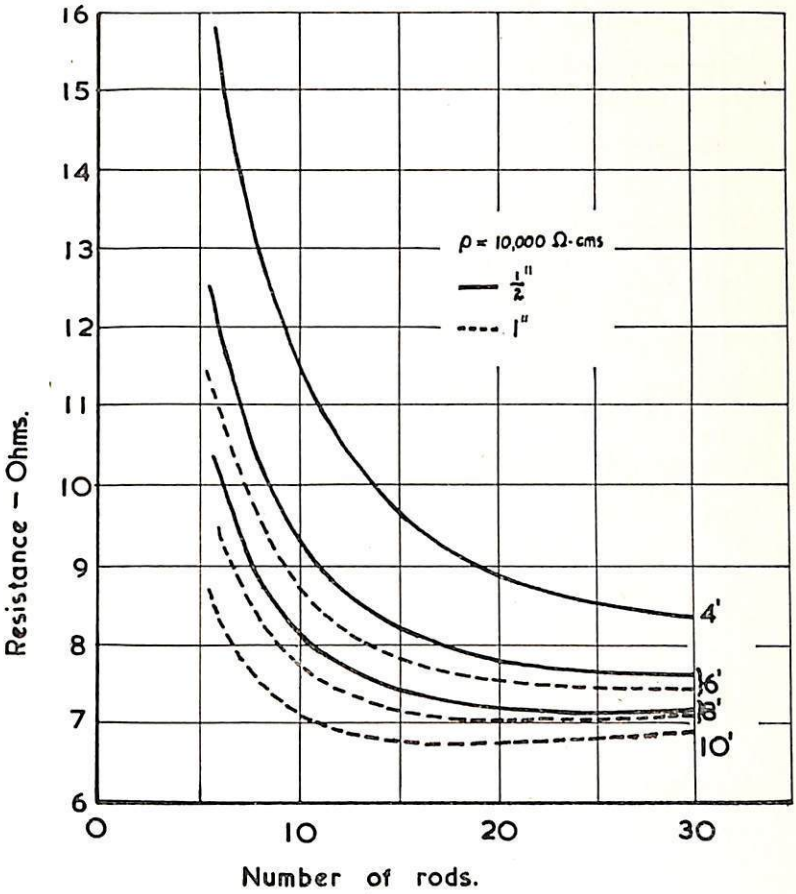


Fig. 7.30. *Rods in a circle* 10 *ft. radius.*

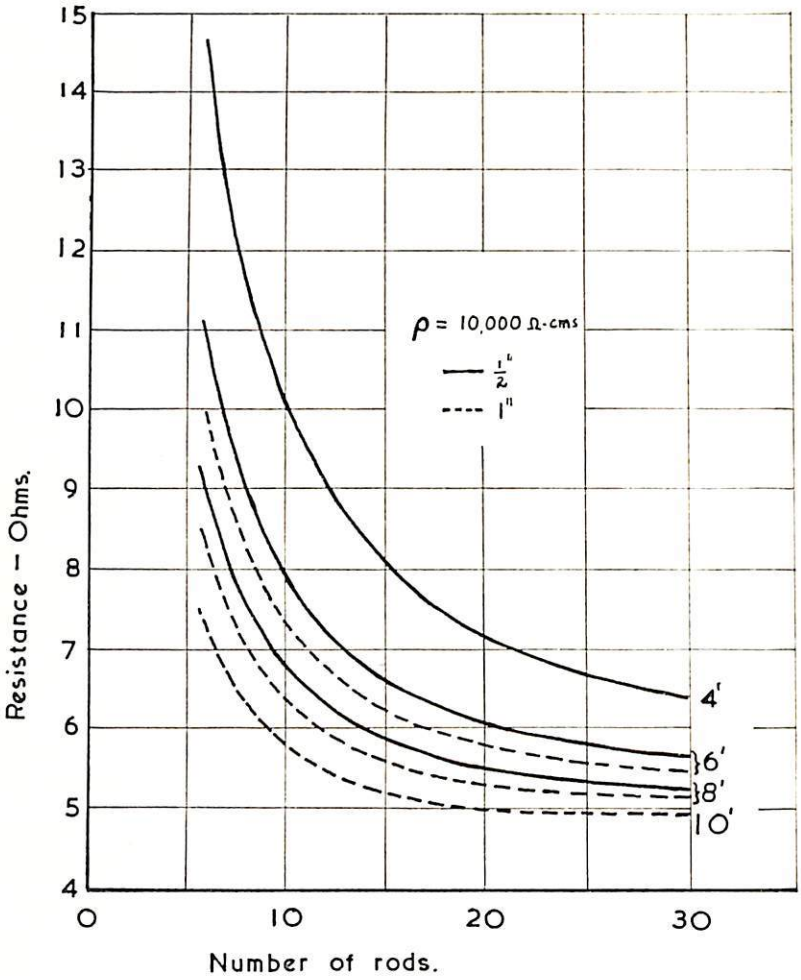


Fig. 7.31. *Rods in a circle* 15 *ft radius.*

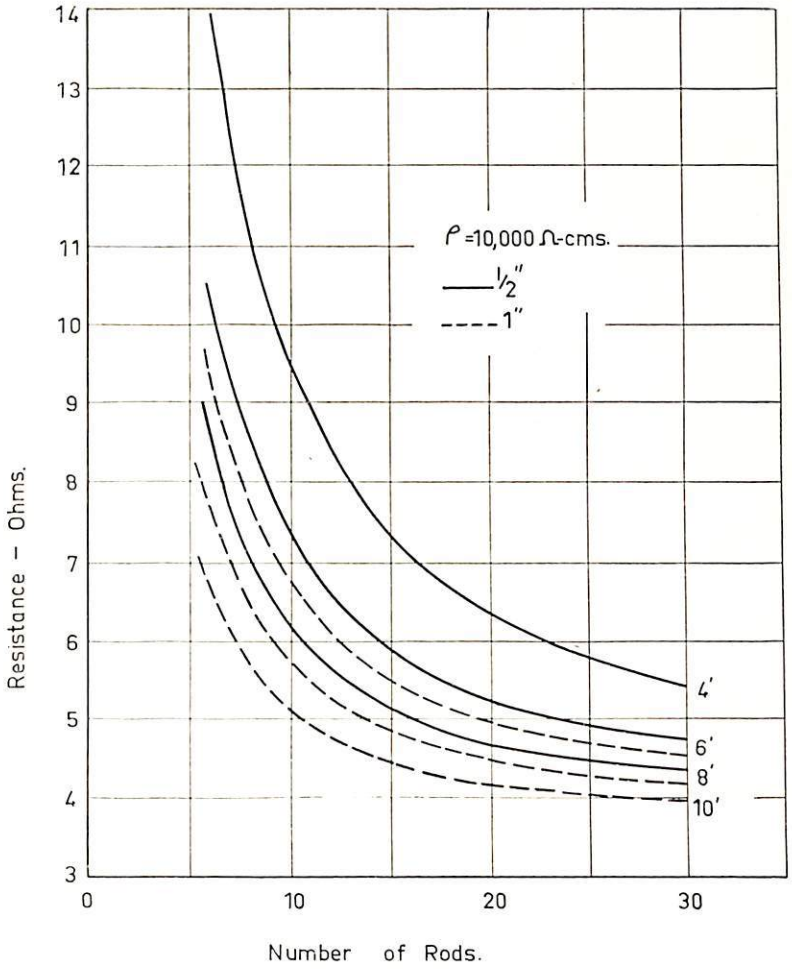
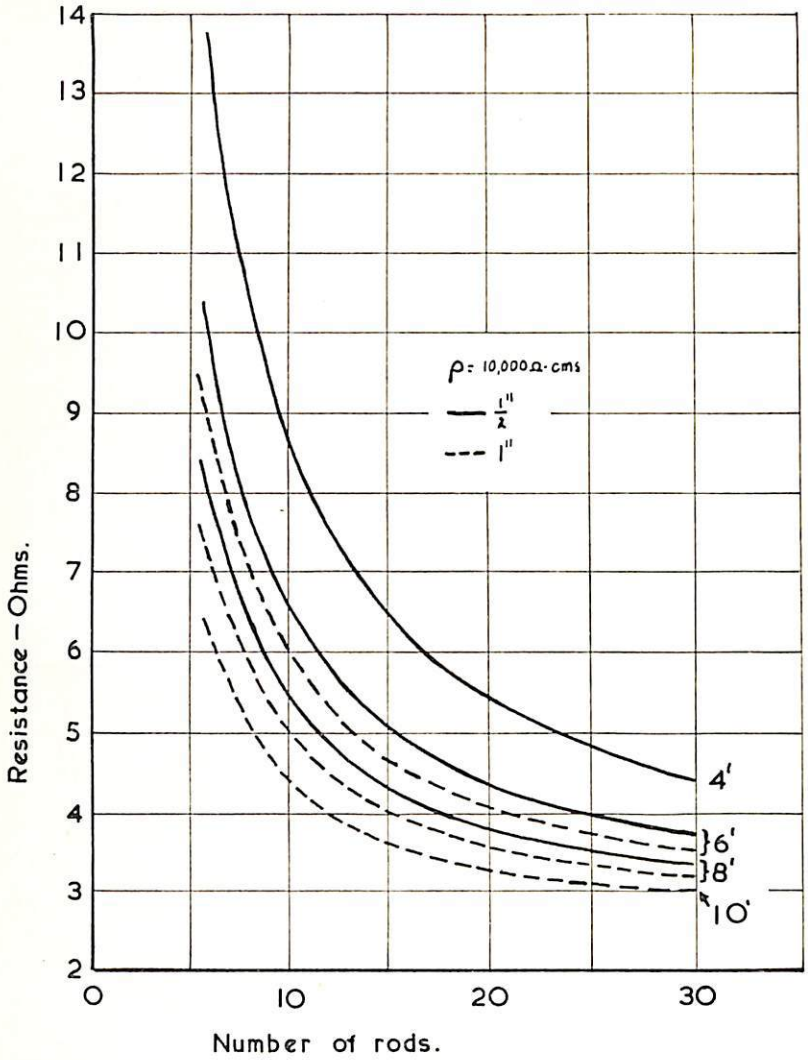


Fig. 7.32. *Rods in a circle* 20 *ft radius.*



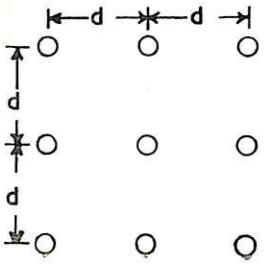


Fig 7.33 (*above*). *Rods in a circle 30 ft radius.*

Fig. 7.34 (*left*). *Rods in parallel in a solid square,*

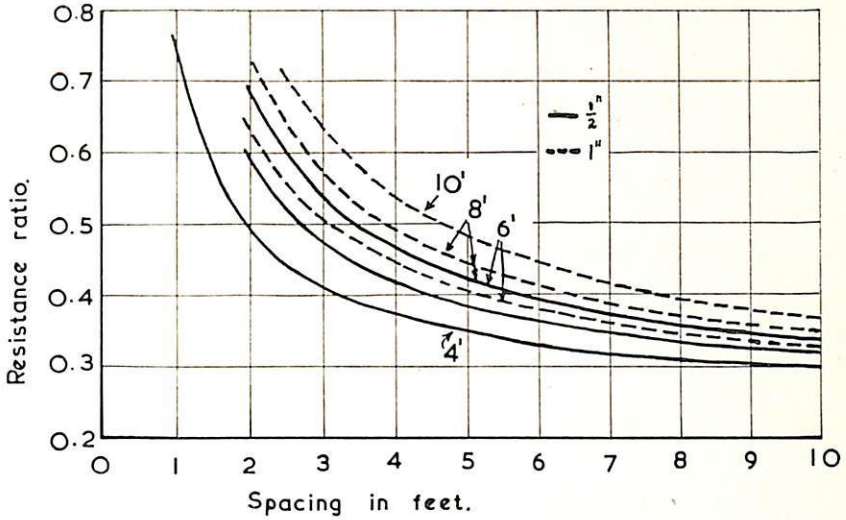


Fig. 7.35. Four rods in a solid square.

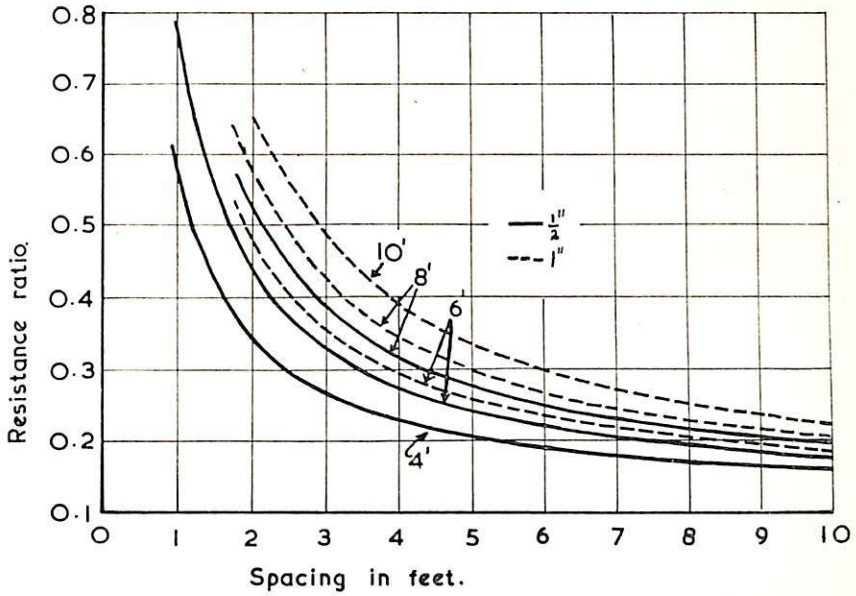


Fig. 7.36. Nine rods in a solid square.

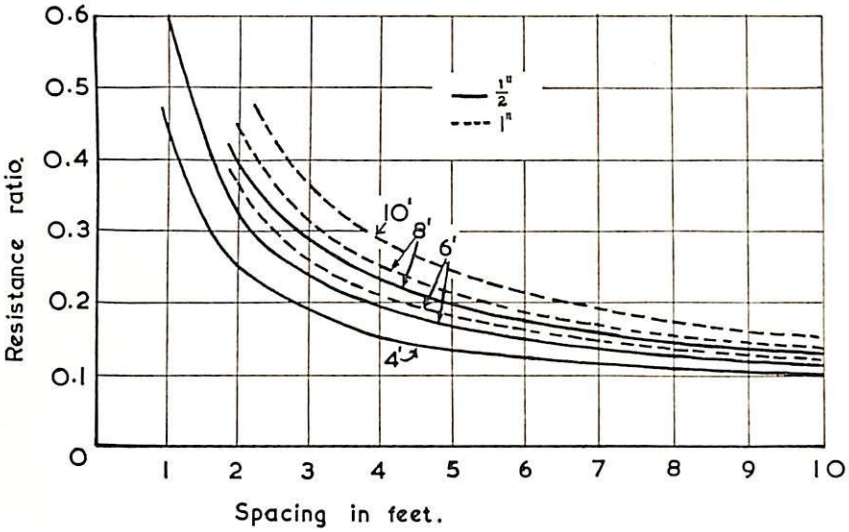


Fig. 7.37. Sixteen rods in a solid square.

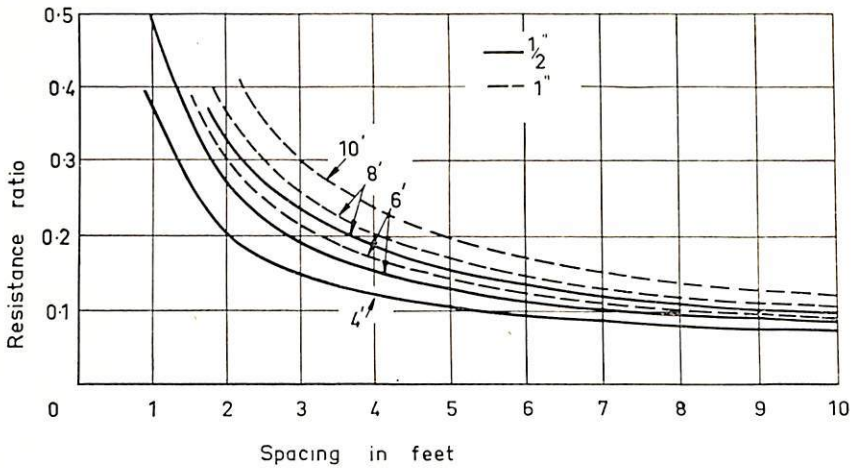


Fig. 7.38. Twenty-five rods in a solid square.

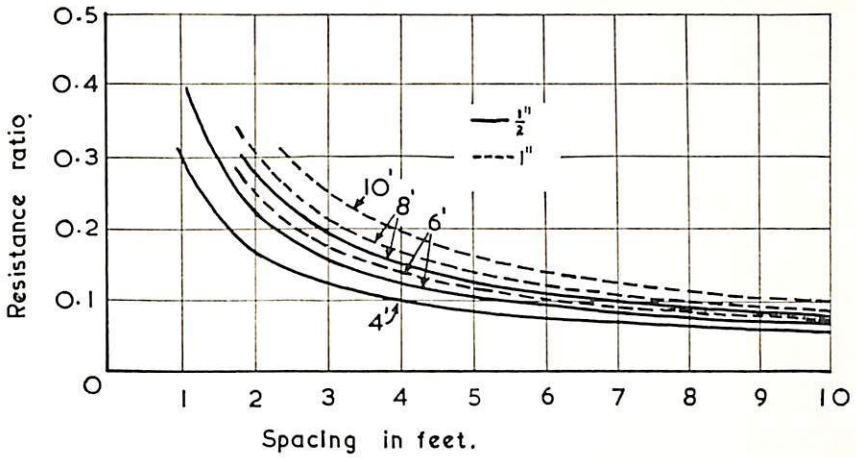


Fig. 7.39. Thirty-six rods in a solid square.

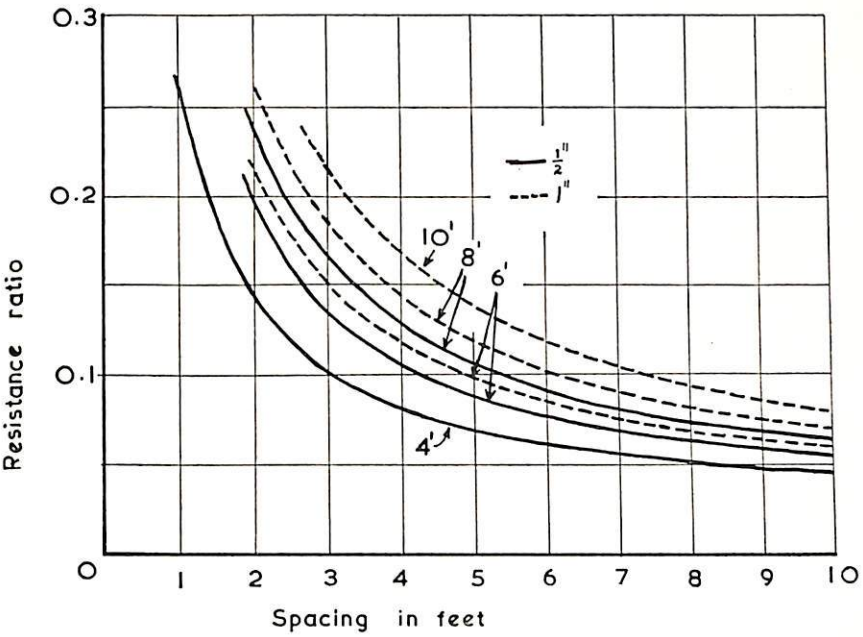


Fig. 7.40. Forty-nine rods in a solid square.

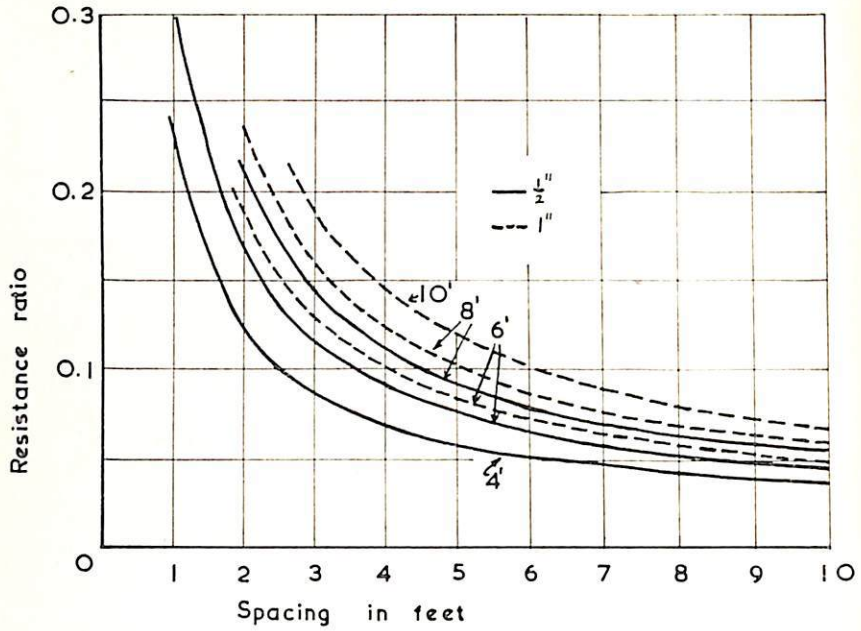


Fig. 7.41. *Sixty-four rods in a solid square.*

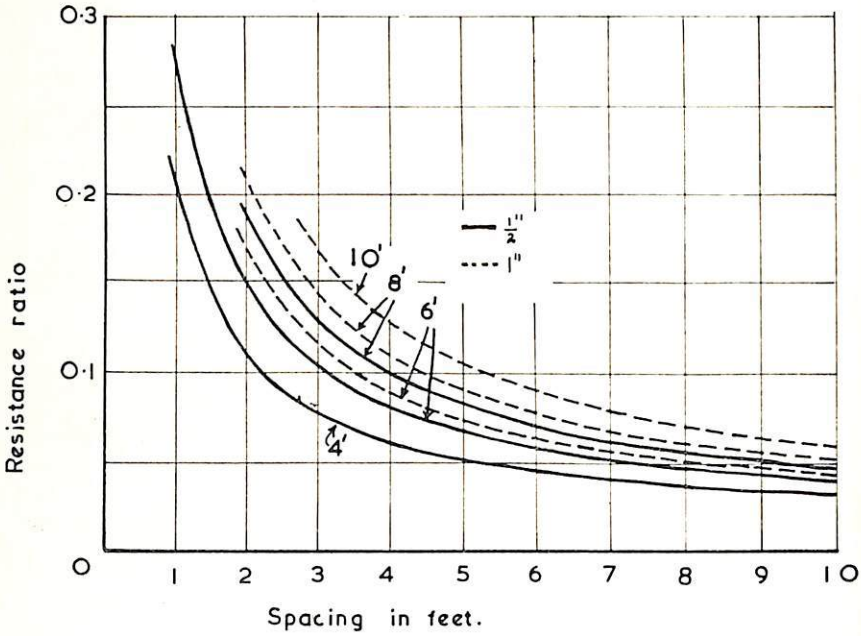


Fig. 7.42. *Eighty-one rods in a solid square.*

TABLE 7.3

d (in)	*Resistance of B*	*Resistance in parallel*	*Average Resistance of one rod*	*Measured Resistance / Resistance of one rod*	α	*Calculated Resistance Ratio*
3·25	360	245	368·4	0·666	0·890	0·945
5·75	325	220	348·6	0·631	0·503	0·752
8·75	355	210	365·6	0·575	0·331	0·666
11·75	348	205	361·8	0·567	0·246	0·623
14·75	310	185	340·2	0·544	0·1961	0·598
17·75	400	208	388·0	0·536	0·163	0·582
23·75	430	210	401·0	0·523	0·122	0·561
35·75	550	225	447·0	0·504	0·081	0·540

It will be seen that the calculated ratio is always higher than the measured value. This is a good point, since it has been pointed out that the method of calculation is only approximate, and it is best for practical reasons that the theory should give pessimistic values.

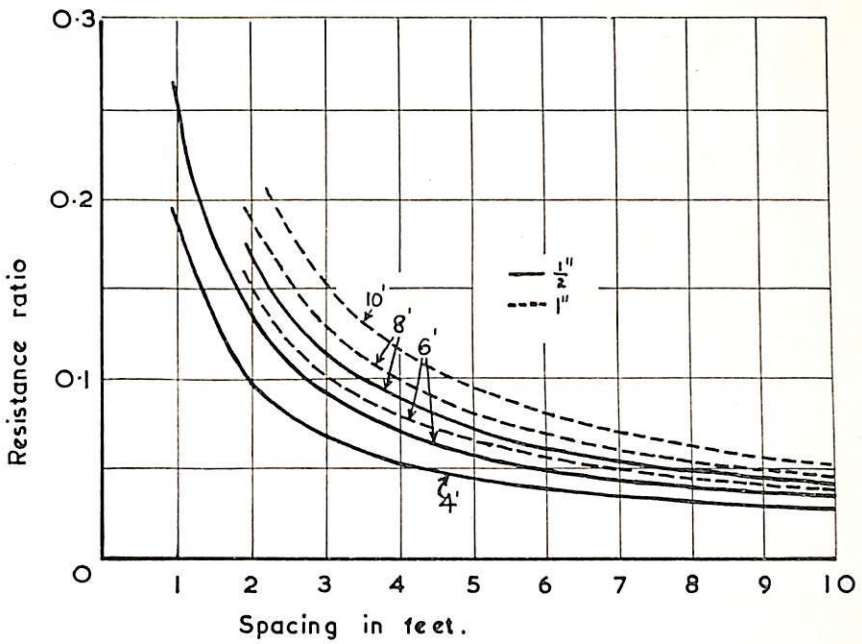


Fig. 7.43. *One hundred rods in a solid square.*

TABLE 7.4

d (*in*)	*A*	*B*	*C*	*Average Resistance of one rod*	*Measured Resistance in parallel*	*Measured Resistance / Resistance of one rod*	α	*Calculated Ratio*
6·0	386	399	345	375	184	0·491	0·482	0·655
9·0	373	377	345	364	165	0·453	0·321	0·547
12·0	364	576	345	406	171	0·421	0·241	0·494
15·0	382	383	345	369	152	0·411	0·192	0·461
18·0	340	404	345	361	145	0·401	0·161	0·431
24·0	384	341	345	356	136	0·392	0·121	0·414

The next test was that of three rods in a triangle. The rods were designated *A*, *B* and *C* and were at the corners of an equilateral triangle of side *d*. The results are given in Table 7.4.

Experiments were then made using a square having a side of 2 ft, all rods still being 12 in long by $\frac{1}{2}$ in diameter. The following are the results.

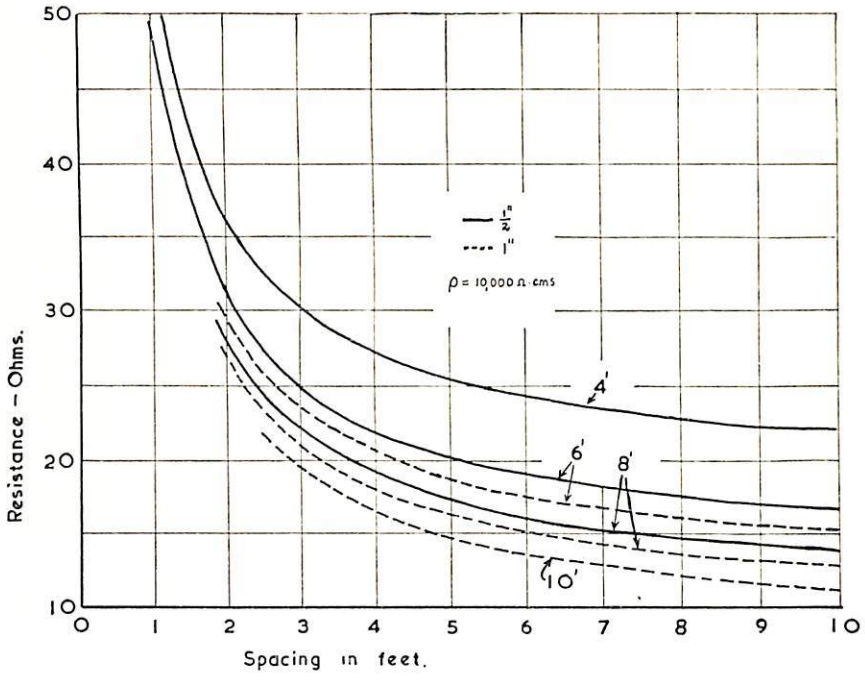


Fig. 7.44. *Four rods in a solid square.*

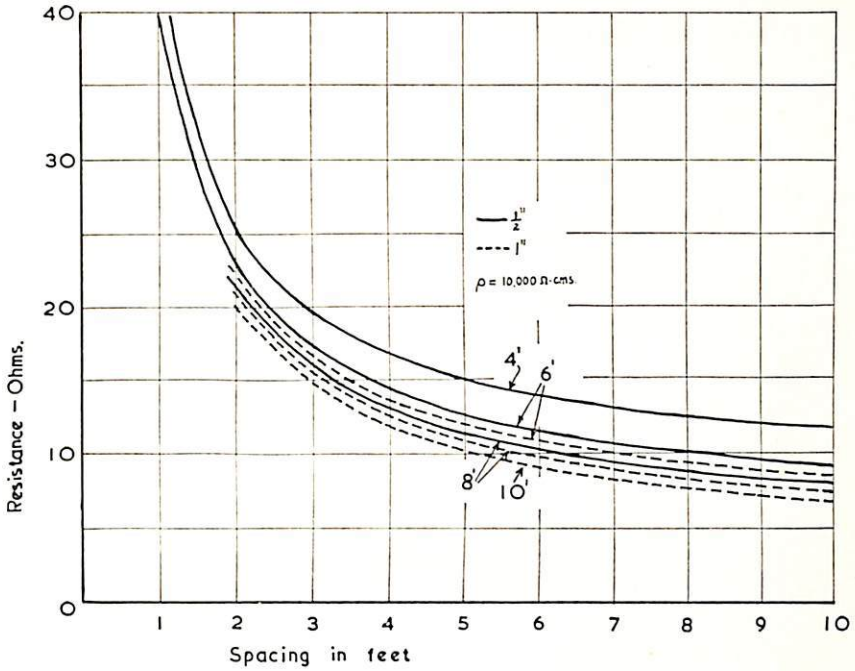


Fig. 7.45. Nine rods in a solid square.

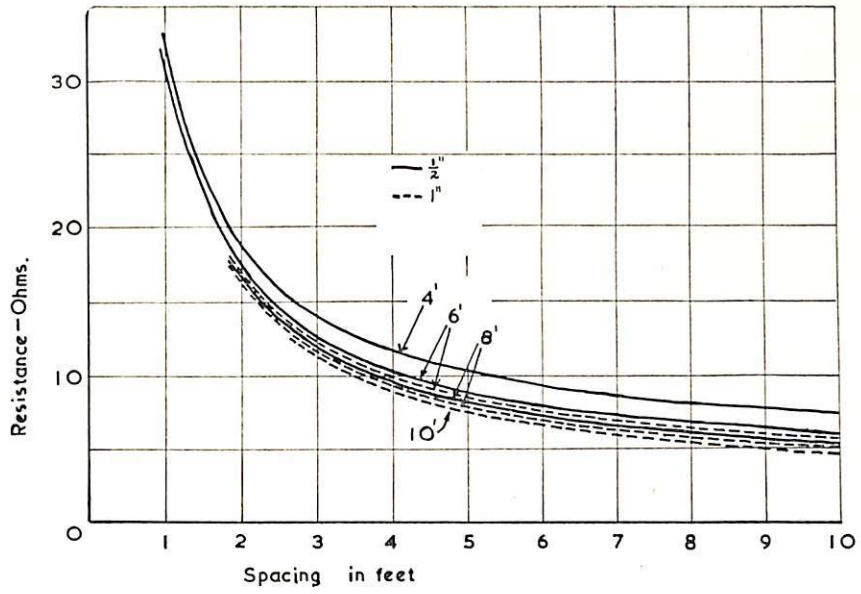


Fig. 7.46. Sixteen rods in a solid square.

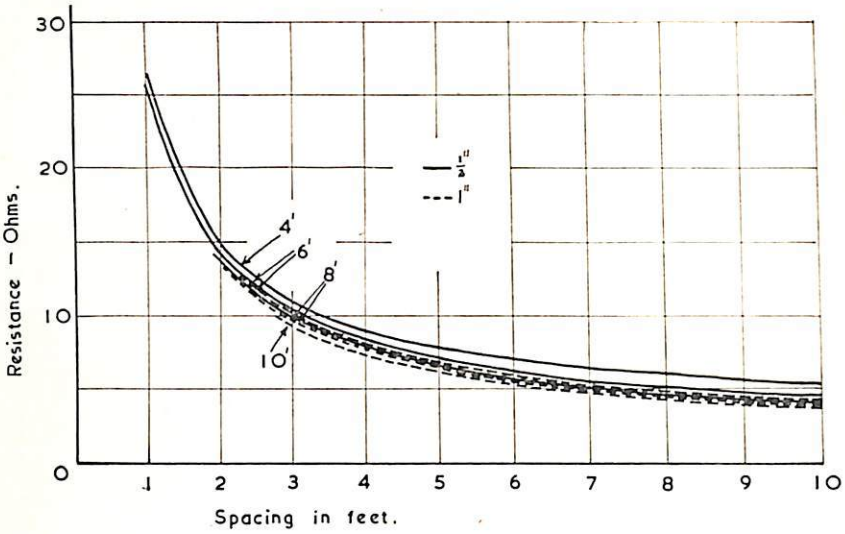


Fig. 7.47. Twenty-five rods in a solid square.

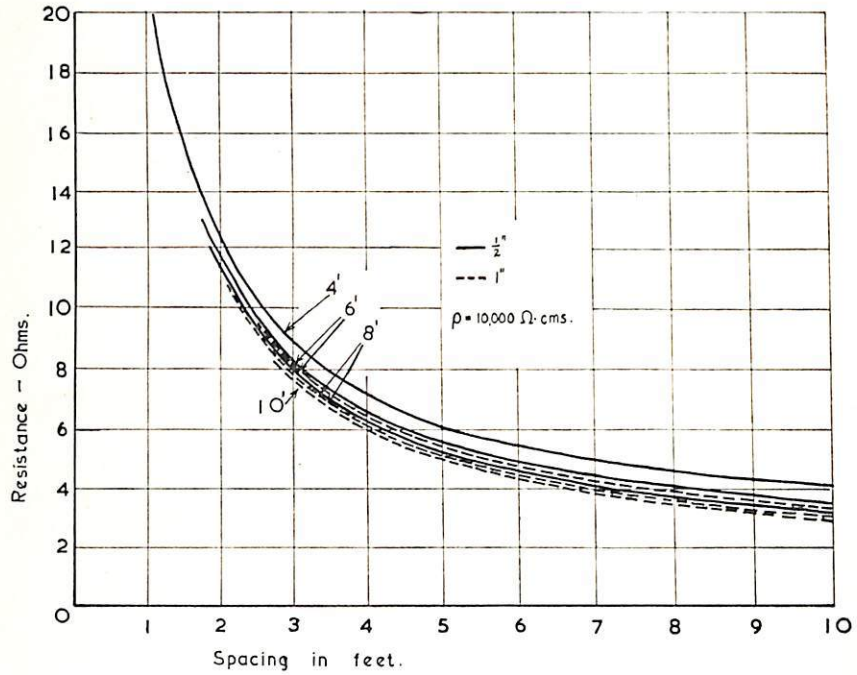


Fig. 7.48. Thirty-six rods in a solid square.

TABLE 7.5

Number of rods	*Arrangement*	*Average resistance of one rod*	*Resistance of rods in parallel*	*Measured Resistance Ratio*	α	*Calculated Ratio*
4	—	358	110·0	0·307	0·121	0·343
8	Hollow square	338	75·2	0·222	0·241	0·253
9	Solid square	337	72·7	0·216	0·241	0·269
12	Hollow square	319	65·5	0·205	0·362	0·246
16	Solid square	318	64·4	0·202	0·362	0·256
16	Hollow square	352	64·4	0·183	0·482	0·244
25	Solid square	332	60·2	0·181	0·482	0·261
20	Hollow square	329	60·5	0·184	0·603	0·245
36	Solid square	307	56·5	0·184	0·603	0·266
24	Hollow square	309	57·6	0·187	0·723	0·247
49	Solid square	225	49·2	0·219	0·723	0·269

It will be noted that the calculated value of the resistance ratio is greater than the actual measured value as it was with the two electrodes in parallel, and as already stated this pessimistic result is an advantage.

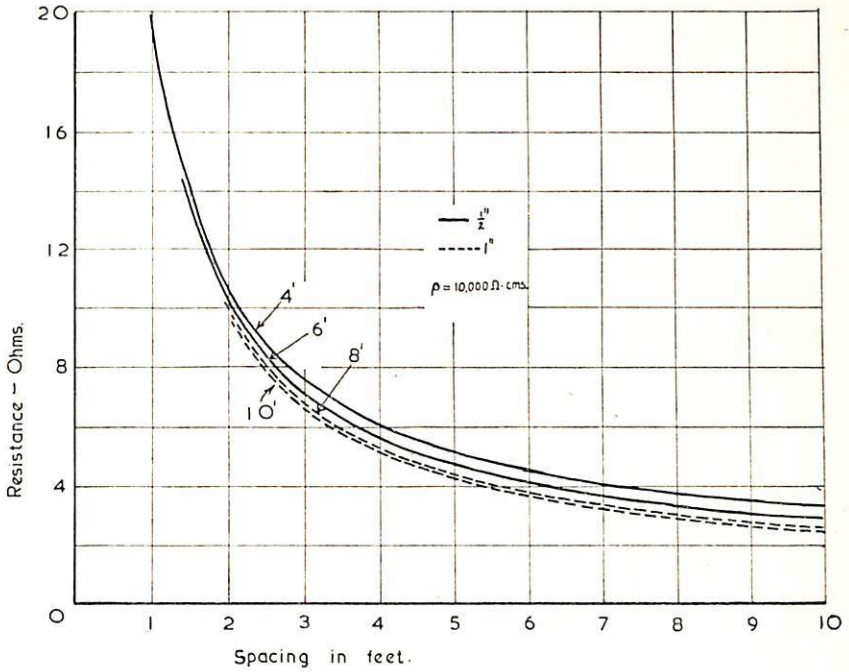


Fig. 7.49. Forty-nine rods in a solid square.

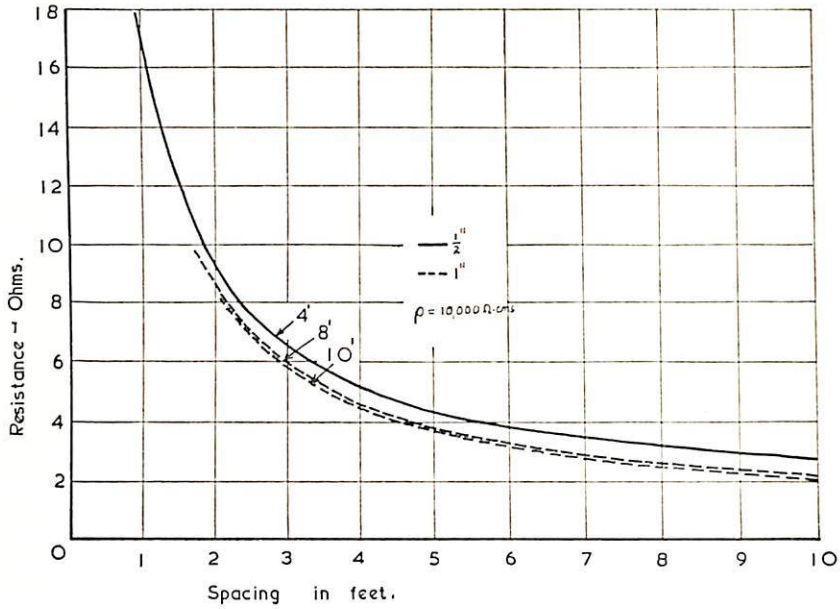


Fig. 7.50 Sixty-four rods in a solid square.

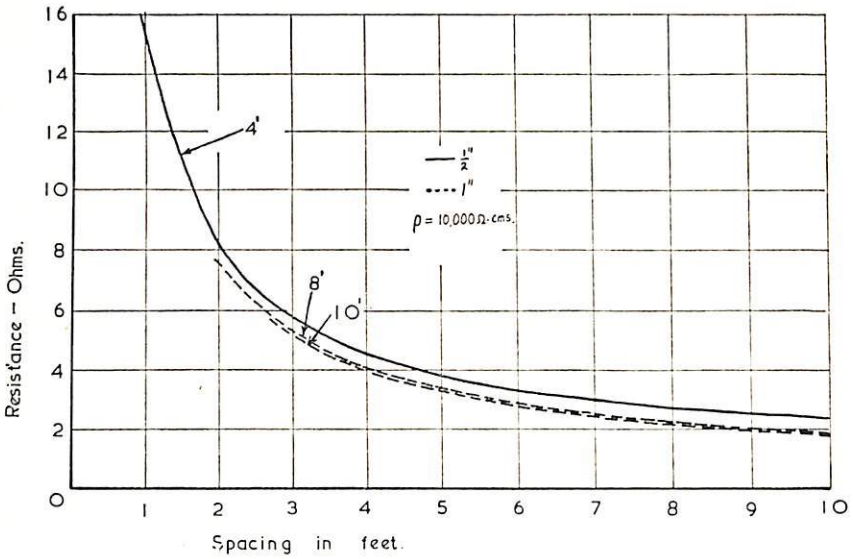


Fig. 7.51. Eighty-one rods in a solid square.

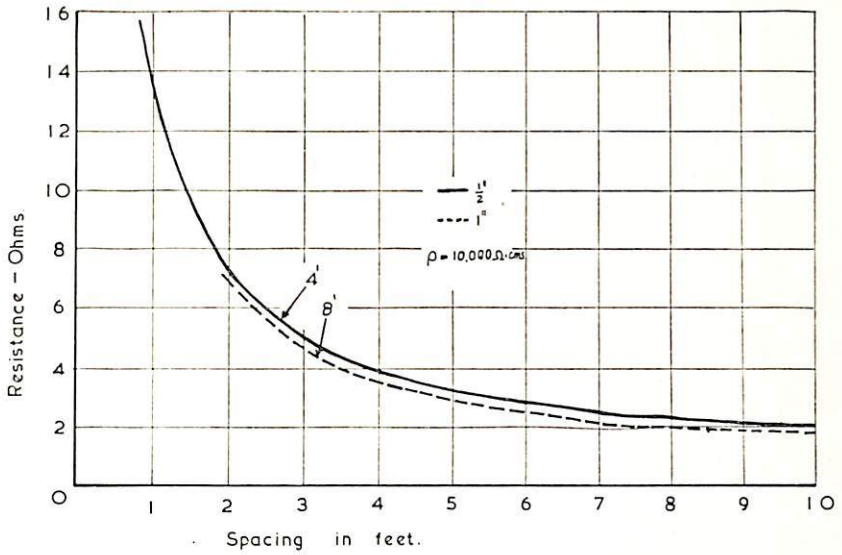


Fig. 7.52. One hundred rods in a solid square.

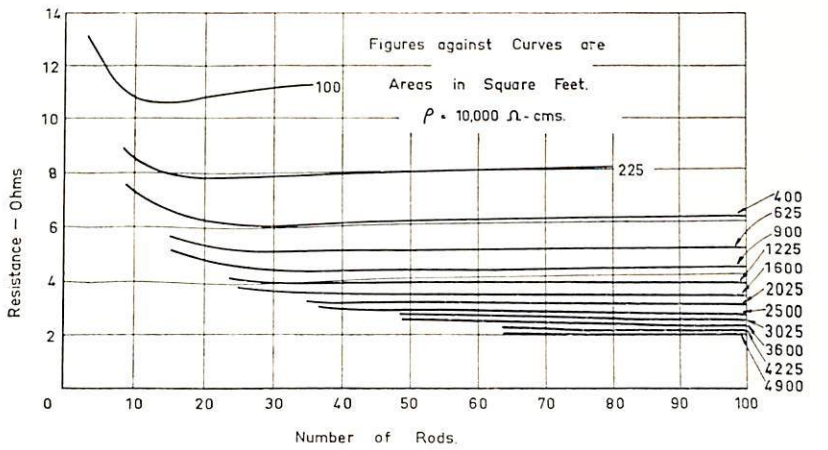


Fig. 7.53. Resistance of paralleled rods in a square. Rods 8 ft long 1 in diameter.

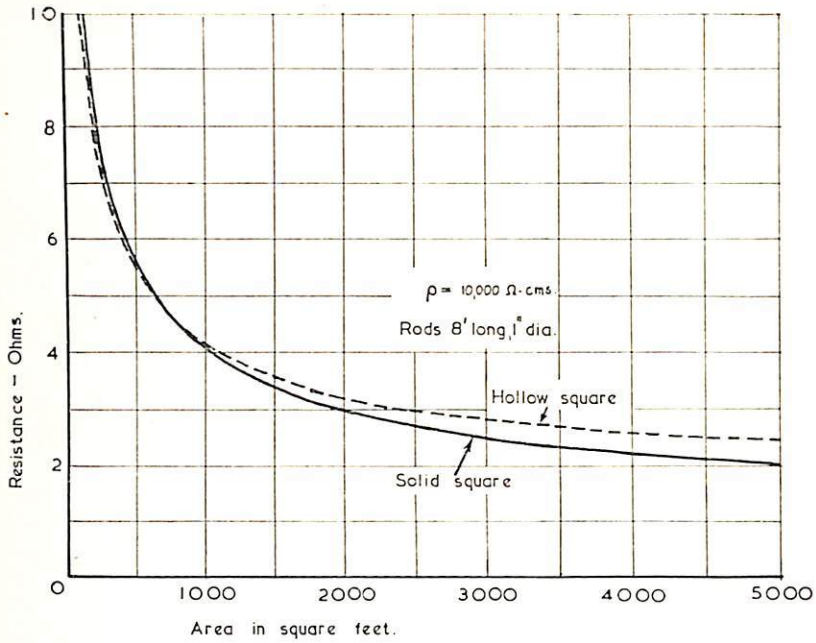


Fig. 7.54. Minimum resistance obtainable with paralleled rods in a square of given area.

Practical Applications

The formulae and curves in this Chapter can be used in two ways. If it is desired to design a multiple-rod electrode, then first the resistivity of the soil can be measured and the curves giving the resistance ratio can be used in conjunction with the calculated resistance of a single rod. This may not be too accurate as it is difficult to determine the true resistivity due to non-homogeneities. This is dealt with in a later Chapter.

The second way is to drive a test rod, measure its resistance, compare this with the theoretical value and, from this, determine the combined resistance for a given array of rods. One point to note in connection with the calculated values given in this Chapter, is that they may be 5–25 per cent higher than the true value.

8

BURIED WIRE, STRIP AND PLATE ELECTRODES

In the previous two chapters the electrode consisted of one or more driven rods. The driven rod is a convenient and economical form of electrode but there are frequently cases where it is not practicable for various reasons. For instance, there may be a layer of rock a few feet below the surface and in cases like this it is necessary to use some other form. In such cases buried wires and strips are used extensively and are also quite often used in place of rods despite the cost of excavation and back-filling after the installation of the wire or strip.

In the past, earth plates were used to a large extent but they are costly to install, and, in many cases, an equivalent electrode could readily be provided by the use of driven rods.

Single Buried Horizontal Wire

First, consider the case of a single horizontal wire, length $2L$, radius a and buried at a depth $S/2$, as shown in Fig. 8.1(a). As before, the resistance to earth can be determined by first calculating the electrostatic capacity. In order to take account of the effect of the surface of the earth, i.e. the depth of burial, it is necessary to consider, in addition to the buried wire itself, its image above the surface of the earth, as in Fig. 8.1(b). Again the capacity can be calculated by assuming a uniform

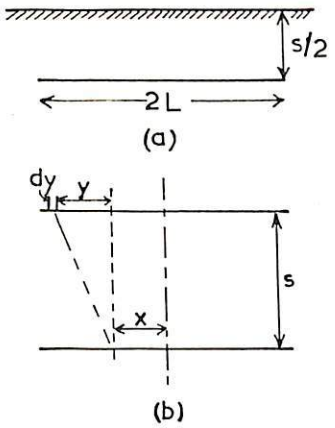


Fig. 8.1. *Buried horizontal wire.*

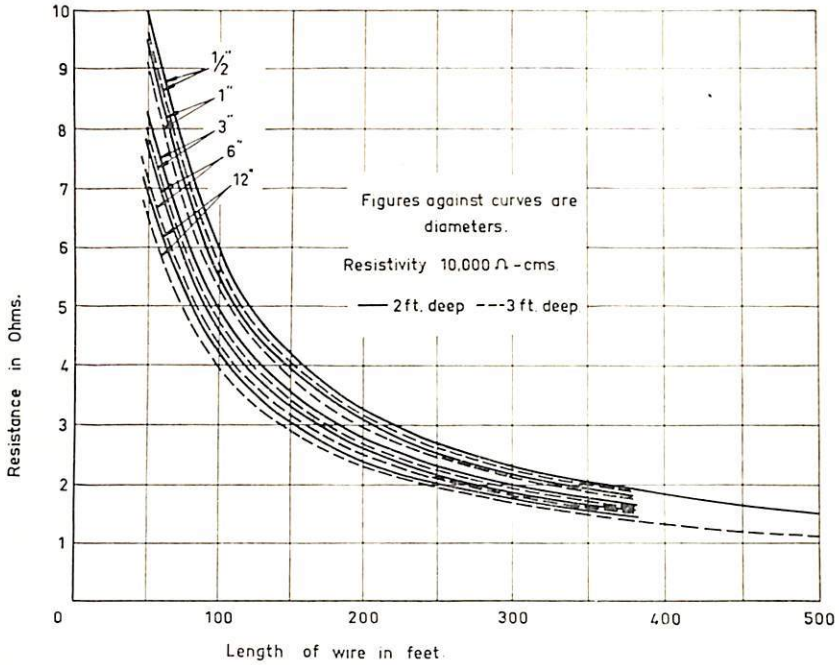


Fig. 8.2. *Resistance of buried horizontal wires.*

distribution of charge, giving an approximate formula, accurate enough for practical purposes.

A uniform charge q per cm length is assumed to exist on both the wire and its image, and it is necessary to calculate the average potential of either wire due to this charge. The potential of the wire due to its own charge follows from the calculations for the driven rod as

$$V_1 = 2q\left(\log_e \frac{4L}{a} - 1\right) \tag{8.1}$$

The average potential of the wire due to the uniform charge on the image can be calculated as follows. The potential at any point on the wire at a distance x from the centre due to element of charge $q\,\mathrm{d}y$ on the image is

$$\frac{q.\mathrm{d}y}{\sqrt{(s^2 + y^2)}}$$

To obtain the potential at this point due to the whole of the image it is necessary to integrate this expression from $y = O$ to $y = (L - x)$ and

from $y = O$ to $y = (L + x)$. The average potential of the wire due to the uniform charge on the image is obtained by multiplying by dx/L and integrating from O to L. This is

$$2q\left\{\log_e \frac{2L + \sqrt{(S^2 + 4L^2)}}{S} + \frac{S}{2L} - \frac{\sqrt{(S^2 + 4L^2)}}{2L}\right\} \tag{8.2}$$

The total potential is

$$2q\left[\log_e \frac{4L}{a} - 1 + \log_e \frac{2L + \sqrt{(S^2 + 4L^2)}}{S} + \frac{S}{2L} - \frac{\sqrt{(S^2 + 4L^2)}}{2L}\right] \tag{8.3}$$

If this is divided by the total charge $4qL$ on the wires, this gives the value of $1/C$ for the two wires and this in turn leads to the following formula for the resistance

$$R = \frac{\rho}{4\pi L}\left[\log_e \frac{4L}{a} - 1 + \log_e \frac{2L + \sqrt{(S^2 + 4L^2)}}{S} + \frac{S}{2L} - \frac{\sqrt{(S^2 + 4L^2)}}{2L}\right] \tag{8.4}$$

For large values of S/L this becomes

$$R = \frac{\rho}{4\pi L}\left(\log_e \frac{4L}{a} - 1\right) + \frac{\rho}{4\pi S}\left(1 - \frac{L^2}{3S^2} + \frac{2}{5}\cdot\frac{L^4}{S^4} \cdots\right) \tag{8.5}$$

and for small values of S/L

$$R = \frac{\rho}{4\pi L}\left(\log_e \frac{4L}{a} + \log_e \frac{4L}{S} - 2 + \frac{S}{2L} - \frac{S^2}{16L^2} + \frac{S^4}{512L^4}\right) \tag{8.6}$$

Values calculated from these formulae are plotted in the curves of Fig. 8.2. These are calculated for a resistivity of 10,000 ohm-cms, and for any other resistivity ρ, the values from the curves must be multiplied

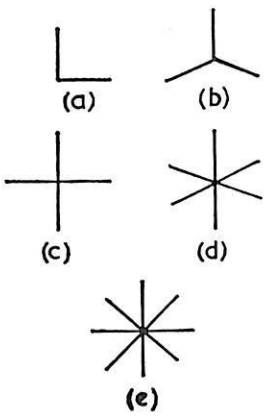


Fig. 8.3. *Arrangements of buried wires.*

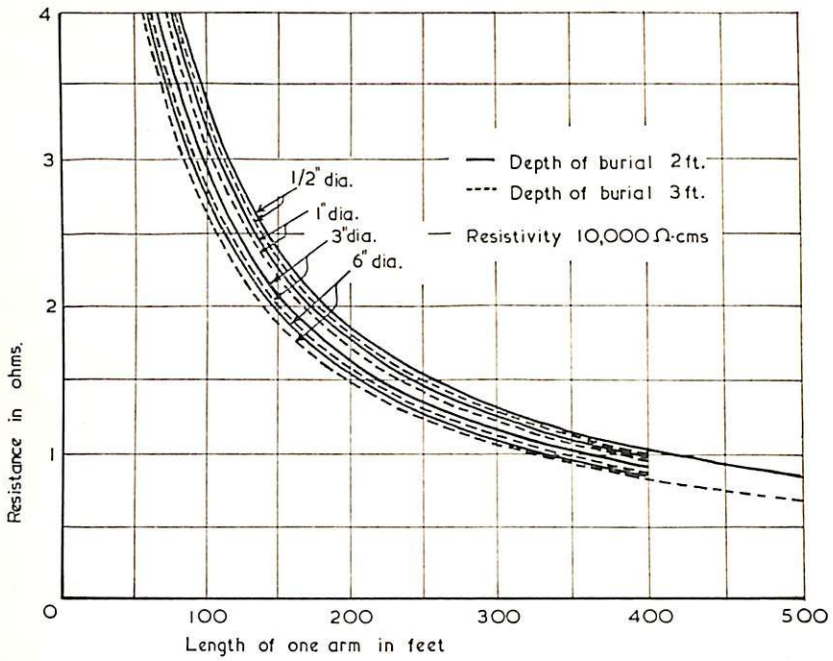


Fig. 8.4. *Resistance of a buried right-angle turn of wire.*

by the ratio $\rho/10{,}000$. These curves indicate the effect of diameter and depth of burial.

Right-angle Turn of Wire

Where space does not permit for sufficient length of wire to be placed in a straight line, it may be possible to arrange the wire in a right-angle turn as in Fig. 8.3(*a*). To calculate the resistance, it is again necessary to consider the image above the surface and a similar calculation to that for the single buried wire leads to the formula:

$$R = \frac{\rho}{4\pi L}\left(\log_e \frac{2L}{a} + \log_e \frac{2L}{S} - 0{\cdot}2373 + 0{\cdot}2146\,\frac{S}{L} + 0{\cdot}1035\,\frac{S^2}{L^2} - 0{\cdot}0424\,\frac{S^4}{L^4}\right) \qquad (8.7)$$

Curves calculated from the formula for various diameters and two depths of burial are given in Fig. 8.4.

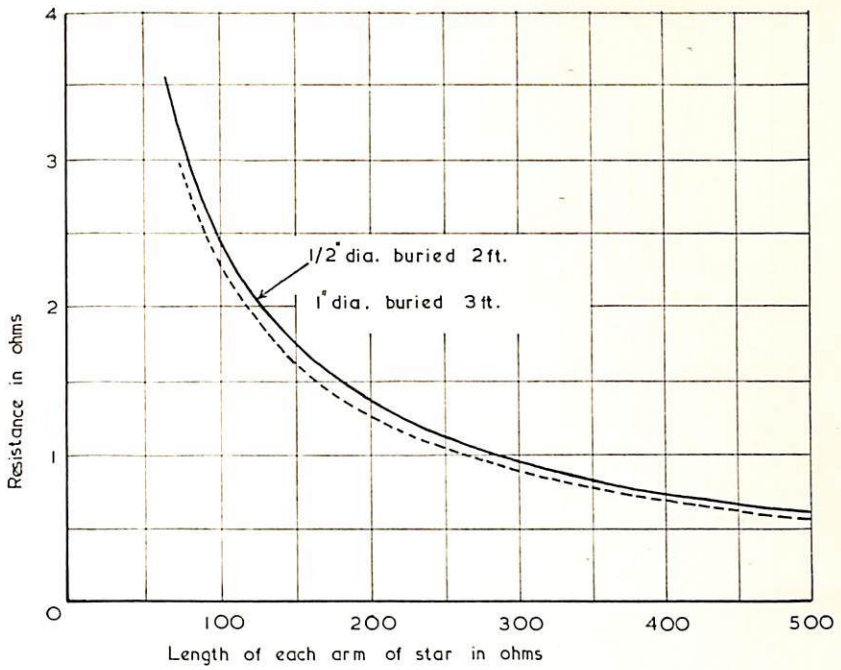


Fig. 8.5. *Resistance of buried wires in the form of a three-arm star.*

Star Arrangement of Wires

To reduce the resistance still further and, where space permits, it is possible to arrange a number of strips in the form of a star as in Fig. 8.3(*b*), (*c*), (*d*) and (*e*). The calculations are carried out in a similar manner and lead to the following.

Three-point star

$$R = \frac{\rho}{6\pi L}\left(\log_e \frac{2L}{a} + \log_e \frac{2L}{S} + 1{\cdot}071 - 0{\cdot}209\,\frac{S}{L} + 0{\cdot}238\,\frac{S^2}{L^2} - 0.054\,\frac{S^4}{L^4}\right) \quad (8.8)$$

Curves calculated from this formula are given in Fig. 8.5.

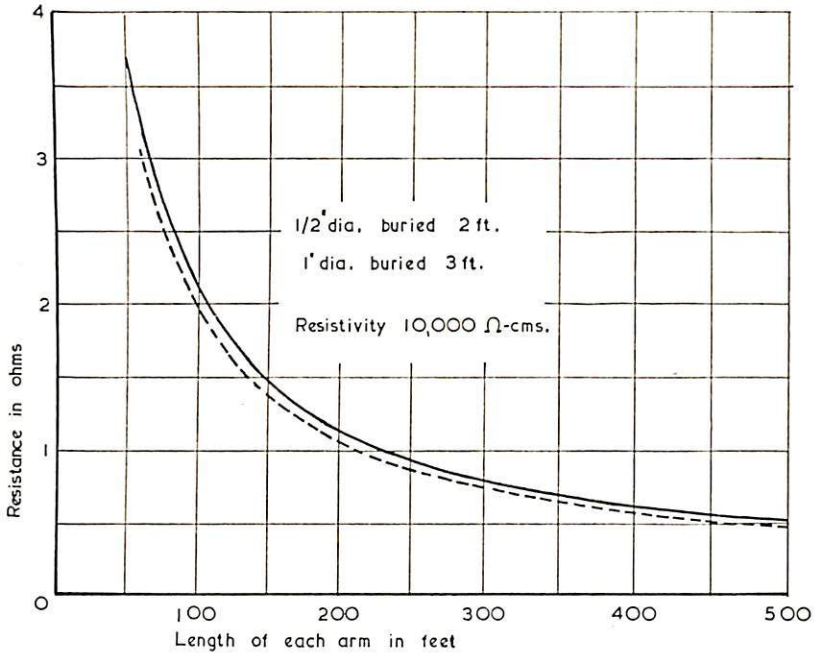


Fig. 8.6. Resistance of buried wires in the form of four-arm star.

Four-point star

$$R = \frac{\rho}{8\pi L}\left(\log_e \frac{2L}{a} + \log_e \frac{2L}{S} + 2{\cdot}912 - 1{\cdot}071\,\frac{S}{L} + 0{\cdot}645\,\frac{S^2}{L^2} - 0{\cdot}145\,\frac{S^4}{L^4}\right) \quad (8.9)$$

Curves calculated from this formula are given in Fig. 8.6.

Six-point star

$$R = \frac{\rho}{12\pi L}\left(\log_e \frac{2L}{a} + \log_e \frac{2L}{S} + 6{\cdot}851 - 3{\cdot}128\,\frac{S}{L} + 1{\cdot}758\,\frac{S^2}{L^2} - 0{\cdot}490\,\frac{S^4}{L^4}\right) \quad (8.10)$$

Eight-point star

$$R = \frac{\rho}{16\pi L}\left(\log_e \frac{2L}{a} + \log_e \frac{2L}{S} + 10{\cdot}98 - 5{\cdot}51\,\frac{S}{L} + 3{\cdot}26\,\frac{S^2}{L^2} - 1{\cdot}17\,\frac{S^4}{L^4}\right) \quad (8.11)$$

TABLE 8.1

Arrangement	*Resistance* (Ω)
Straight wire	1·38
Right angle	1·42
Three-point star	1·46
Four-point star	1·55
Six-point star	1·96
Eight-point star	2·28

Comparison of Various Arrangements

Curves giving the comparative resistance of the various arrangements of wires are given in Fig. 8.7; these are for wires 1 in diameter buried 3 ft. It is interesting to compare the resistances for a given length of wire arranged in various arrangements. In the example chosen, the length of wire is 500 ft, 1 in diameter, buried 3 ft and the figures in Table 8.1 give the resistances obtained with various arrangements.

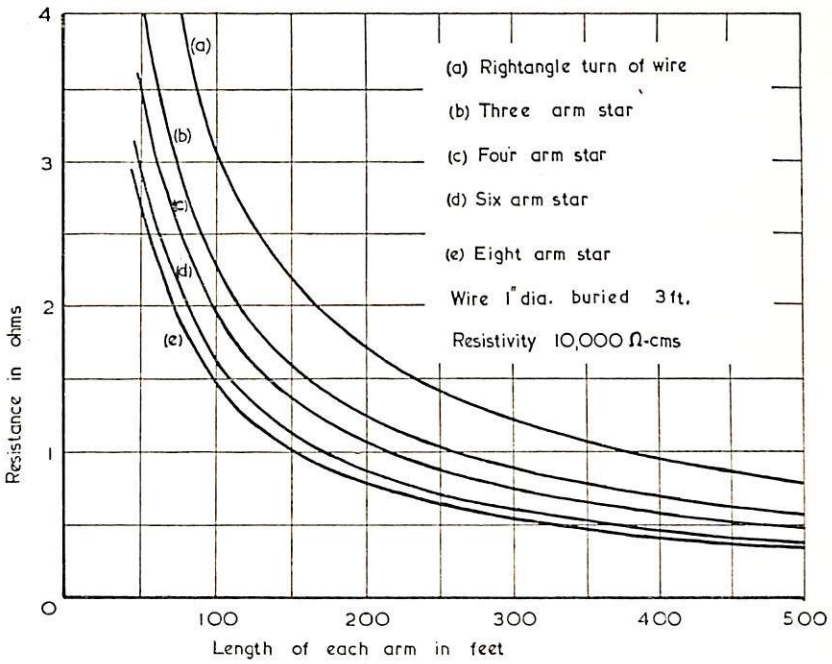


Fig. 8.7. *Resistance of various arrangements of buried wires.*

TABLE 8.2

Arrangement	*Resistance* (Ω)	*Remarks*	*Total wire* (*ft.*)
Straight wire	2·98	Wire 200 ft long	200
Right angle	2·31	Each limb 141 ft long	282
Three-point star	2·27	Each arm 100 ft long	300
Four-point star	1·95	Each arm 100 ft long	400
Six-point star	1·62	Each arm 100 ft long	600
Eight-point star	1·48	Each arm 100 ft long	800

This suggests that for a given length of wire the single straight wire is the best.

On the other hand suppose an area of 100 ft radius is available the figures in Table 8.2 give the comparative resistance.

In this case the lowest resistance is obtained with the eight-point star, but 800 ft of wire is necessary, which is four times the amount in a single straight wire resulting in half the resistance.

As before, all calculations have been made for a resistivity of 10,000 Ω cms, and for any other resistivity ρ all figures in the curves and tables must be multiplied by the ratio $\rho/10{,}000$.

Buried Horizontal Strip

If strip is used instead of wire, an approximation can be made to the resistance by replacing the strip with a wire having the same circumference and using formula (8.4). If a somewhat more accurate formula is required it can be obtained in the following way.

The capacitance of an isolated strip conductor of length $2L$ cms large compared with the width a cms or thickness b cms can be shown to be

$$\frac{1}{C} = \frac{1}{L}\left[\log_e \frac{4L}{a} + \frac{a^2 - \pi ab}{2(a+b)^2}\right] \tag{8.12}$$

The thickness b should be less than about one eighth of the width a.

Taking into account the effect of the image, the capacity becomes

$$\frac{1}{C} = \frac{1}{2L}\left[\log_e \frac{4L}{a} + \frac{a^2 - \pi ab}{2(a+b)^2} + \log_e \frac{4L}{S} - 1 + \frac{S}{2L} - \frac{S^2}{16L^2} + \frac{S^4}{512L^4}\right] \tag{8.13}$$

and this gives a formula for the resistance:

$$R = \frac{\rho}{4\pi L}\left[\log_e \frac{4L}{a} + \frac{a^2 - \pi ab}{2(a+b)^2} + \log_e \frac{4L}{S} - 1 + \frac{S}{2L} - \frac{S^2}{16L^2} + \frac{S^4}{512L^4}\right] \tag{8.14}$$

Curves calculated from this formula are given in Fig. 8.8. For comparison, curves are also given for two buried wires having approximately the same circumference as the strips, and it will be seen that the wire formula gives a slightly lower resistance than the strip formula.

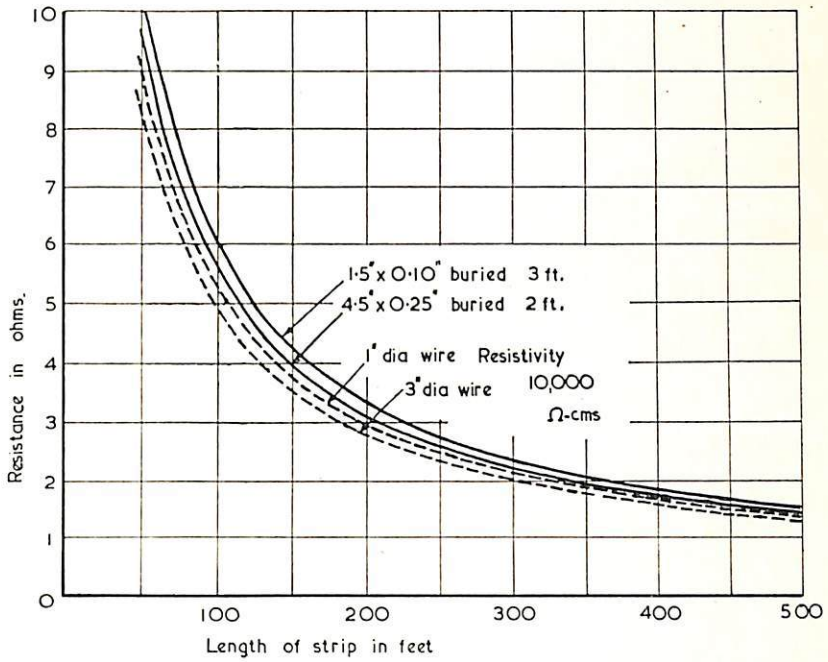


Fig. 8.8. *Resistance of buried strip.*

Practical Formulae for Buried Wires

All the formulae given in this Chapter are in c.g.s. units and the equivalent practical formulae are as follows.

In all formulae, l is the length of one arm in feet, d is the diameter of the wire in inches, t is the depth of burial in feet, δ is the conversion factor from feet to centimetres.

Single wire

$$R = \frac{\rho}{2\pi\delta l}\left[\log_e \frac{48l}{d} + \log_e \frac{l}{t} - 2 + \frac{2t}{l} - \left(\frac{t}{l}\right)^2 + \frac{1}{2}\left(\frac{t}{l}\right)^4\right] \tag{8.15}$$

Right-angle turn

$$R = \frac{\rho}{4\pi\delta l}\left(\log_e \frac{48l}{d} + \log_e \frac{l}{t} - 0{\cdot}2375 + 0{\cdot}4292\,\frac{t}{l} - 0{\cdot}4140\left(\frac{t}{l}\right)^2 - 0{\cdot}6784\left(\frac{t}{l}\right)^4\right] \tag{8.16}$$

Three-point star

$$R = \frac{\rho}{6\pi\delta l}\left[\log_e \frac{48l}{d} + \log_e \frac{l}{t} + 1{\cdot}071 - 0{\cdot}418\,\frac{t}{l} + 0{\cdot}952\left(\frac{t}{l}\right)^2 - 0{\cdot}864\left(\frac{t}{l}\right)^4\right] \tag{8.17}$$

Four-point star

$$R = \frac{\rho}{8\pi\delta l}\left[\log_e \frac{48l}{d} + \log_e \frac{l}{t} + 2{\cdot}912 - 2{\cdot}142\left(\frac{t}{l}\right) + 2{\cdot}580\left(\frac{t}{l}\right)^2 - 2{\cdot}320\left(\frac{t}{l}\right)^4\right] \tag{8.18}$$

Six-point star

$$R = \frac{\rho}{12\pi\delta l}\left[\log_e \frac{48l}{d} + \log_e \frac{l}{t} + 6{\cdot}851 - 6{\cdot}256\left(\frac{t}{l}\right) + 7{\cdot}032\left(\frac{t}{l}\right)^2 - 7{\cdot}84\left(\frac{t}{l}\right)^4\right] \tag{8.19}$$

Eight-point star

$$R = \frac{\rho}{16\pi\delta l}\left[\log_e \frac{48l}{d} + \log_e \frac{l}{t} + 10{\cdot}98 - 11{\cdot}02\left(\frac{t}{l}\right) = 13{\cdot}04\left(\frac{t}{l}\right)^2 - 18{\cdot}72\left(\frac{t}{l}\right)^4\right] \tag{8.20}$$

All the formulae (8.15) to (8.20) are of the general form.

$$R = \frac{\rho}{2\pi l_t \delta}\left[\log_e \frac{48l}{d} + f.\left(\frac{t}{l}\right)\right] \tag{8.21}$$

where l_t is the total length of wire and $f(t/l)$ is a function of t/l which is different for each arrangement of wires. The values of this function for different values of t/l and various arrangements of buried wires are given in the curves of Fig. 8.9. Given the value of this function, the resistance of any arrangement of buried wires can be obtained from

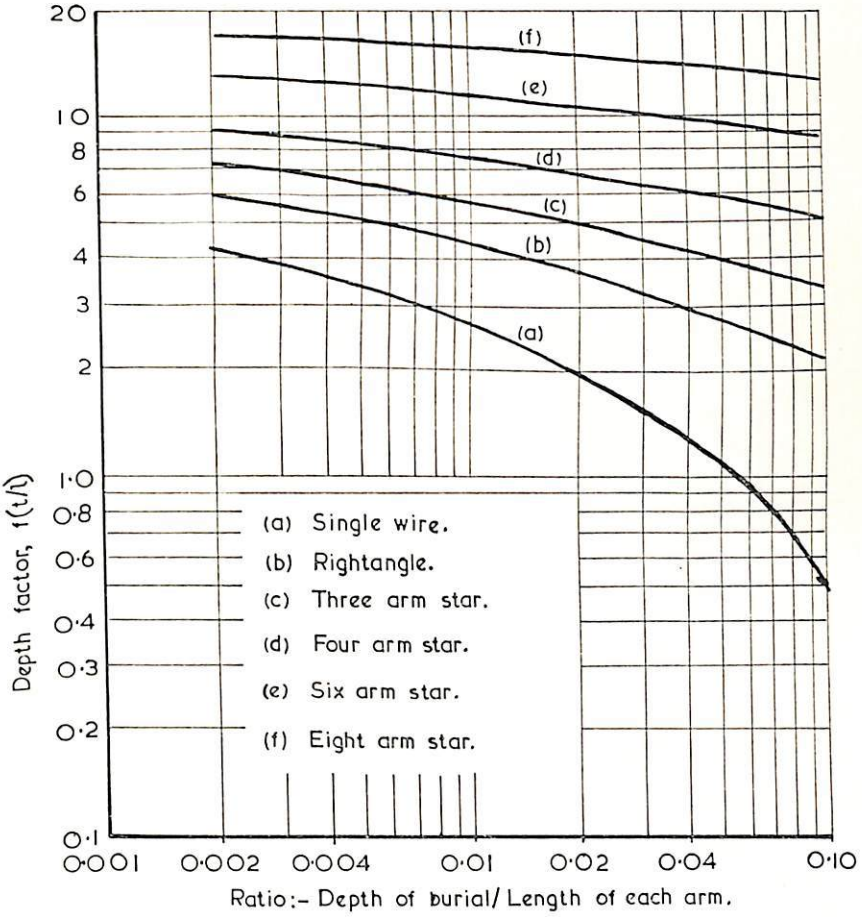


Fig. 8.9. *Depth factor for buried wires.*

Fig. 8.10. If a straight-edge is laid across this nomogram, passing through the values of l/d and $f(t/l)$, it will pass through a value in the centre scale, which is the resistance multiplied by the *total* length of wire in the system. So, if this value is divided by the total length of wire, the result is the required resistance. This is the value with a resistivity of

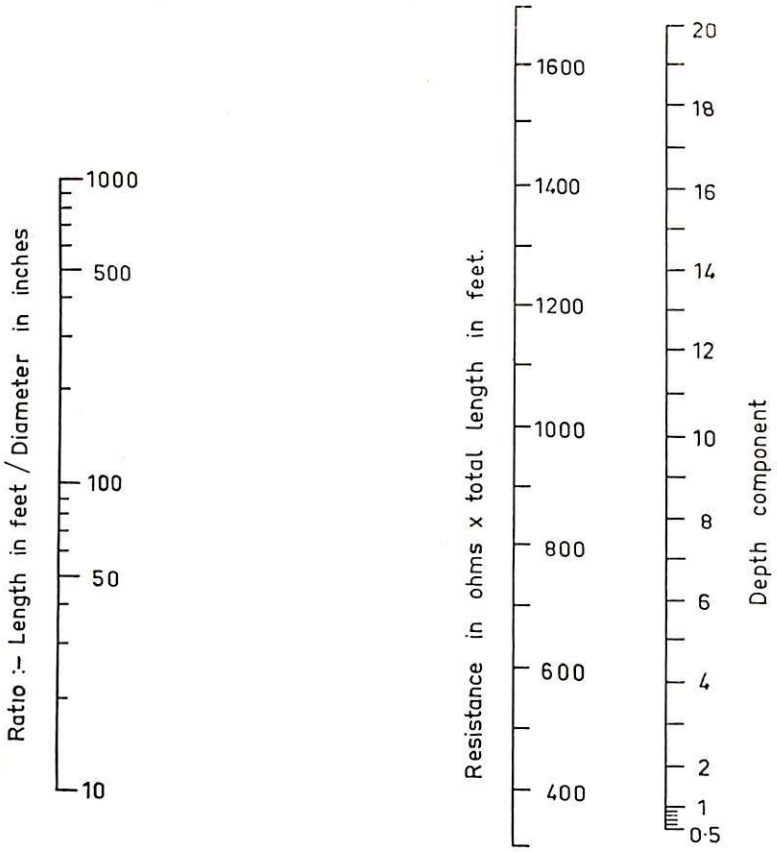


Fig. 8.10. *Nomogram to give resistance of buried wires.*

10,000 ohm-cms, and for any resistivity ρ, the value must be multiplied by the factor $\rho/10{,}000$.

Buried Ring of Wire

An interesting form, if perhaps somewhat less practical than some of the others, is a buried ring of wire. It can be shown that the capacitance of a ring, together with its image at a distance S,

$$\frac{1}{C} = \frac{1}{\pi D}\left(\log_e \frac{8D}{d} + \log_e \frac{4D}{S}\right) \tag{8.22}$$

where D is the diameter of the ring, and d is the diameter of the wire. This leads to a formula for the resistance:

$$R = \frac{\rho}{2\pi^2 D}\left(\log_e \frac{8D}{d} + \log_e \frac{4D}{S}\right) \tag{8.23}$$

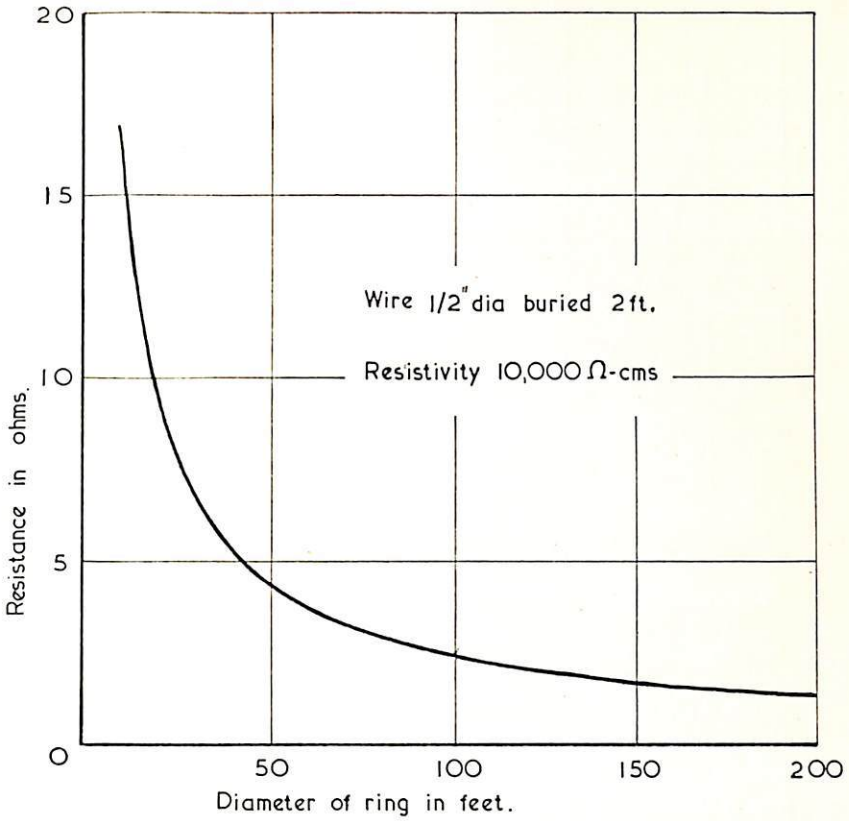


Fig. 8.11. *Resistance of buried ring of wire.*

Curves calculated from this formula are given in Fig. 8.11.

Earth Plates

In the past considerable use has been made of buried plates, but in general these are not so efficient or economical as the rod and strip electrodes already described.

It is not easy to calculate the resistance of a plate although several attempts have been made. An approximation can, however, be made by considering an isolated circular plate. It is known that the capacity of such a plate is given by

$$\frac{1}{C} = \frac{\pi}{2a}$$

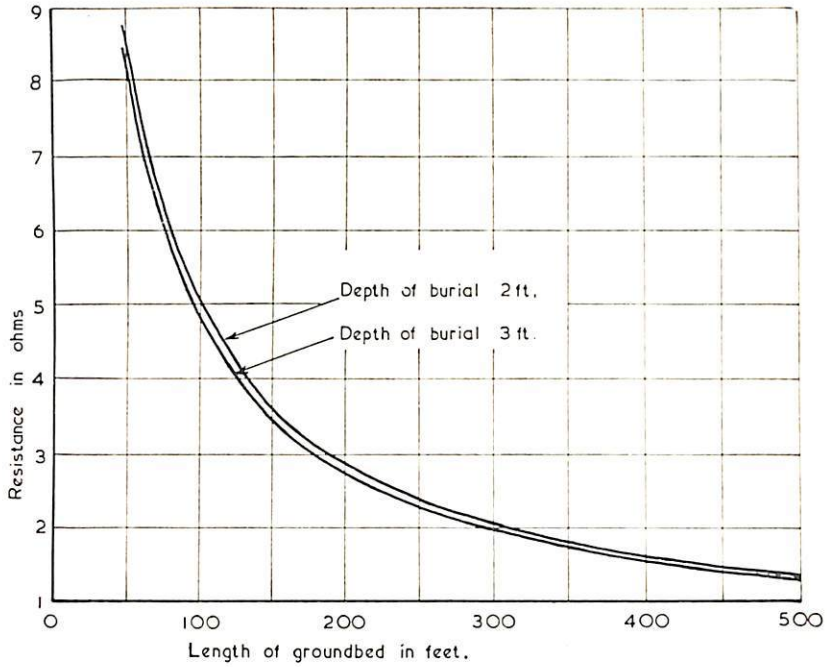


Fig. 8.12. *Resistance of groundbeds* 12 *in. square.* $\rho = 10{,}000$ *ohm-cms.*

where a is the radius of the plate. From this, the resistance is

$$R = \frac{\rho}{2\pi C} = \frac{\rho}{4a} \tag{8.24}$$

In order to generalise this, suppose $A = \pi a^2$, the area of the plate, then

$$R = \frac{\rho}{4}\sqrt{\frac{\pi}{A}} \tag{8.25}$$

If A is in square feet and $\rho = 10{,}000$ ohm-cms, then

$$R = \frac{145{\cdot}4}{\sqrt{A}} \tag{8.26}$$

Thus a plate 6 ft square would have a resistance of 24·2 Ω and one 3 ft square 48·5 Ω. To obtain a low resistance it is obviously necessary to use a number in parallel.

Groundbeds for Cathodic Protection

In connection with cathodic protection schemes it is necessary to install groundbeds, as they are called. When designing the complete installation, a knowledge of the resistance of the groundbed is essential. These groundbeds are usually beds of coke about 12 in square with a metal centre, buried about 3 ft and quite often several hundred feet long.

The resistance of a single groundbed in homogeneous soil can be calculated by the use of formula (8.15) by replacing the square bed by a circular one having the same circumference. The resistance of a 12 in square groundbed calculated from this formula, for two depths of burial, is given in Fig. 8.12. The resistance of a bed, say 150 ft long, 3 ft

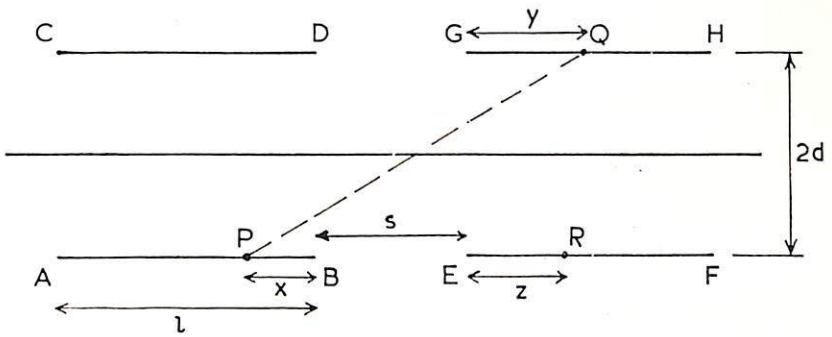


Fig. 8.13. *Two groundbeds in parallel.*

deep, is 3·49 Ω, while for one 300 ft long it is 1·99 Ω, which is more than one half of the value for 150 ft. So, it may be possible by using two 150 ft beds in parallel, properly spaced, to obtain a resistance lower than that of a single 300 ft bed.

It is possible to calculate the combined resistance of two beds in parallel. In Fig. 8.11, AB and EF are the two groundbeds in question. In order to carry out the calculations it is necessary to consider the images CD and GH above the surface of the earth. As before, it is assumed that there is a uniform charge q per cm length on both beds and their images.

The charges in the bed AB and its image CD give rise to an average

potential over AB equal to V_1. The total charge is $2ql$ and, if AB is the only groundbed, then the capacity is given by

$$\frac{1}{C} = \frac{V}{2ql}$$

and the resistance is $R = \dfrac{\rho V}{4\pi ql}$

Now consider the effect of the second groundbed EF and its image GH, and first the potential at any point P, distant x from B.

(1) Potential due to GH.

At any point Q there is a charge $q\,\mathrm{d}y$ and the potential at P due to this is

$$\frac{q.\mathrm{d}y}{[4d^2 + (x + y + S)^2]^{\frac{1}{2}}}$$

Total potential at P due to GH is then

$$\int_0^l \frac{q.\mathrm{d}y}{[4d^2 + (x + y + S)^2]^{\frac{1}{2}}} = q[\log_e\{l + x + S + \sqrt{4d^2 + (l + x + S)^2}\}$$

$$- \log_e\{(x + S) + \sqrt{4d^2 + (x + S)^2}\}] \qquad (8.27)$$

(2) Potential due to EF.

At any point R there is a charge $q.\mathrm{d}z$ and the potential at P due to this is

$$\frac{q.\mathrm{d}z}{x + S + z}$$

The total potential at P due to EF is

$$\int_0^l \frac{q.\mathrm{d}z}{(x + S + z)} = q[\log_e(l + x + S) - \log_e(x + S)] \qquad (8.28)$$

The total potential at P due to EF and its image GH is

$$q[\log_e\{l + x + S + \sqrt{4d^2 + (l + S + x)^2}\} - \log_e\{(x + S)$$

$$+ \sqrt{4d^2 + (x + S)^2} + \log_e(l + x + S) - \log_e(x + S)]$$

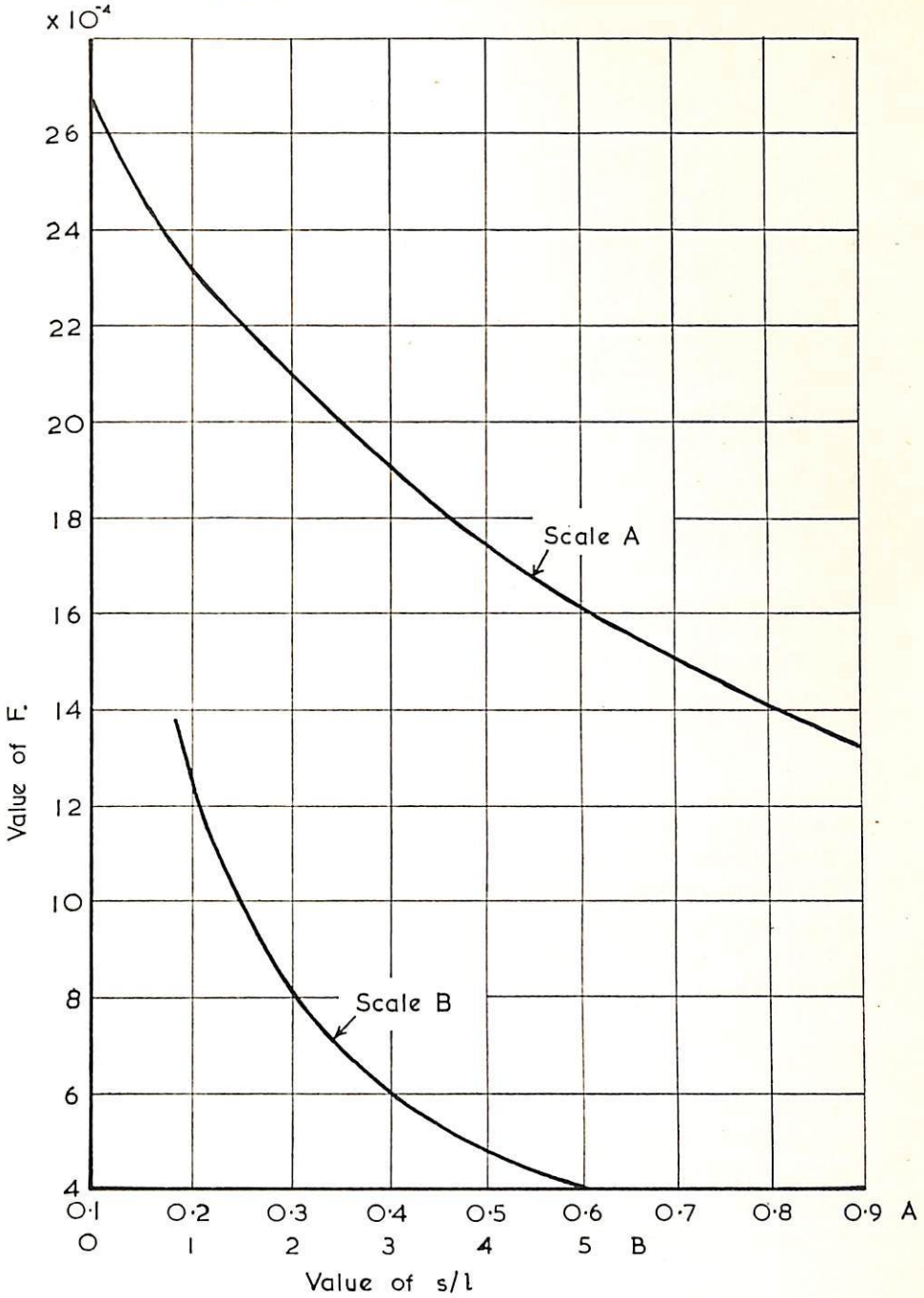


Fig. 8.14. *Values of F for two groundbeds in parallel.*

The average potential of AB due to EF and GH is accordingly

$$\frac{q}{l}\int_0^l [\log_e\{l + x + S + \sqrt{4d^2 + (l + S + x)^2}\} - \log_e\{(x + S) + \sqrt{4d^2 + (x + S)^2} + \log_e(l + x + S) - \log_e(x + S)]\, dx$$

$$= \frac{q}{l}\left[l.\log_e \frac{\{2l + S + \sqrt{4d^2 + (2l + S)^2}\}^2\{2l + S\}^2}{\{l + S + \sqrt{4d^2 + (2l + S)^2}\}^2(l + S)^2} + S\log_e \frac{\{2l + S + \sqrt{4d^2 + (2l + S)^2}\}^2(2l + S)\{S + \sqrt{4d^2 + S^2}\}S}{\{l + S + \sqrt{4d^2 + (2l + S)^2}\}^2(l + S)^2} - \sqrt{4d^2 + (2l + S)^2} + 2\sqrt{4d^2 + (l + S)^2} - \sqrt{4d^2 + S^2}\right]$$

$$= q.B \text{ say} \qquad (8.29)$$

The total potential of $AB = (V_1 + qB)$ and the total charge is $4ql$. So,

$$\frac{1}{C} = \frac{V_1 + qB}{4ql}$$

The resistance is

$$R = \frac{\rho(V_1 + qB)}{8\pi ql} = \frac{\rho V_1}{8\pi ql} + \frac{\rho B}{8\pi l} \qquad (8.30)$$

The first term is one half the resistance of one isolated strip and so the quantity $\rho B/8\pi ql$ is the amount by which the resistance of two strips in parallel exceeds half that of one strip.

The quantity $\rho B/8\pi l$ can be written as $\rho/l\ F$ and as in general S/l is small, F can be written as

$$F = \frac{1}{8\pi}\left[4\log_e\left(2 + \frac{S}{l}\right) + \frac{2S}{l}.\log_e\left(2 + \frac{S}{l}\right)\frac{S}{l} - 4\left(1 + \frac{S}{l}\right)\log_e\left(1 + \frac{S}{l}\right)\right] \qquad (8.31)$$

The values of F for various values of S/l are given in Fig. 8.14. So, from (8.30) and (8.31), the resistance of two groundbeds in parallel, may be calculated.

These curves of Fig. 8.14 can also be used in conjunction with the curves for a buried wire to give the resistance of two wires in parallel.

9

EARTH ELECTRODES IN NON-HOMOGENEOUS SOILS

In the previous chapters the calculations of the resistances of various forms of earth electrode are, as has been stated, based on the assumption that the soil is perfectly homogeneous, and so that a single value of soil resistivity is adequate. Unfortunately this is very seldom the case in practice and some type of non-homogeneity is always present. Quite frequently this takes the form of a series of layers which are more or less parallel.

As the resistance of an earth electrode is entirely due to the resistance of the earth surrounding it, the presence of these layers can affect the resistance, even though the electrode is completely buried in one layer only. This, of course, adds to the difficulties of designing an earth electrode in advance, since a knowledge of soil resistivity is essential. To do this properly, it is thus necessary to make a resistivity survey of the area in which the electrode is to be placed and to analyse the results of this so that the presence of any important non-homogeneity can be detected.

It is possible to make approximate calculations of the effect of some kinds of non-homogeneity on the resistance of some simple forms of earth electrode.

Two-layer Case—Driven Rod in Upper Layer

It frequently happens that a very good approximation to the practical conditions can be obtained by assuming that the earth can be considered as consisting of two layers as shown in Fig. 9.1. The upper layer has a

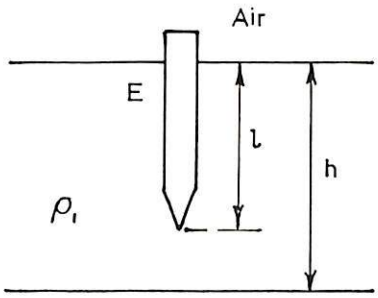


Fig. 9.1. *Driven rod in two-layer soil.*

resistivity of ρ_1 and the thickness of this layer, i.e. the depth to the second layer, is h. The resistivity of the second layer is ρ_2 and the reflection coefficient is defined as $(\rho_2 - \rho_1)/(\rho_2 + \rho_1)$. A driven rod E is inserted in the upper layer to a length l as shown. The effect of the second layer on the resistance of the rod may be calculated approximately by using the average potential method and combining this with the method of images.

It is first necessary to replace the arrangement of Fig. 9.1 by a completely symmetrical arrangement as shown in Fig. 9.2, in which there is a layer of resistivity ρ_1 and thickness $2h$ sandwiched between two semi-

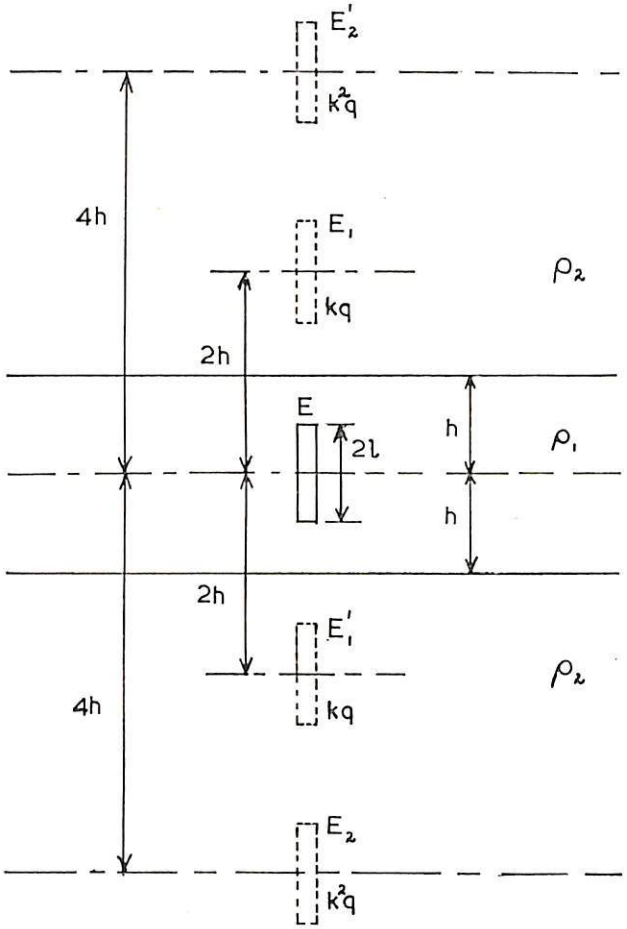


Fig. 9.2. Driven rod with images.

infinite layers of resistivity ρ_2. The rod is combined with its image above the surface of the earth giving a source of length $2l$ and carrying a charge q per cm length. Now, it is known that the potential at any point in the system can be obtained by calculating that due to the charge itself and adding to it the potential due to its images in the interfaces between layers. This can be extended to cover a charged body such as the rod. Again there are two interfaces; so the result is similar to that of the optical case of an object placed between parallel layers, resulting in an infinite number of images.

The first four images are shown in Fig. 9.2. The first pair are E_1 and E_1' both at a distance $2h$ from the centre of the source. It can be shown that the charge in these two must be taken as kq per cm length, where k is the reflection coefficient, $(\rho_2 - \rho_1)/(\rho_2 + \rho_1)$. The second pair of images are E_2 and E_2' at a distance $4h$ from the centre of the source rod, and the charge on these will be k^2q. Similarly the nth pair of image will be at a distance $2nh$ and the charge will be k^nq.

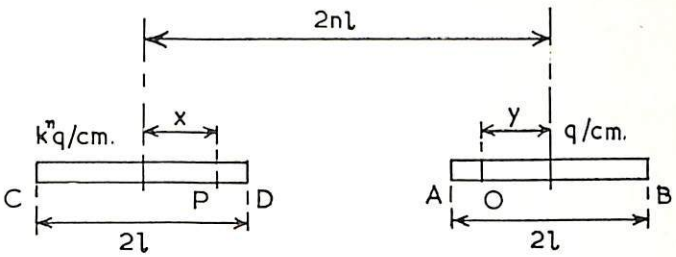


Fig. 9.3. Driven rod and nth image.

One of the nth images is shown in Fig. 9.3. The problem resolves itself into the calculation of the average potential over the source rod due to the charge on this image and summing the result for the infinite series of images.

Consider a point O on the rod at a distance q from the centre and an element $\mathrm{d}x$ in the image at a distance x from the centre. This element will carry a charge $k^nq.\mathrm{d}x$, and the potential at O due to this will be

$$\frac{k^nq.\mathrm{d}x}{(2nh - x - y)}$$

The total potential at O due to the whole charge on the image is

$$\int_{x=-l}^{x=+l} \frac{k^nq.\mathrm{d}x}{(2nh - x - y)} = k^nq\{\log_e(2nh + l - y) - \log_e(2nh - l - y)\}$$

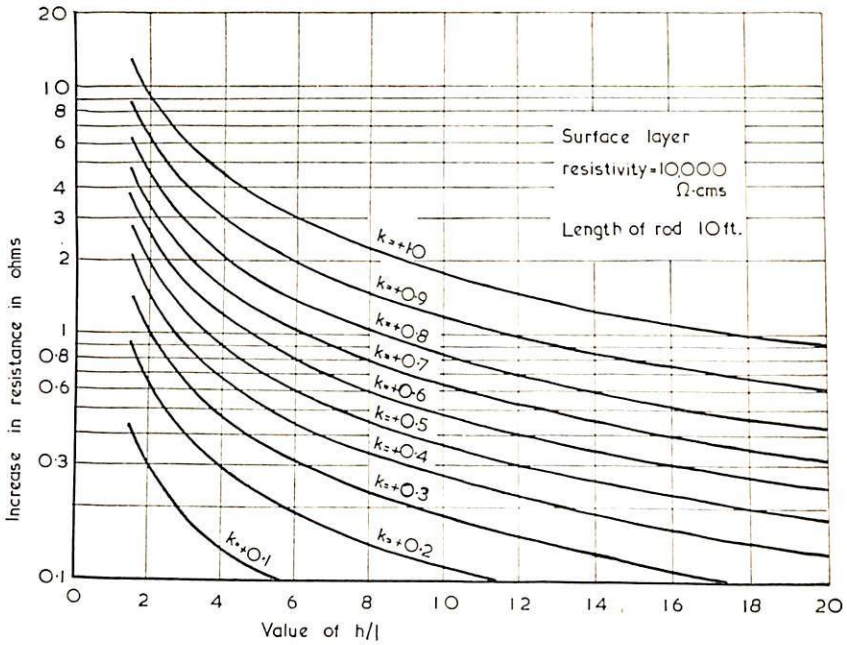


Fig. 9.4. Driven rods. Increase in resistance due to lower layer of higher resistivity.

So the average potential of the rod due to the image charge is

$$\frac{1}{2l}\int_{y=-l}^{y=+l} k^n q\{\log_e(2nh + l - y) - \log_e(2nh - l - y)\}\,dy$$

$$= \frac{k^n q}{2}\,[\log_e(2nh + 2l) - \log_e(2nh - 2l)] \quad (9.1)$$

Bearing in mind that there are two images at the distance $2nh$, carrying a charge $k^n q$ per cm length, the total potential due to the series of images is

$$\sum_{n=1}^{n=\infty} k^n q[\log_e(2nh + 2l) - \log_e(2nh - 2l)] \quad (9.2)$$

From the calculations for a single driven rod, the potential of the rod due to its own charge is

$$2q\left\{\log_e \frac{4l}{a} - 1\right\}$$

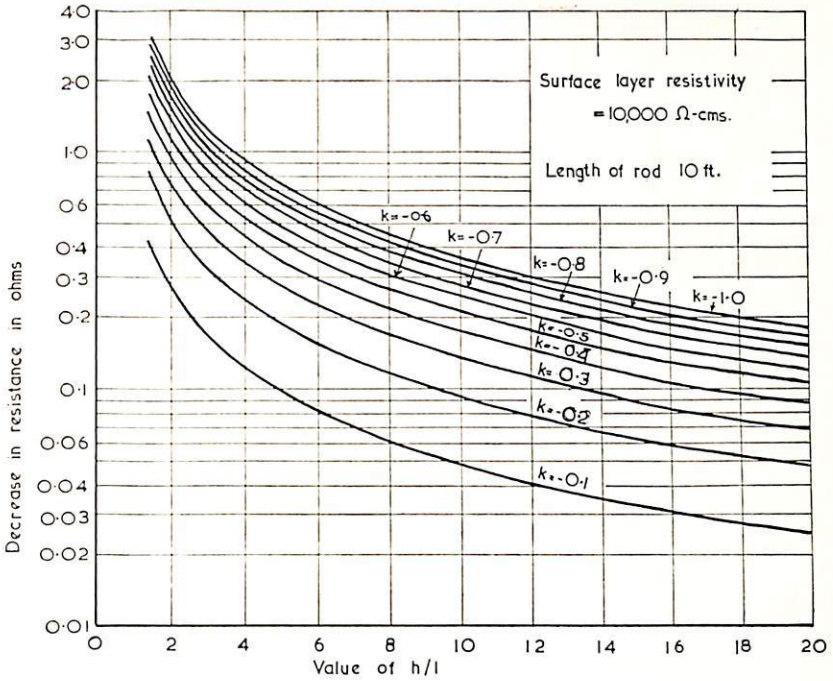


Fig. 9.5. Driven Rods. Decrease in resistance due to lower layer of lower resistivity.

where a is the radius of the rod. The total potential of the rod is

$$2q\left\{\log_e \frac{4l}{a} - 1\right\} + \sum_{n=1}^{n=\infty} k^n q\{\log_e(2nh + 2l) - \log_e(2nh - 2l)\}$$

Hence the value of the capacity of the combination is

$$\frac{1}{C} = \frac{1}{l}\left\{\log_e \frac{4l}{a} - 1\right\} + \sum_{n=1}^{n=\infty} \frac{k^n}{2l} \log_e \frac{(2nh + 2l)}{(2nh - 2l)}$$

and so the resistance of the rod becomes

$$R = \frac{\rho_1}{2\pi l}\left[\left\{\log_e \frac{4l}{a} - 1\right\} + \sum_{n=1}^{n=\infty} \frac{k^n}{2} \log_e \frac{\{nh/l + 1\}}{\{nh/l - 1\}}\right] \qquad (9.3)$$

The first term in this expression is the resistance of a rod of length l driven into soil of resistivity ρ_1, and so the second term represents the additional resistance due to the second layer. If this is R_a, then

$$R_a = \frac{\rho}{2\pi l} \sum_{n=1}^{n=\infty} \frac{k^n}{2} \log_e \frac{\{nh/l + 1\}}{\{nh/l - 1\}} \qquad (9.4)$$

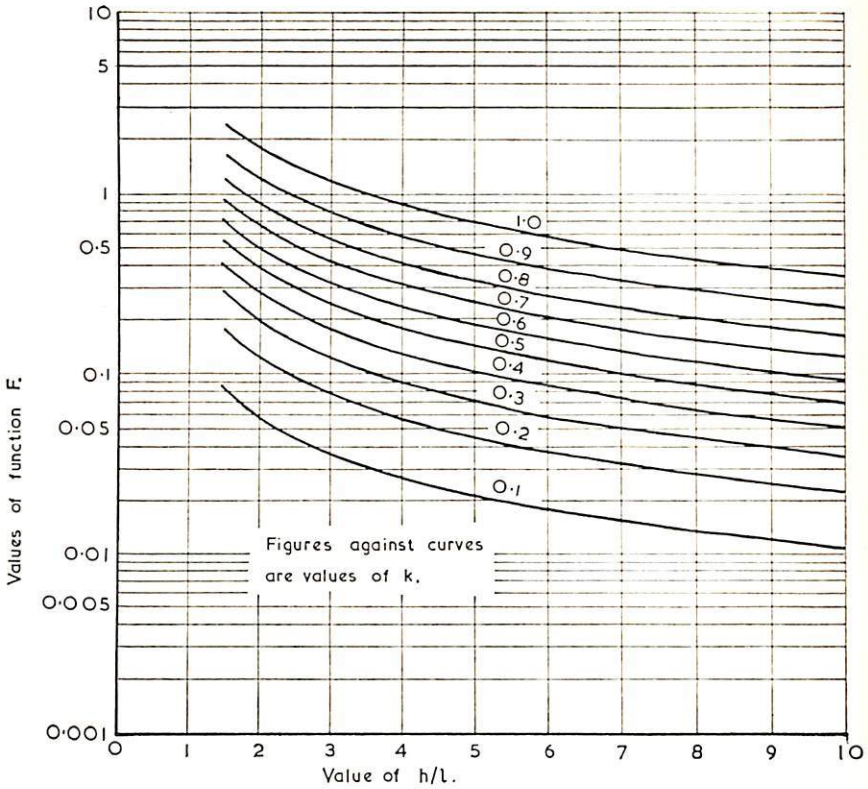


Fig. 9.6. Values of function F for positive values of k.

Curves giving the value of this additional resistance for various values of h/l are given in Figs. 9.4 and 9.5. Note that in Fig. 9.5, the additional resistance when k is negative is also negative. These curves have been calculated for a 10 ft rod and a surface resistivity of 10,000 ohm-cms.

To facilitate calculation, the curves in Figs. 9.6 and 9.7 give the values of the factor F, where

$$F = \sum_{n=1}^{n=\infty} \frac{k^n}{2} \log_e \frac{\{nh/l + 1\}}{\{nh/l - 1\}} \tag{9.5}$$

Driven Rod in Upper Layer—Alternative Method

There is an alternative method of calculating the effect of the lower layer, based on a solution of Laplace's equation for the special conditions involved. In this it is assumed that the electrode can be replaced by a line source of current, I per cm length.

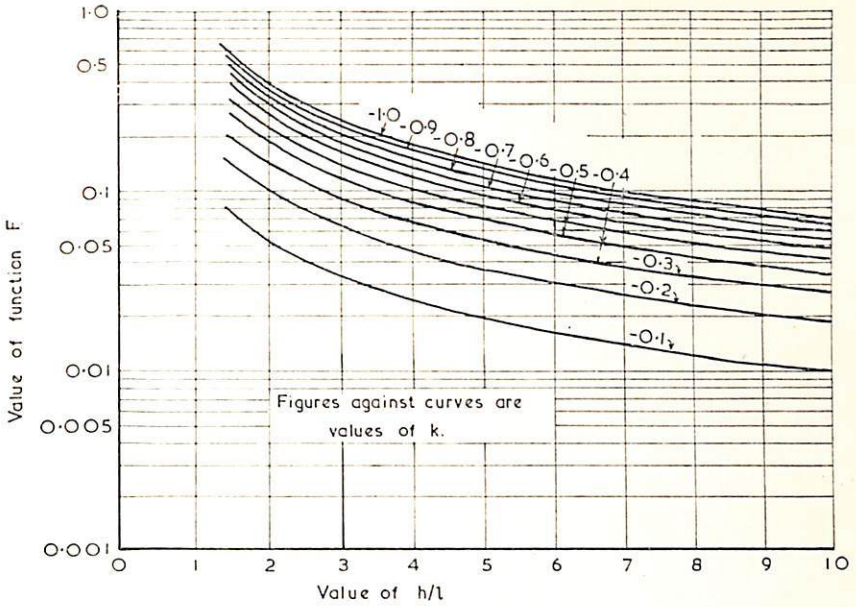


Fig. 9.7. Values of function F for negative values of k.

Consider a point source O in the upper layer at a depth d from the surface as in Fig. 9.8. Let this be the centre of a system of cylindrical co-ordinates. As explained when dealing with resistivity calculations, the potential in the upper layer can be written

$$V_1 = \frac{I\rho_1}{4\pi}\left[\int_0^\infty J_0(\lambda r)e^{+\lambda z}.d\lambda + \int_0^\infty \phi_1(\lambda)J_0(\lambda r)e^{-\lambda z}.d\lambda + \int_0^\infty \psi_1(\lambda)J_0(\lambda r)e^{+\lambda z}.d\lambda\right] \quad (9.6)$$

where J_0 is the Bessel function of the first kind of order zero. It should

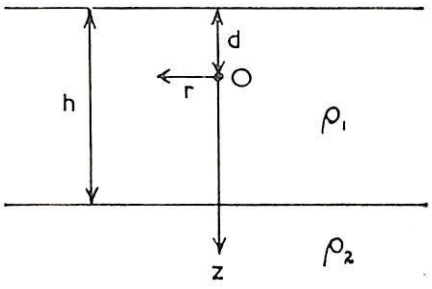


Fig. 9.8. Driven rod in upper layer.

be noted that the co-ordinate z is positive in a downward direction from O and negative in an upward direction. In the first term the negative sign must be taken in the exponent for positive values of z, and the positive sign for negative values of z. A similar expression can be written for the potential V_2 in the lower layer but, as in this case z is always positive, the negative sign is taken for the exponent in the first term, and the first two terms can be combined into one. Thus

$$V_2 = \frac{I\rho_1}{4\pi}\left[\int_0^\infty \phi_2(\lambda)J_0(\lambda r)e^{-\lambda z}.d\lambda + \int_0^\infty \psi_2(\lambda)J_0(\lambda r)e^{+\lambda z}.d\lambda\right] \quad (9.7)$$

The four functions, $\phi_1(\lambda)$, $\psi_1(\lambda)$, $\phi_2(\lambda)$ and $\psi_2(\lambda)$ are determined by the boundary conditions which are:

(1) As z approaches + infinity, V_2 must approach zero. This immediately makes $\psi_2(\lambda) = 0$

(2) At the surface, $z = -d$, $dV/dz = 0$

(3) At the interface $z = (h - d)$ $V_1 = V_2$

and

$$\frac{1}{\rho_1}.\frac{dV_1}{dz} = \frac{1}{\rho_2}.\frac{dV_2}{dz}$$

Applying these conditions to equations (9.6) and (9.7) leads to the following expressions for the function of λ

$$\phi_1(\lambda) = \frac{e^{2\lambda(h-d)} + k}{e^{2\lambda h} - k} \quad (9.8)$$

$$\psi_1(\lambda) = k\frac{(e^{2\lambda d} + 1)}{(e^{2\lambda h} - k)} \quad (9.9)$$

$$\phi_2(\lambda) = \frac{(1 + k)(e^{2\lambda h} + e^{2\lambda(h-d)})}{(e^{2\lambda h} - k)} \quad (9.10)$$

If these values are substituted in equations (9.6) and (9.7) and the integrals expanded, the expression for the potential in the upper layer becomes

$$V_1 = \frac{I\rho_1}{4\pi}\left[\frac{1}{(r^2 + z^2)^{1/2}} + \frac{1}{\{r^2 + (z + 2d)^2\}^{1/2}} + \sum_{n=1}^{n=\infty}\left\{\frac{k^n}{[r^2 + \{2(nh + d) + z\}^2]^{1/2}} + \frac{k^n}{[r^2 + \{2nh + z\}^2]^{1/2}} + \frac{k^n}{[r^2 + \{2(nh - d) - z\}^2]^{1/2}} + \frac{k^n}{[r^2 + \{2nh - z\}^2]^{1/2}}\right\}\right] \quad (9.11)$$

In the case of a rod of radius a and length l, the potential of the surface of the rod is obtained by putting $r = a$ and $z = d$. This leads to

$$V = \frac{I\rho_1}{2\pi}\left[\frac{1}{(a^2+d^2)^{\frac{1}{2}}} + \sum_{n=1}^{n=\infty}\left\{\frac{k^n}{[a^2+(2nh-d)^2]^{\frac{1}{2}}} + \frac{k^n}{[a^2+(2nh+d)^2]^{\frac{1}{2}}}\right\}\right] \quad (9.12)$$

Now the current I in the above expression is the current i flowing from a small element of the rod. To obtain the average potential over the whole rod it is necessary to integrate this from $d = 0$ to $d = l$ and to divide the result by l. This leads to

$$V = \frac{I\rho_1}{2\pi l}\left[\log_e\left\{\frac{l+\sqrt{a^2+l^2}}{a}\right\} + \sum_{n=1}^{n=\infty} k^n \log_e \frac{2nh+l+\sqrt{a^2+(2nh+l)^2}}{2nh-l+\sqrt{a^2+(2nh-l)^2}}\right] \quad (9.13)$$

If a can be considered small, this leads to an expression for the resistance

$$R = \frac{\rho_1}{2\pi l}\left[\log_e \frac{2l}{a} + \sum_{n=1}^{n=\infty} k^n \log_e\left\{\frac{1+\frac{l}{2nh}}{1-\frac{l}{nh}}\right\}\right] \quad (9.14)$$

This differs somewhat from the expression (9.3) obtained by the image method and the difference is similar to that observed between the average potential and ellipsoid formula for a single rod. It is not necessary to make any detailed calculations from (9.14) since both formulae are approximate only and, in any case, the practical conditions never agree exactly with those assumed for the calculations.

Driven Rod Penetrating Two Layers

The length of the driven rod l is, in this case, greater than h as in Fig. 9.9. Owing to the change in resistivity in the soil surrounding the rod, it is no longer possible to assume that the current flows uniformly from the rod all along its length. It is, however, a reasonable assumption that the current density is inversely proportional to the resistivity.

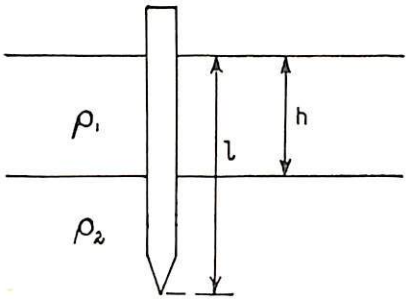


Fig. 9.9.
Driven rod penetrating two layers.

Let σ_1 and σ_2 be the current per unit length, σ_1 in the upper layer and σ_2 in the lower layer.

Now

$$\sigma_1\rho_1 = \sigma_2\rho_2$$

and

$$\sigma_1 h + \sigma_2(l - h) = I$$

Solving these equations gives

$$\left.\begin{aligned}\sigma_1 &= \frac{I\rho_2}{\rho_2 h + \rho_1(l-h)}\\ \sigma_2 &= \frac{I\rho_1}{\rho_2 h + \rho_1(l-h)}\end{aligned}\right\} \tag{9.15}$$

Now the potential at the surface of the rod on the surface of the earth due to the current flow from that part of the rod in the upper layer is from equation (9.12)

$$\frac{I\rho_1\rho_2}{2\pi\{\rho_2 h + \rho_1(l-h)\}}\left[\frac{1}{(a^2+d^2)^{1/2}} + \sum_{n=1}^{n=\infty}\left\{\frac{k^n}{\{a^2+(2nh-d)^2\}^{1/2}} + \frac{k^n}{\{a^2+(2nh+d)^2\}^{1/2}}\right\}\right] \tag{9.16}$$

It can be shown by a similar analysis to that above, that the potential at the surface of the rod due to a source I in the lower layer is

$$\frac{I\rho_2}{2\pi}\sum_{n=1}^{n=\infty}\frac{(1-k)k^n}{\{a^2+(2nh+d^2)^2]^{1/2}}$$

Replacing I by the value of σ_2 leads to

$$\frac{I\rho_1\rho_2}{2\pi[\rho_2 h + \rho_1(l-h)]}\sum_{n=1}^{n=\infty}\frac{(1-k)k^n}{[a^2+(2nh+d)^2]^{1/2}} \tag{9.17}$$

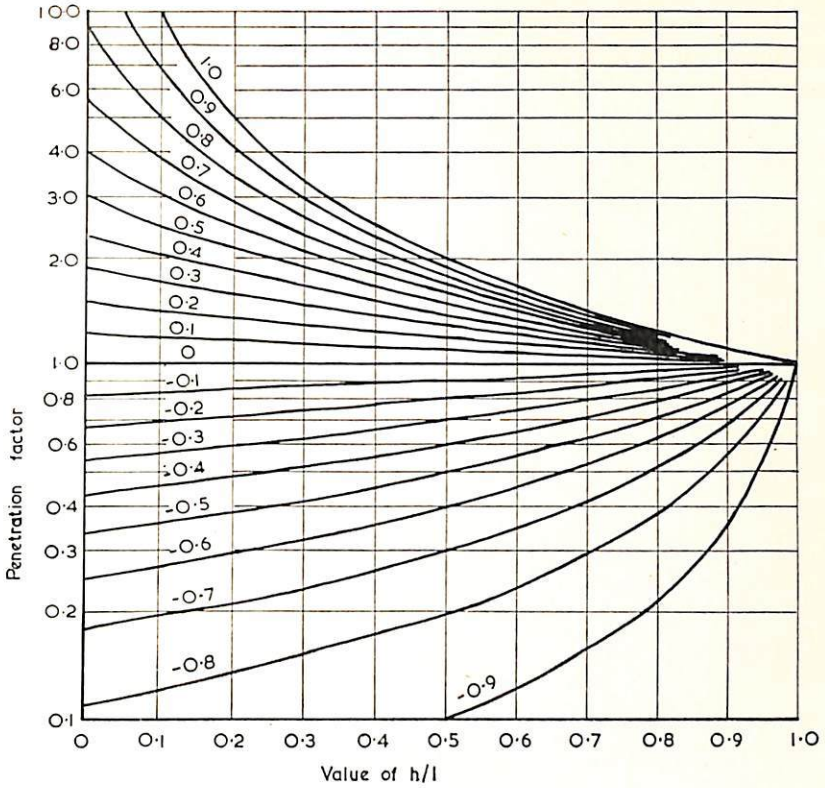


Fig. 9.10. *Driven rod in two layers. Penetration factor.*

The average potential over the whole rod is thus

$$\frac{I\rho_1\rho_2}{2\pi\{\rho_2 h+\rho_1(l-h)\}l}\int_0^h\left\{\frac{1}{(a^2+d^2)^{\frac{1}{2}}}+\sum_{n=1}^{n=\infty}\frac{k^n}{\{a^2+(2nh-d)^2\}^{\frac{1}{2}}}\right.$$

$$\left.+\frac{k^n}{\{a^2+(2nh+d)^2\}^{\frac{1}{2}}}\right\}^{dd}+\frac{I\rho_1\rho_2}{2\pi\{\rho_2 h+\rho_1(l-h)\}l}\int_h^l\left\{\sum_{n=1}^{n=\infty}\right.$$

$$\left.\frac{(1-k)k^n}{[a^2+(2nh+d)^2]^{\frac{1}{2}}}\right\}^{dd}$$

These lead to an expression for the resistance

$$R=\frac{\rho_1}{2\pi l}\cdot\frac{(1+k)}{\left\{(1-k)+2k\dfrac{h}{l}\right\}}\left[\log_e\frac{2l}{a}+\sum_{n=1}^{n=\infty}k^n\log_e\frac{(2nh+l)}{\{(2n-2)h+l\}}\right] \tag{9.18}$$

$$=F[R_1+R_a] \tag{9.19}$$

Where R_1 is the resistance of the rod, driven in uniform soil of resistivity ρ_1, R_a is an additional resistance due to the second layer and F is a penetration factor.

The values of the penetration factor F are given in Fig. 9.10 and the values of R_a are given in Figs. 9.11 and 9.12.

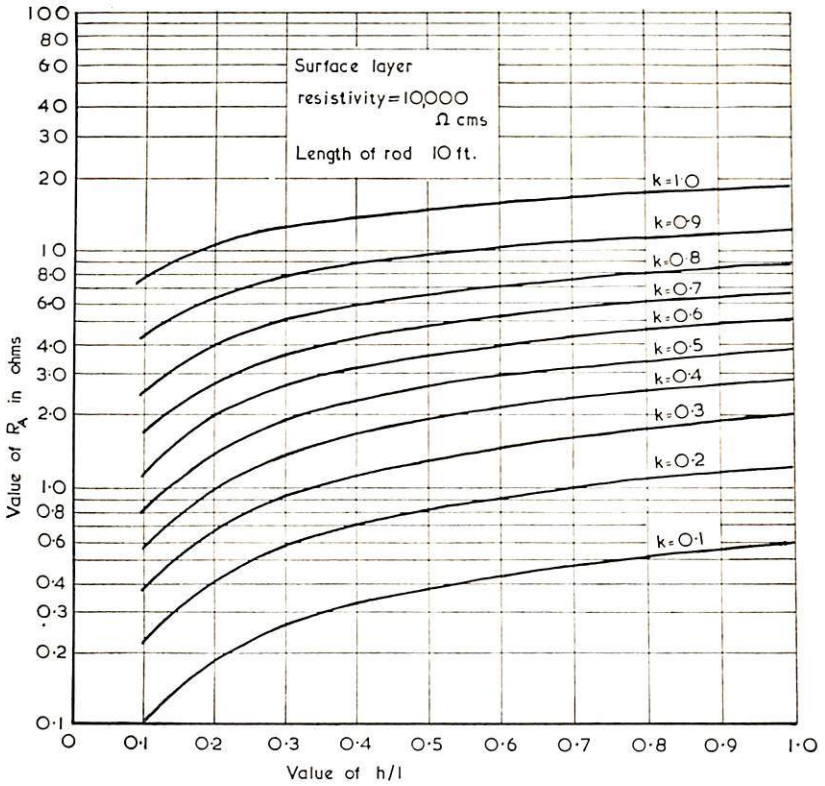


Fig. 9.11. Driven rod penetrating two layers. Additional resistance R_A.

Driven Rod near Vertical Fault

Vertical discontinuities sometimes occur and an example of this can be given in Fig. 9.13 where a driven rod is in close proximity to a vertical fault. Here soils of two resistivities ρ_1 and ρ_2 are separated by a vertical interface and a rod of length l is driven in the soil ρ_1 at a distance d from the interface. The easiest way to deal with this problem is by the method of images.

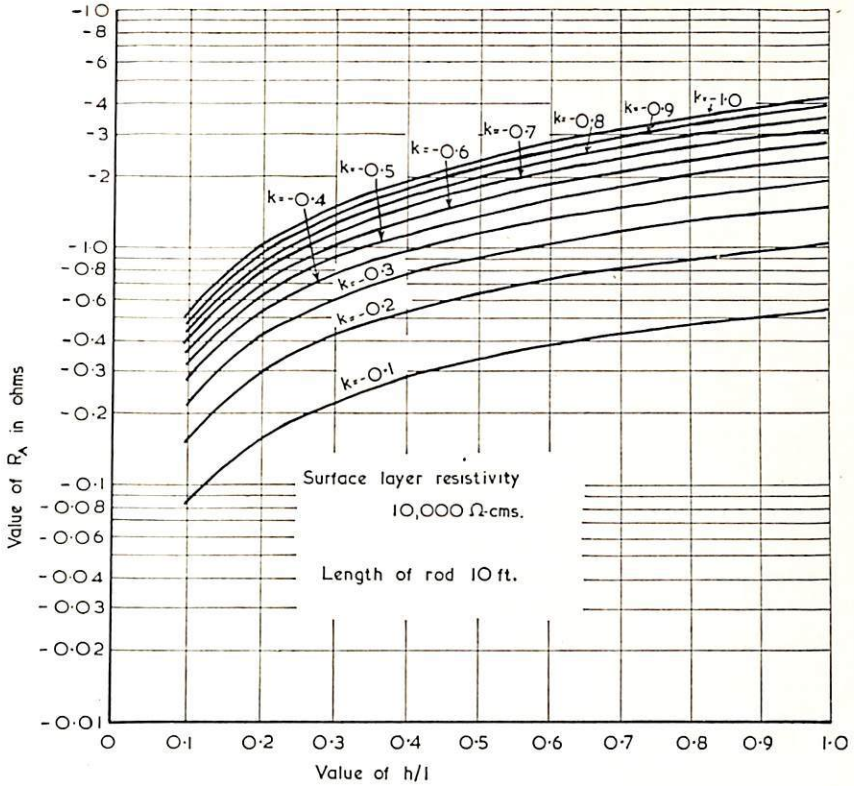


Fig. 9.12. *Driven rod penetrating two layers. Additional resistance* R_A.

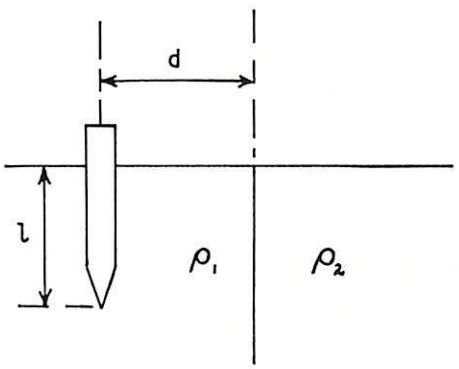


Fig. 9.13. *Driven rod near vertical fault.*

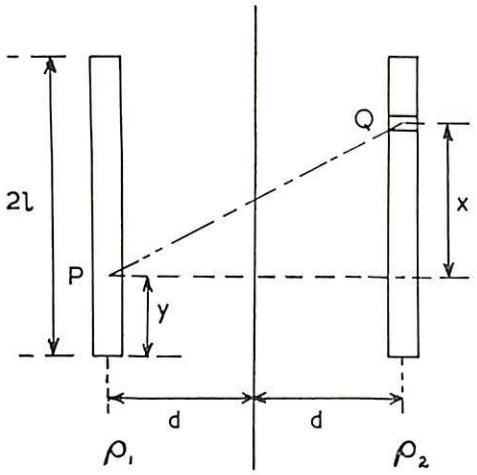


Fig. 9.14. Driven rod in Fig. 9.13 replaced by cylinder.

In Fig. 9.14 the rod has been replaced by a cylinder of length $2l$ in a medium of resistivity ρ_1. There will be an image in the medium ρ_2 at a distance d from the interface. If there is a charge of q per cm length on the cylinder, there will be a charge kq per cm length on the image.

The potential at the point P due to the element of charge $kq\,\mathrm{d}x$ at Q is

$$\frac{kq\,\mathrm{d}x}{\{(x-y)^2+4d^2\}^{\frac{1}{2}}}$$

and so the potential at P due to the charge on the image is

$$\int_0^{2l} \frac{kq\,\mathrm{d}x}{[(x-y)^2+4d^2]^{\frac{1}{2}}} = kq[\log_e\{2l-y+\sqrt{(2l-y)^2+4d^2}\} - \log_e\{-y+\sqrt{y^2+4d^2}\}]$$

The average potential over the rod due to the image is

$$\frac{1}{2l}\int_0^{2l} kq[\log_e\{2l-y+\sqrt{(2l-y)^2+4d^2}\} - \log_e\{-y+\sqrt{y^2+4d^2}\}\,\mathrm{d}y$$

$$= kq\left[\log_e\left\{\frac{\sqrt{4l^2+4d^2}+2l}{\sqrt{4l^2+4d^2}-2l}\right\} + \frac{2d}{l} + \frac{\sqrt{4l^2+4d^2}}{l}\right] \qquad (9.20)$$

This leads to an expression for the additional resistance due to the image, i.e. due to the fault,

$$R_a = \frac{\rho_1 k}{2\pi l}\left[\log_e\left\{\frac{\sqrt{1+(d/l)^2}+1}{\sqrt{1+(d/l)^2}-1}\right\} + \frac{2d}{l} - 2\sqrt{1+\left(\frac{d}{l}\right)^2}\right] \qquad (9.21)$$

Values calculated from (9.21) for positive values of k are plotted in the curves of Fig. 9.15. These show increases in resistance for positive values of k and the same values hold for equivalent negative values of k, except that, in this case, the curves represent a reduction in resistance. The calculations are for a resistivity of 10,000 ohm-cms for ρ_1 and for a length of 10 ft. They can be used for other values by changing in proportion to actual values of resistivity and length, provided the correct value of d/l is chosen.

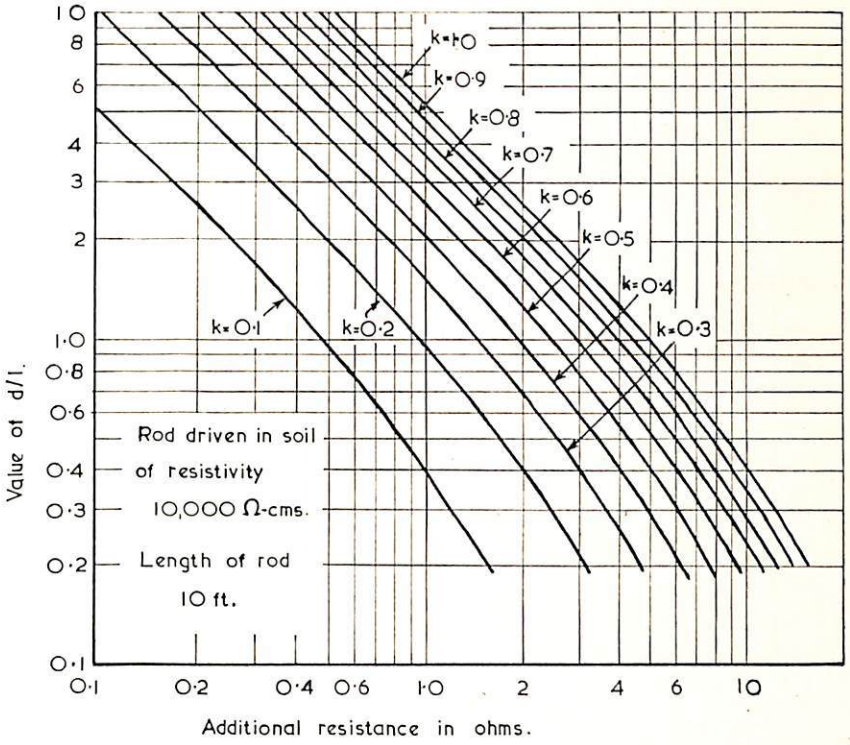


Fig. 9.15. Driven rod. Additional resistance due to neighbouring fault.

Two-layer Case—Buried Strip in Upper Layer

Another important case is illustrated in Fig. 9.16. In this, a strip of length l is buried at a depth d in a medium of resistivity ρ_1 and thickness h overlaying a second layer of resistivity ρ_2. This can be analysed by

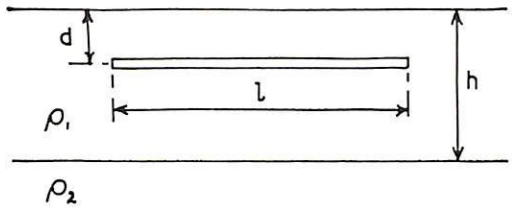


Fig. 9.16.
Buried strip in two-layer soil.

similar methods to those used for the driven rods. As an example of the results obtained the additional resistance when the strip or wire is very close to the surface is given by

$$R_a = \frac{\rho_1}{2\pi l} \sum_{n=1}^{n=\infty} k^n \left\{ 4 \log_e \frac{1 + \sqrt{\left(\frac{2nh}{l}\right)^2 + 1}}{2n\frac{h}{l}} + \frac{8nh}{l} - 4\sqrt{\left(\frac{2nh}{l}\right)^2 + 1} \right\} \tag{9.22}$$

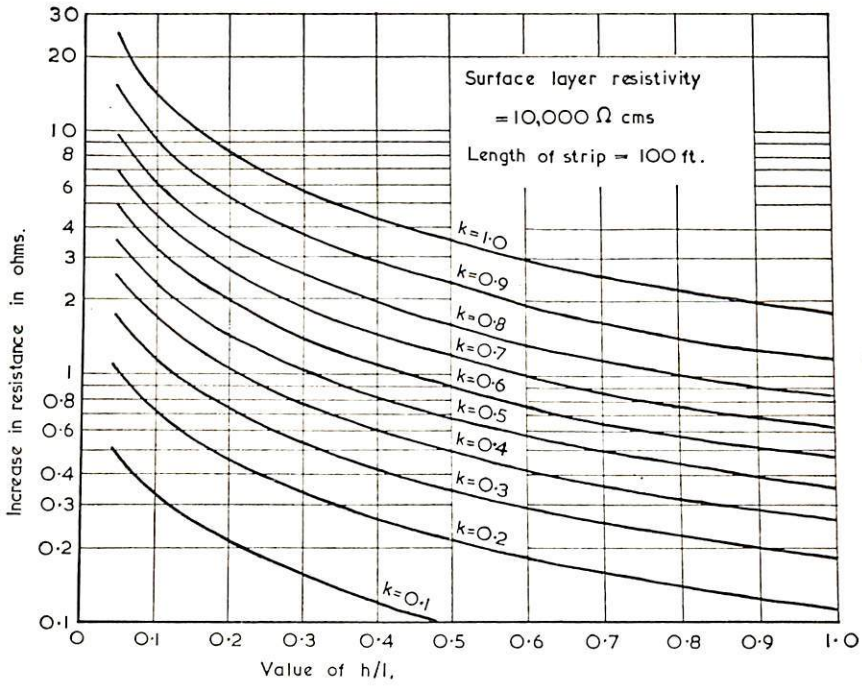


Fig. 9.17. *Buried strip. Increase in resistance due to lower layer of higher resistivity.*

Curves calculated from this formula are given in Figs. 9.17 and 9.18 for a strip 100 ft long and a resistivity of 10,000 ohm-cms. For other values of length and resistivity the values may be changed in proportion provided the correct value of h/l is chosen.

A few calculations have shown that for small values of d/l, i.e. the depth of burial is small compared with the length, the departure from the values given by (9.22) is slight and so this formula can be used for most practical cases.

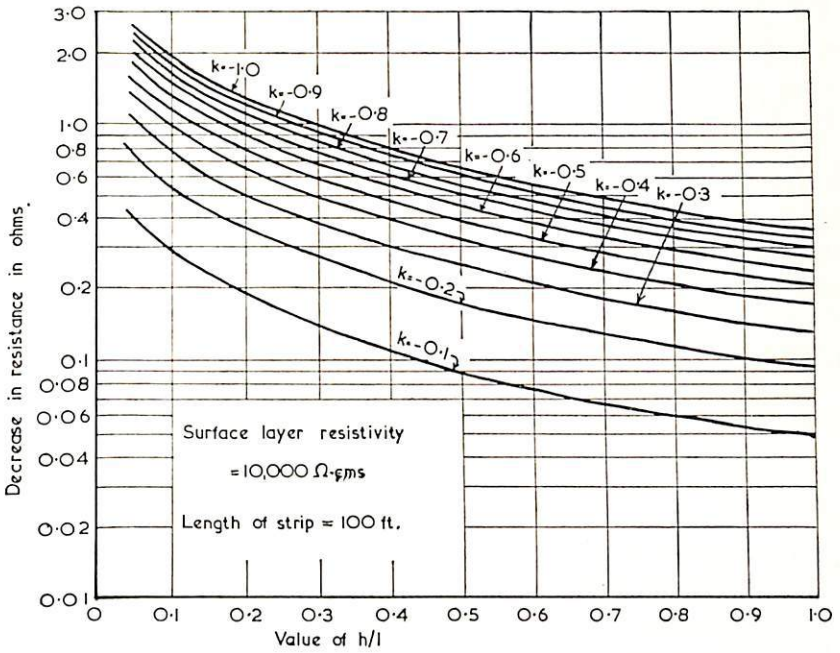


Fig. 9.18. *Buried strip. Decrease in resistance due to lower layer of lower resistivity.*

Summary

Formulae have been given in this Chapter for the change in resistance of driven rods and buried strips for certain forms of non-homogeneity. It is necessary to have some knowledge of the non-homogeneity and of the resistivities involved. The resistance of the appropriate electrode can be calculated for homogeneous conditions and then the additional resistance, which may actually be negative, may also be calculated.

10

THE MEASUREMENT OF AN EARTH ELECTRODE RESISTANCE

The resistance of an earth electrode is unique in one respect in that only one end, the metal of the electrode itself, is definite. As already explained, the resistance consists practically entirely of that of a body of earth surrounding the electrode and while, theoretically, this extends to an infinite distance, in practice the greater proportion, say 98 per cent, of the total is contained within a finite distance.

There is no point in trying to obtain a very high accuracy in such a measurement since one of the major factors in the resistance is the resistivity of the soil and this is liable to variations with temperature and moisture content among other things. If the resistance is measured on a summer day it may differ considerably from the value obtained in winter. An accuracy of 2 per cent is more than adequate for most purposes.

The area surrounding the electrode and containing say 98 per cent of the total resistance is the resistance area of that electrode and, in making any form of measurement, two facts must be borne in mind. First, the whole of the required resistance area must be included in the measurement and second, if any auxiliary earth electrodes are used, it is essential to see that the resistance area of the auxiliary electrode does not overlap that of the main electrode.

Test with a Known Earth

The simplest way to make the necessary measurement is to find the combined resistance of the electrode in question and one of known or

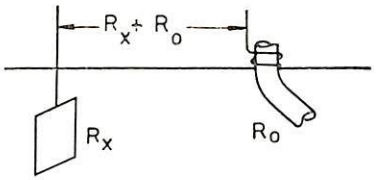


Fig. 10.1. (*top*). *Test with a known earth.*

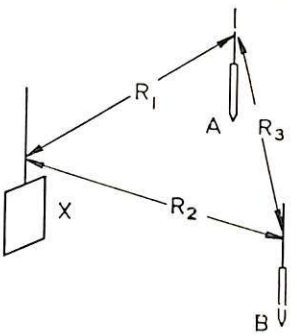


Fig. 10.2. (*right*). *The three-point method.*

negligible resistance, as illustrated in Fig. 10.1. There are two objections to this comparatively simple method. The first is that earth electrodes of known resistance are rare, although it is sometimes possible to find a water supply system having a negligible resistance to earth. The increasing use of insulated pipes, however, makes it more difficult to find a water supply system which forms a suitable earth electrode. The second objection is the difficulty of ensuring that the resistance areas do not overlap. Again, if a water supply system is used, this may be very extensive and, as a result, have a very large resistance area making it very difficult to meet the condition that the resistance areas do not overlap. This method is very rarely used nowadays.

The Three-point Method

A logical extension of the method described above is to insert two auxiliary electrodes in the earth, to measure the combined resistances in pairs and to deduce from these the required resistance. This is illustrated in Fig. 10.2, where X is the electrode under test and A and B are the auxiliaries. Then

$$\left.\begin{aligned} R_1 &= X + A \\ R_2 &= X + B \\ R_3 &= A + B \end{aligned}\right\} \tag{10.1}$$

from which it follows that

$$X = \frac{R_1 + R_2 - R_3}{2} \tag{10.2}$$

This would appear to be a straightforward method but again there are objections. It is probably not too difficult to ensure that the resistance areas do not overlap; it is only necessary to increase the spacing between the electrodes until the measured resistances become reasonably constant. The main objection is that the effect of any observational or experimental error is necessarily magnified by the form of equation (10.2). It is nearly certain that A and B will be large compared with X, and so that R_3 is approximately twice R_1 and R_2. So any errors in R_1, R_2 or R_3 may produce very large errors in the calculated value of X. It is, in fact, quite a simple matter to establish a case where quite small experimental errors can produce a negative value for X. Thus, this method cannot be considered reliable.

The Fall-of-Potential Method

The method which is almost universally employed at the present time is known as the fall-of-potential method and is illustrated in

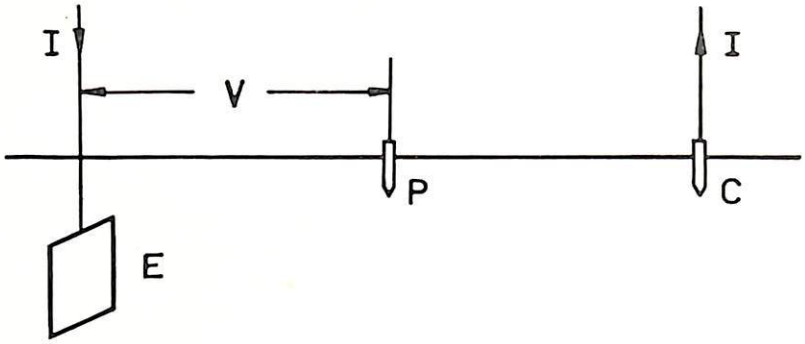


Fig. 10.3. *Fall-of-potential method.*

Fig. 10.3. In this E is the earth electrode under test and P and C are two auxiliary electrodes placed at suitable distances from E. A known current is passed between the electrodes E and C, and the potential difference between E and P is measured. If the current is I and the potential difference V, the quotient V/I will give a resistance which under suitable conditions will give the required resistance of E.

It is evident that in this method, the resistances of the auxiliary electrodes are not included. The electrode C is in the main path and its resistance is one of the factors determining the value of the current I, but this in turn determines the value of V, so that the quotient V/I is independent of the resistance of C. Again, the resistance of the auxiliary electrode P is part of the circuit used to measure the potential difference

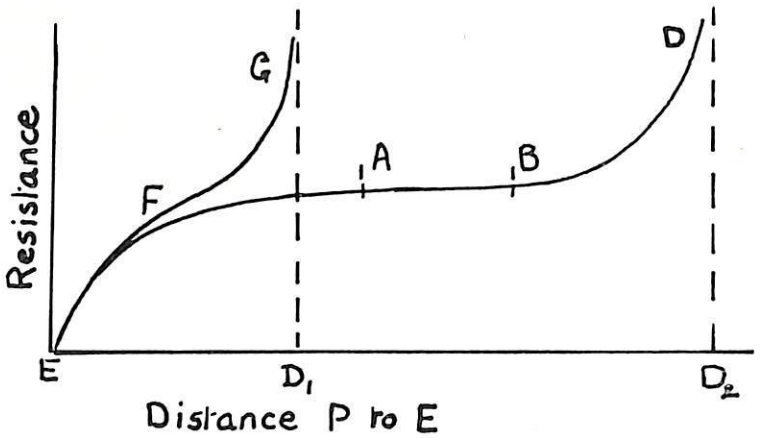


Fig. 10.4. *Earth resistance curves.*

V and, if this is a null balancing system so that at balance no current is passing through P, then the resistance of this electrode can have no effect on the measured value. If, on the other hand, a high resistance voltmeter or its equivalent is used to measure P, then it may be possible to make this resistance so high that the values of P normally encountered in practice have no effect, or to arrange to measure P and apply a suitable correction.

Earth-resistance Curves

It is, of course, essential to see that the resistance areas do not overlap and this is readily checked in this case by plotting an earth-resistance curve. Such curves are illustrated in Fig. 10.4.

If the current electrode is, for example, at a distance D_1 from the electrode under test, this distance being such that the resistance areas

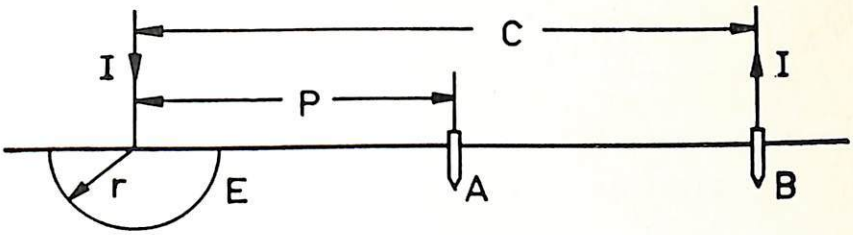


Fig. 10.5. *Hemispherical electrode.*

overlap then, if a series of measurements are made with the potential electrode at various distances from E, a curve of the type EFG will result. The measured resistance is increasing steadily as the potential electrode is moved along the line joining the earth electrode to the current electrode. If, on the other hand, the current electrode is placed at a distance D_2 such that the resistance areas do not overlap, a curve of the type $EFABD$ will result, with a part AB which is practically horizontal. The resistance corresponding to this horizontal section is the true resistance of the electrode.

The conditions necessary to obtain the required result can be demonstrated in the following manner. Suppose the electrode under test can be considered to be a hemisphere as in Fig. 10.5. The use of the hemisphere is discussed further later. The current electrode is B set at a distance C from the centre of the hemisphere and the potential electrode is A at a distance P. Let a current I enter at E and leave at B.

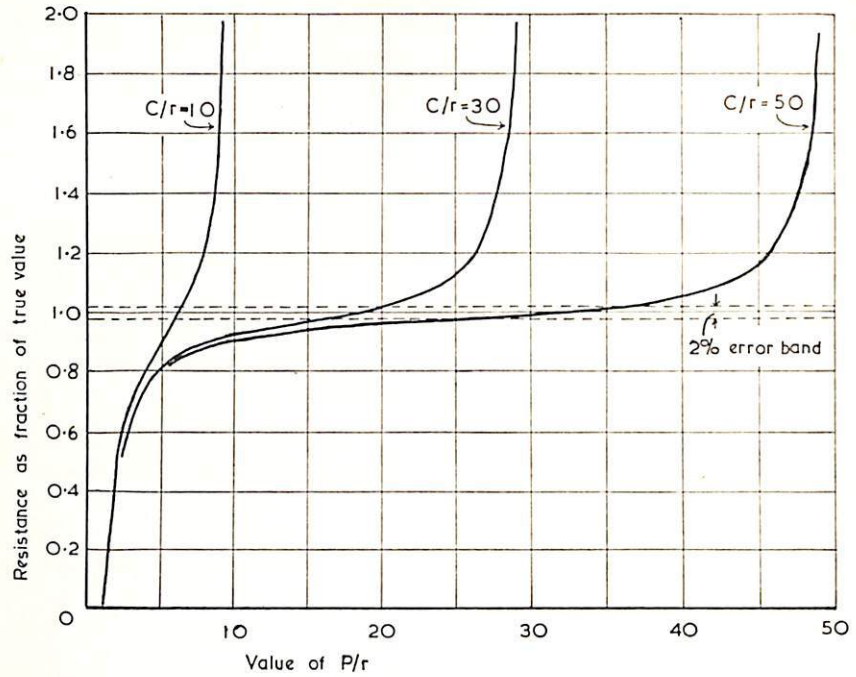


Fig. 10.6. *Calculated earth resistance curves for hemisphere.*

Then the potential at E due to the current I entering at E is $I\rho/2\pi r$, and that due to the current leaving at B is $-I\rho/2\pi(C-r)$ so that the total potential of E is

$$\frac{I\rho}{2\pi}\left[\frac{1}{r}-\frac{1}{(C-r)}\right] \tag{10.3}$$

Again the potential at A due to the current I entering at E is $I\rho/2\pi P$ and that due to the current I leaving at B is $-I\rho/2\pi(C-P)$, so that the total potential at A is

$$\frac{I\rho}{2\pi}\left[\frac{1}{P}-\frac{1}{(C-P)}\right] \tag{10.4}$$

Thus the potential difference between E and A is

$$V=\frac{I\rho}{2\pi}\left[\frac{1}{r}-\frac{1}{C-r}-\frac{1}{P}+\frac{1}{C-P}\right] \tag{10.5}$$

So the measured resistance

$$R = \frac{V}{I} = \frac{\rho}{2\pi}\left[\frac{1}{r} - \frac{1}{C-r} - \frac{1}{P} + \frac{1}{C-P}\right] \tag{10.6}$$

The true resistance of the hemisphere is $\rho/2\pi r$, and so the measured resistance expressed as a fraction of the true value is

$$\frac{\frac{1}{r} - \frac{1}{C-r} - \frac{1}{P} - \frac{1}{C-P}}{\frac{1}{r}} = 1 - \left[\frac{r}{C-r} + \frac{r}{P} - \frac{r}{C-P}\right]$$

$$= 1 - \left[\frac{1}{\frac{C}{r} - 1} + \frac{1}{\frac{P}{r}} - \frac{1}{\frac{C}{r} - \frac{P}{r}}\right] \tag{10.7}$$

Now let $C/r = c$ and $P/r = p$ then the ratio

$$\frac{\text{Measured resistance}}{\text{True resistance}} = 1 - \left[\frac{1}{c-1} + \frac{1}{p} - \frac{1}{c-p}\right] \tag{10.8}$$

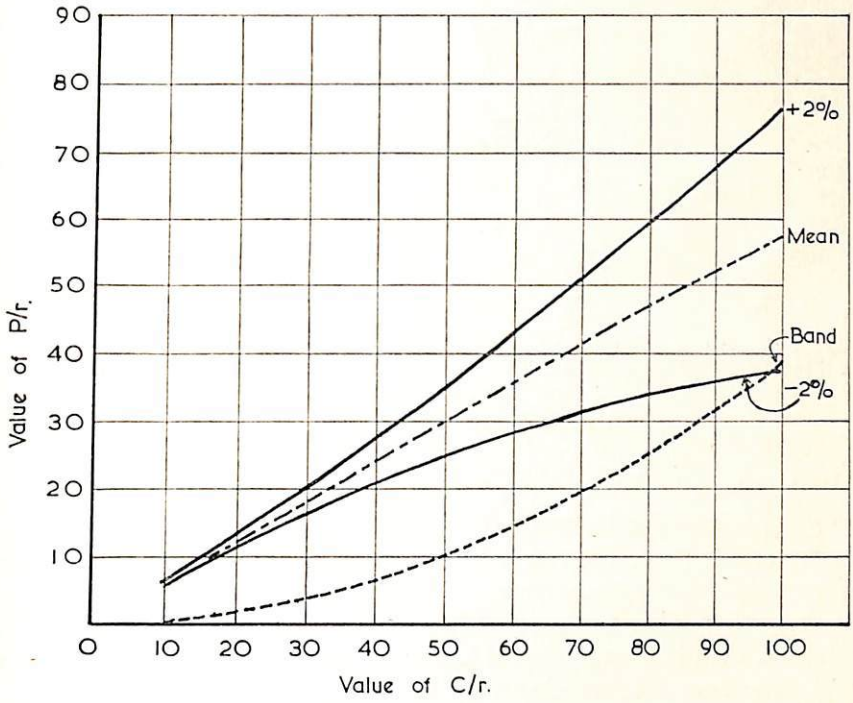


Fig. 10.7. *Values for errors of* ± 2 *per cent.*

and the expression in the bracket represents the fractional error due to the values of C and P chosen for the measurement. The curves given in Fig. 10.6 have been calculated from equation (10.8) for values of $c = C/r$ of 10, 30 and 50 and show clearly the result obtained when C is too small.

Calculation of Spacings for a given Error

In making an actual test it is usual practice to place the current and potential electrodes at distances which are considered to be satisfactory, to make a measurement, then to move the potential electrode 10 ft nearer the electrode E, then 10 ft nearer the electrode C and to make two additional measurements. If the three values so obtained do not differ from their mean value by more than an amount representing the required accuracy, then this mean is taken as the required value. The necessary spacings can be calculated for the hemispherical electrode for a given accuracy in the following manner.

Suppose the required fractional error is x.

Then

$$\frac{1}{c-1} + \frac{1}{p} - \frac{1}{c-p} = x$$

let

$$x - \frac{1}{c-1} = k$$

then

$$\frac{1}{p} - \frac{1}{c-p} = k$$

or

$$kp^2 - p(2 + kc) + c = 0$$

Solving this equation gives

$$p = \frac{(2 + kc) \pm \sqrt{(4 + k^2c^2)}}{2k} \qquad (10.9)$$

Values have been calculated for values of x, corresponding to errors of ± 2 per cent for various values of C/r, and these are given in the curves of Fig. 10.7. The two full line curves give the values of P/r for a given value of C/r and errors of ± 2 per cent. Now, if the practical method of moving the potential electrode 10 ft either way is to be adopted, then it is necessary to know the band of values of P/r within which the error does not exceed ± 2 per cent. This will be the difference between the two full line curves in Fig. 10.7 and is the dotted line curve

marked "Band". The second dotted line curve marked "Mean" gives the value of P/r at the centre of the band. The manner in which these curves can be used is as follows.

Since the potential electrode is moved a total of 20 ft, the band has the value $20/r$, and if for example r is 2 ft, then the band of P/r is 10.

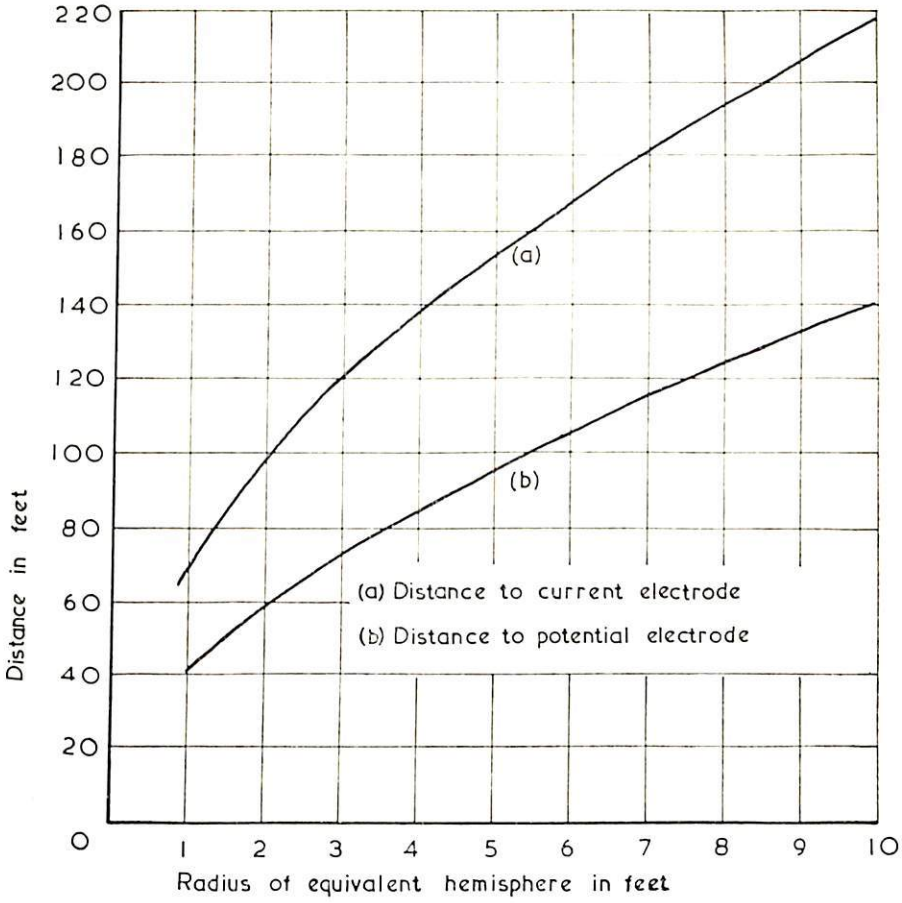


Fig. 10.8. *Spacing of auxiliary electrodes for* 2 *per cent error.*

From the curves of Fig. 10.7 the value of C/r is 49·4 and the mean P/r is 30. So the required spacings are 98·8 ft for the current electrode and 60 ft for the potential electrode.

The Equivalent Hemisphere

It would be very difficult to carry out calculations such as those in (10.6) for any other shape of earth electrode, but these calculations and

TABLE 10.1. RODS 8 FT LONG, 1 IN DIA. IN SOLID SQUARE FORMATION

Distance between rods-feet	*4 rods*		*9 rods*		*16 rods*		*25 rods*		*36 rods*	
	Equivalent hemisphere	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*
2	1·94	0·65	2·47	0·44	3·22	0·38	3·90	0·34	4·65	0·33
3	2·49	0·59	3·38	0·40	4·50	0·35	5·53	0·33	6·69	0·32
4	2·90	0·51	4·13	0·37	5·62	0·33	7·02	0·31	8·56	0·30
5	3·21	0·45	4·78	0·34	6·62	0·31	8·34	0·30	10·20	0·29
6	3·46	0·41	5·33	0·31	7·51	0·30	9·56	0·28	11·81	0·28
7	3·67	0·37	5·83	0·29	8·29	0·28	10·64	0·27	13·23	0·27
8	3·84	0·34	6·25	0·28	9·02	0·27	11·71	0·26	14·63	0·26
9	3·98	0·31	6·62	0·26	9·71	0·25	12·66	0·25	15·93	0·25
10	4·10	0·29	6·95	0·25	10·29	0·24	13·16	0·23	17·02	0·24

TABLE 10.1. (*continued*)

Distance between rods-feet	*49 rods*		*64 rods*		*81 rods*		*100 rods*	
	Equivalent hemisphere	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*
2	5·35	0·32	6·11	0·31	6·81	0·30	7·59	0·30
3	7·74	0·30	8·86	0·30	10·00	0·30	11·10	0·29
4	9·91	0·29	11·41	0·29	12·91	0·29	14·81	0·28
5	12·01	0·28	13·89	0·28	15·64	0·28	17·53	0·28
6	13·90	0·27	16·18	0·27	18·33	0·27	20·60	0·27
7	15·72	0·26	18·32	0·26	20·80	0·26	23·45	0·26
8	17·41	0·26	20·35	0·26	23·15	0·26	26·25	0·26
9	19·00	0·25	22·35	0·25	25·50	0·25	28·85	0·25
10	20·40	0·24	24·05	0·24	27·50	0·24	30·60	0·24

TABLE 10.2. RODS 8 FT LONG, 1 IN DIA. IN HOLLOW SQUARE FORMATION

Distance between rods-feet	*8 rods*		*12 rods*		*16 rods*		*20 rods*	
	Equivalent hemisphere	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*
1	1·61	0·57	1·97	0·47	2·39	0·42	2·79	0·39
2	2·81	0·50	3·54	0·42	4·31	0·38	5·07	0·36
3	3·76	0·44	4·81	0·38	5·91	0·35	6·97	0·33
4	4·51	0·40	5·86	0·34	7·25	0·32	8·58	0·30
5	5·13	0·36	6·74	0·32	8·40	0·30	9·98	0·28
6	5·65	0·33	7·49	0·29	9·40	0·28	11·24	0·27
7	6·08	0·31	8·15	0·27	10·23	0·26	12·22	0·25
8	6·46	0·29	8·69	0·26	10·99	0·24	13·23	0·23
9	6·79	0·27	9·21	0·24	11·69	0·23	14·04	0·22
10	7·06	0·25	9·65	0·23	12·28	0·22	14·81	0·21

TABLE 10.2. (*continued*)

Distance between rods-feet	*24 rods*		*28 rods*		*32 rods*		*36 rods*	
	Equivalent hemisphere	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*	*Equivalent hemisphere*	*Hemisphere Diagonal*
1	3·18	0·37	3·56	0·36	3·94	0·33	4·31	0·33
2	5·81	0·34	6·53	0·33	7·23	0·32	7·96	0·31
3	8·22	0·33	9·03	0·30	10·06	0·30	11·06	0·29
4	9·90	0·29	11·22	0·28	12·46	0·28	13·77	0·27
5	11·53	0·27	13·10	0·26	14·62	0·26	16·10	0·25
6	12·98	0·26	14·77	0·25	16·49	0·24	18·19	0·24
7	14·23	0·24	16·22	0·23	18·09	0·23	20·80	0·23
8	15·37	0·23	17·49	0·22	19·66	0·22	21·75	0·21
9	16·42	0·22	18·68	0·21	20·90	0·21	23·22	0·20
10	17·31	0·20	20·40	0·21	22·15	0·20	24·45	0·19

TABLE 10.3. RODS 8 FT LONG, 1 IN DIA. IN A CIRCLE

Number of rods	*Radius 10 ft*		*Radius 15 ft*		*Radius 20 ft*		*Radius 30 ft*	
	Equivalent hemisphere	*Hemisphere Diameter*	*Equivalent hemisphere*	*Hemisphere Diameter*	*Equivalent hemisphere*	*Hemisphere Diameter*	*Equivalent hemisphere*	*Hemisphere Diameter*
6	6·29	0·32	6·32	0·21	7·74	0·17	7·23	0·12
8	6·96	0,35	7·41	0·25	6·44	0·20	8·96	0·15
10	7·36	0·37	8·21	0·27	5·71	0·23	10·40	0·17
12	7·57	0·38	8·78	0·29	5·24	0·25	11·60	0·19
14	7·69	0·39	9·18	0·31	4·93	0·27	12·52	0·21
16	7·75	0·39	9·49	0·32	4·71	0·28	13·38	0·22
18	7·75	0·39	9·69	0·32	4·56	0·29	14·05	0·23
20	7·76	0·39	9·86	0·33	4·44	0·29	14·51	0·24
22	7·72	0·39	9·98	0·33	4·35	0·30	15·10	0·25
24	7·70	0·39	10·06	0·34	4·29	0·30	15·50	0·26
26	7·67	0·38	10·10	0·34	4·23	0·31	15·86	0·26
28	7·65	0·38	10·13	0·34	4·19	0·31	16·18	0·27
30	7·60	0·38	10·16	0·34	4·04	0·32	16·42	0·27

TABLE 10.4. STAR ARRANGEMENT OF WIRES 1 IN DIA., BURIED 3 FT

Length of each arm-feet	*3 arm*		*4 arm*		*6 arm*		*8 arm*	
	Equivalent hemisphere	*Hemisphere Diameter*	*Equivalent hemisphere*	*Hemisphere Diameter*	*Equivalent hemisphere*	*Hemisphere Diameter*	*Equivalent hemisphere*	*Hemisphere Diameter*
50	12·91	0·129	14·81	0·148	17·56	0·176	19·10	0·191
100	23·00	0·115	26·90	0·135	32·2	0·161	35·35	0·177
150	32·50	0·109	38·20	0·128	46·1	0·154	51·2	0·171
200	38·50	0·106	49·10	0·123	59·6	0·149	66·2	0·166
250	50·50	0·101	59·6	0·119	73·0	0·146	81·2	0·163
300	59·10	0·099	69·9	0·117	86·0	0·144	95·6	0·160
400	75·70	0·095	90·2	0·113	111·2	0·139	125·1	0·157
500	92·20	0·092	110·2	0·110	136·3	0·137	153·1	0·153

TABLE 10.5. RING OF WIRE 1 IN BURIED 3 FT DEEP DIA

Diameter of ring, ft	*Equivalent hemisphere*	*Hemisphere*
		Diameter
10	3·59	0·359
20	6·27	0·314
30	8·60	0·287
40	11·03	0·276
60	15·28	0·255
80	19·42	0·243
100	23·50	0·235
120	27·45	0·229
150	33·30	0·222
200	42·60	0·213

TABLE 10.6. HORIZONTAL WIRE 1 IN DIA., BURIED 3 FT DEEP

Length of wire, ft	*Equivalent hemisphere*	*Hemisphere*
		Length
50	5·74	0·115
100	9·97	0·0997
150	13·87	0·0926
200	17·58	0·0879
250	21·15	0·0846
300	24·55	0·0818
400	31·35	0·0784
500	39·00	0·0780

curves can be applied to any electrode by replacing it by its equivalent hemisphere, i.e. by a hemisphere having the same total resistance.

The Tables 10.1–10.6 give examples of the radii of the equivalent hemispheres for some of the electrode arrangements discussed in the preceding chapters.

From these tables and the curves of Fig. 10.7 it is possible to determine the necessary spacings for an accuracy of ± 2 per cent. If, for example, the earth electrode consists of a hollow square of 24 rods, with a spacing of 8 ft, then from Table 10.2, the radius of the equivalent hemisphere is 15·37 ft. Now, the curves in Fig. 10.8 and 10.9 have been obtained from Fig. 10.7 and give the spacings necessary to produce an error not exceeding ± 2 per cent.

It often happens that the earth electrode system is not so organised as the system discussed, and it is then necessary to determine the radius of the equivalent hemisphere. An examination of the tables suggests that, except in a few cases, the radius of the equivalent hemisphere is less than 0·5 of the maximum dimensions of the electrode system. If then the hemisphere is taken as 50 per cent of the maximum dimension

and the electrode spacings are based on this then, in general, any error in determining the hemisphere will tend to make the measurement more accurate. For instance, there might be a sub-station contained inside an earthed fence 100 ft square, with a number of rods and buried cables inside. Then the maximum dimension is the diagonal of the square which is 141 ft. The equivalent hemisphere is 50 per cent of this, say 70 ft. Then from Fig. 10.9 the required spacings will be 590 ft for the current electrode, and 370 ft for the potential electrode.

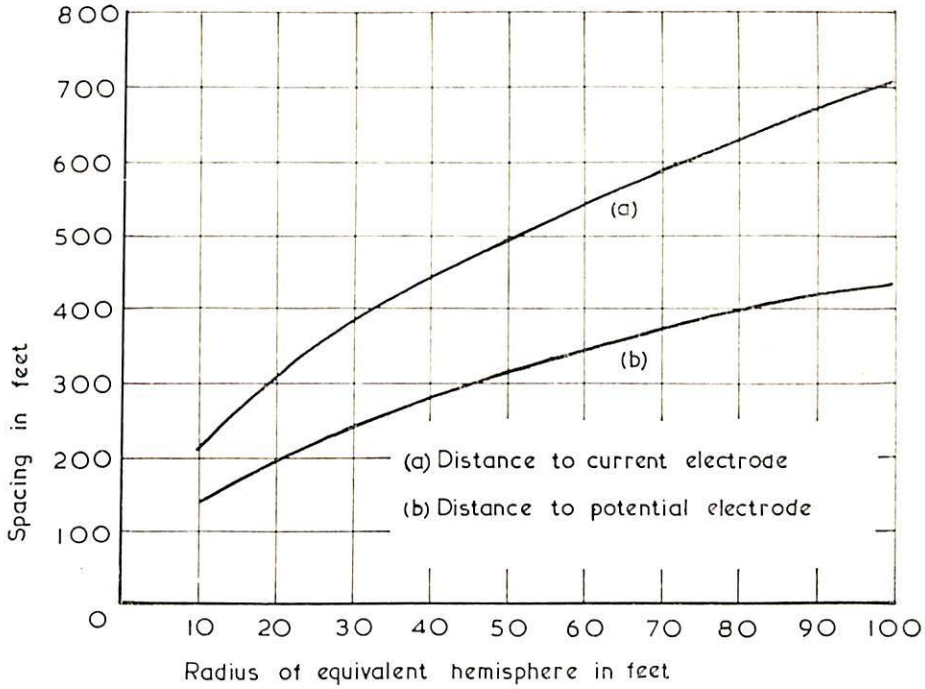


Fig. 10.9. *Spacing of auxiliary electrodes for* 2 *per cent error.*

Alternative Arrangement of Electrodes

It has sometimes been suggested that if there is insufficient space available for the correct electrode spacings, the potential electrode may be placed on the opposite side of the earth electrode to the current electrode. This unfortunately may give very erroneous results and there is no simple way of checking, during the test, that such errors are present. Suppose the earth electrode is represented again by a hemisphere as in Fig. 10.10, with the current electrode *B* on one side, and the potential electrode *A* on the other.

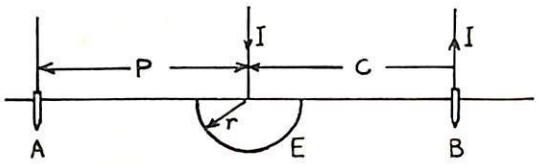


Fig. 10.10.
Alternative arrangement of electrodes.

The potential of the electrode E due to the current I is $I\rho/2\pi r$ and that due to the current $-I$ at B is $-I\rho/2\pi(C-r)$ and the total is thus

$$\frac{I\rho}{2\pi}\left[\frac{1}{r}-\frac{1}{(C-r)}\right]$$

The potential of the electrode A due to the current I at E is $I\rho/2\pi P$ and that due to the current $-I$ at B is $-I\rho/2\pi(C+P)$ and the total is

$$\frac{I\rho}{2\pi}\left[\frac{1}{P}-\frac{1}{C+P}\right]$$

The potential difference between E and A is therefore

$$V=\frac{I\rho}{2\pi}\left[\frac{1}{r}-\frac{1}{C-r}-\frac{1}{P}+\frac{1}{C+P}\right] \tag{10.10}$$

So that the resistance is

$$R=\frac{V}{I}=\frac{\rho}{2\pi}\left[\frac{1}{r}-\frac{1}{C-r}-\frac{1}{P}+\frac{1}{C+P}\right]$$

and the ratio of this to the true resistance $\rho/2\pi r$ is

$$\frac{\frac{1}{r}-\frac{1}{C-r}-\frac{1}{P}+\frac{1}{C-P}}{\frac{1}{r}}$$

$$=1-\frac{1}{\frac{C}{r}-1}-\frac{\frac{1}{P}}{\frac{r}{}}+\frac{1}{\frac{C}{r}+\frac{}{r}}$$

If $C/r = c$ and $P/r = p$, then this resistance ratio becomes

$$1 - \frac{1}{c-1} - \frac{1}{p} + \frac{1}{c+p} \tag{10.11}$$

Now the error will be

$$\frac{1}{c-1} + \frac{1}{p} - \frac{1}{c+p} = x \text{ say}$$

If as before

$$x - \frac{1}{c-1} = k$$

then

$$\frac{1}{p} - \frac{1}{c+p} = k$$

Some p and c are positive numbers $1/p$ is greater than $1/(c+p)$ and so k must be positive. Hence x must be greater than $1/(c-1)$. So if the error is not to exceed 2 per cent, i.e. $x = 0{\cdot}02$ than $(c-1)$ must be greater than 50 or c must be greater than 51. This may easily be a greater spacing than that required when the electrodes are in line. This is illustrated further in Fig. 10.11, which shows the earth resistance curves for both arrangements of electrodes for the case $c = 30$. It is evident from these curves that there is a band of values of p in the right-hand curve, from 16·5 to 20·3, within which the error in the measurement falls within the limits of ± 2 per cent. The left-hand curve never reaches the 2 per cent line, and in fact the minimum error is represented by the fraction $1/(c-1)$ in this case 1/29, i.e. 0·0345 or 3·5 per cent, and this is reached only when p approaches infinity.

There is a physical explanation for this failure of this system of electrodes to give a correct answer. The potential of E is reduced by the current leaving at B, and in the usual arrangement with the electrodes in line, this is counteracted by the increase in potential of A as it approaches B. This does not happen when the auxiliary electrodes are on opposite sides of E, as an increase in P means that electrode B is further away from A and the potential at B due to A is actually decreasing.

This arrangement of electrodes should not therefore be used, since if the area available is insufficient for the normal arrangement it will certainly be insufficient for the alternative.

Further Analysis based on Equivalent Hemisphere

Formula (10.6) gives the measured resistance of a hemispherical electrode for any value of P and C. In most cases r can be neglected in comparison with C, and so the equation can be rewritten:

$$R = \frac{\rho}{2\pi}\left[\frac{1}{r} - \frac{1}{C} - \frac{1}{P} + \frac{1}{C-P}\right]$$

$$= \frac{\rho}{2\pi r}\left[1 - \frac{1}{c} - \frac{1}{p} + \frac{1}{(c-p)}\right] \qquad (10.12)$$

As stated previously the true resistance $R_\infty = \rho/2\pi r$ and so

$$\frac{R}{R_\infty} = 1 - \frac{1}{c} - \frac{1}{p} + \frac{1}{(c-p)} \qquad (10.13)$$

Now for any given value of c, there is obviously a value of p for which the ratio $R/R_\infty = 1$, and this value must be such that

$$-\frac{1}{c} - \frac{1}{p} + \frac{1}{c-p} = 0$$

i.e.

$$-p(c-p) - c(c-p) + cp = 0$$

or

$$p^2 + cp - c^2 = 0$$

and

$$p = \frac{-c \pm \sqrt{5c^2}}{2}$$

Since p must be positive

$$p = c\left[\frac{\sqrt{5}-1}{2}\right] = 0{\cdot}6180c$$

or

$$P = 0{\cdot}618C \qquad (10.14)$$

This is an important result and means that no matter what value of C is used, the true resistance is obtained when $P = 61{\cdot}8$ per cent of c. This result is based on the ideal hemispherical electrode, but the idea may be extended to any earthing system in the following way. It can safely be assumed that the true resistance of any earthing system can be written as $R = \rho/2\pi f(D)$, where $f(D)$ represents some function of the dimensions

and arrangement of the various parts of the earthing system. Similarly the potential of the system due to a current I passing through it may be written as $I\rho/2\pi f(D)$. So, by a similar argument to the above, the actual measured resistance will be given by an equation similar to (10.12) with r replaced by $f(D)$. The major functional difficulty will be to determine the exact point in the earthing system which should be taken as the centre from which to make measurements of P and C. This problem is considered again later.

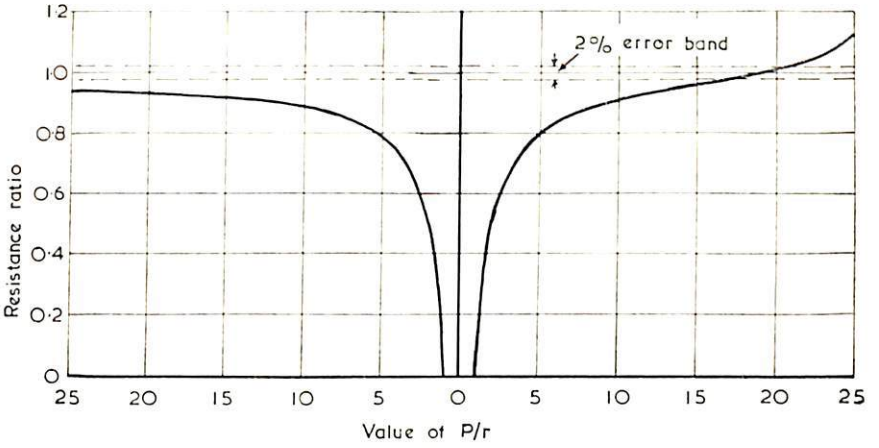


Fig. 10.11. *Comparison of electrode arrangements.*

Tests to Confirm the 61·8 per cent Rule

In order to test this theory it was decided to make a series of tests under controlled conditions. The first of these was made on a rod 3ft long, $\frac{1}{2}$ in. diameter, using distances to the current electrode of 10, 20, 40 and 80 ft. The earth resistance curves obtained are given in Fig. 10.12. Applying the theory that the correct resistance is given when $P = 0{\cdot}618C$ results in the figures given in Table 10.7.

The average of the figures in the last column is 57·7 ohms and the maximum variation from the average is less than 1 per cent. It is known that the radius of the equivalent hemisphere of this rod is 0·561 ft. Now at the mid-point $P = C/2$, the measured resistance is from equation (10.12)

$$R = \frac{\rho}{2\pi r}\left[1 - \frac{1}{c}\right]$$

TABLE 10.7

Distance to current electrode (ft)	*Distance to potential electrode (ft)*	*Resistance Ohms*
10	6·18	58·0
20	12·36	57·4
40	24·72	58·0
80	49·44	57·2

and so the measured values at the mid-points can be corrected. The figures from this calculation are given in Table 10.8.

The average of the figures in the last column is 57·1 ohms and this agrees within approximately 1 per cent with the average figure from Table 10.7.

The single rod is a fairly good approximation to the hemisphere and so tests were also made on a number of more complex systems. Among these was a group of rods, consisting of 40 arranged in a hollow square of side 5ft, each rod being 12 in. long and $\frac{1}{2}$ in. diameter. The earth resistance curves obtained for this are given in Fig. 10.13. An application of the 61·8 per cent rule to these gives the figures in Table 10.9.

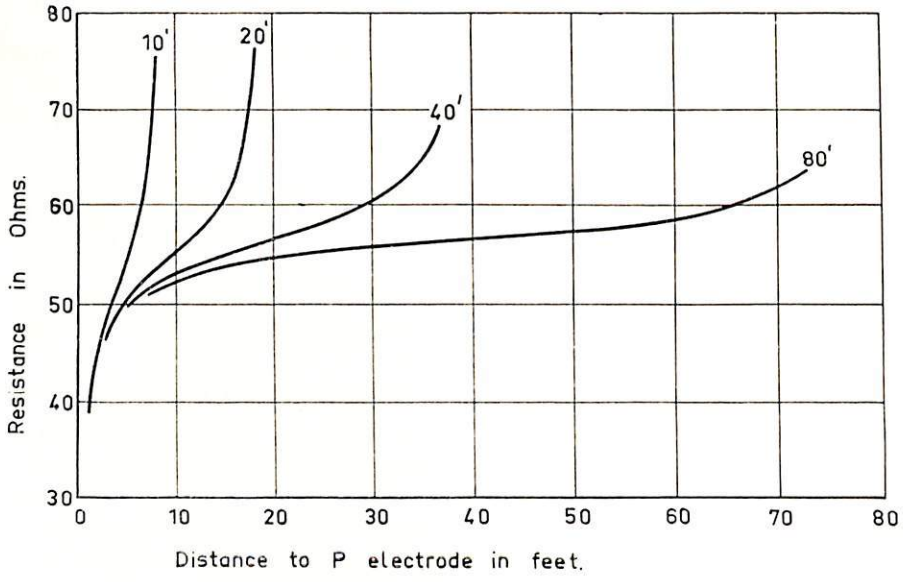


Fig. 10.12. *Resistance curves of one rod,* 3 *ft long,* $\frac{1}{2}$ *in. diameter.*

TABLE 10.8

Distance to current electrode (ft)	*Resistance at mid-point, ohms*	$c = \frac{C}{r}$	$1 - \frac{1}{c}$	*Corrected resistance, ohms*
10	54·1	17·83	0·9449	57·2
20	55·2	35·63	0·9720	56·8
30	56·7	53·50	0·9860	57·5
40	56·4	71·30	0·9930	56·8

The average of the figures in the last column is 18·5 ohms and the maximum variation from the average is 5·4 per cent. It is interesting to note that the figure for $C = 40$ ft is high compared with the others and the reason for this is not known. If this figure is ignored, the average of the remaining three is 18·2 ohms and the maximum variation from this average is 2·2 per cent.

Tests at an actual Sub-Station

Measurements were also made at an actual sub-station. This was a fairly large sub-station, containing a number of earth electrodes and surrounded by an earthed fence approximately 83 ft by 100 ft. The earth resistance curves are given in Fig. 10.14. The rather odd figures for the current electrode spacings are due to the fact that measurements were made from a point outside the fence and then corrected so that the zero point became the centre of the rectangle formed by the fence. It will be noted that even with the maximum electrode spacing used there is no horizontal position in the earth resistance curves.

If the 61·8 per cent rule is applied to these curves the figures obtained are those given in Table 10.10.

TABLE 10.9

Distance to current electrode (ft)	*Distance to potential electrode (ft)*	*Resistance in ohms*
20	12·36	18·6
40	24·72	19·5
80	49·44	18·0
130	80·34	18·0

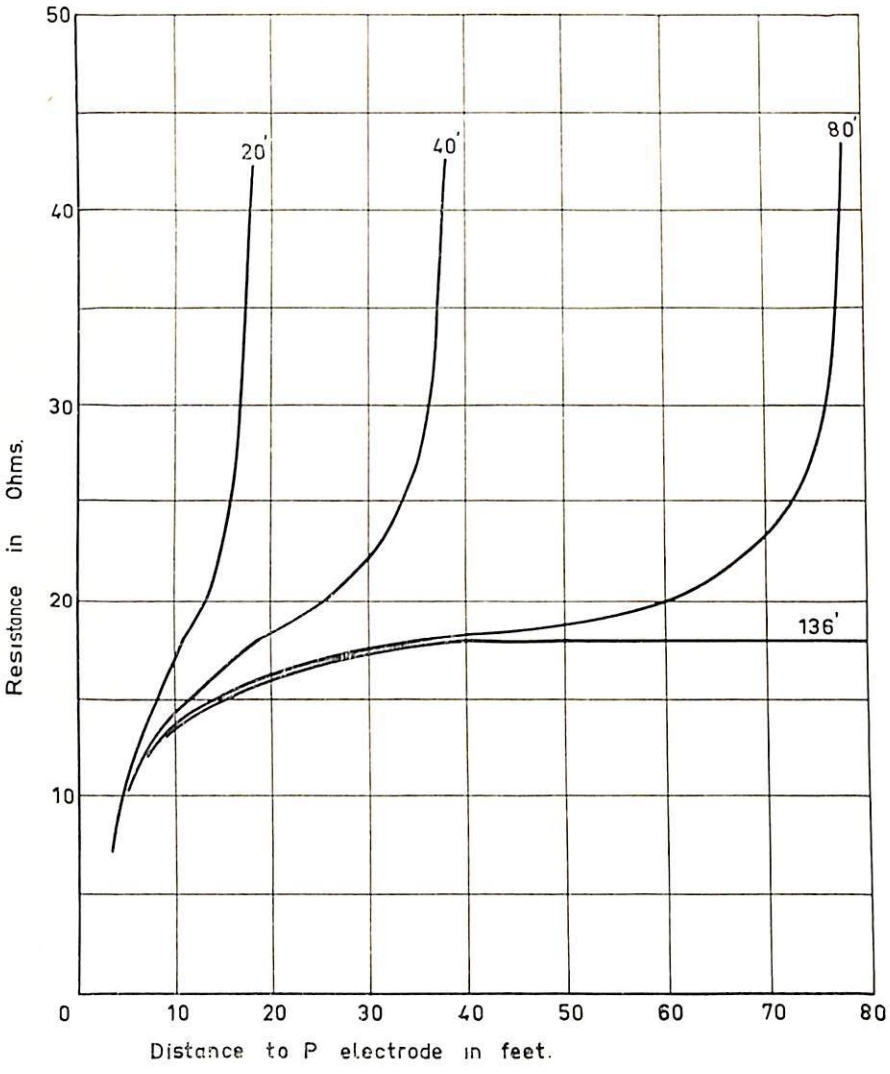


Fig. 10.13. *Resistance curves of* 40 *rods in* 5 *ft hollow square.*

TABLE 10.10

Distance to current electrode (ft)	*Distance to potential electrode (ft)*	*Resistance in ohms*
171	105·7	0·231
251	155·1	0·216
351	216·9	0·216
431	266·4	0·213

The average of the figures in the last column is 0·219 ohms and the maximum variation from this average is 5·5 per cent. It is notable that as the distance to the current electrode is increased, the resistance obtained by this method decreases. This will be referred to again later.

Difficulties with Large Earthing Systems

There are two major difficulties in the application of this method to large earthing systems. The first which is not confined to large systems but applies equally to small ones, is that the theory is based on homogeneous soil. As the distance to the current electrode is increased, the degree of penetration of the testing current into the ground will change and the effect of any non-homogeneities in the soil may be different for various values of the current electrode spacing. This may possibly be the reason for the one odd result in Table 10.9 and for the decreasing values in Table 10.10.

The second difficulty is a practical one and consists in deciding where the true centre lies, i.e. the point from which measurements to the potential and current electrodes should be made. In an earthing system containing a large number of electrodes of various types this may become practically impossible. One way out of this difficulty would be to make the electrode spacings sufficiently high, so that errors in deciding this centre point are not important. As, however, this would frequently involve spacings running into thousands of feet, it is very seldom that this would be a possible solution.

Suggested Method for determining Earth Resistance when True Centre is not known

The general equation for the measured resistance of a hemisphere is (10.12). This can be rewritten

$$R = K\left[\alpha - \frac{1}{P} - \frac{1}{C} + \frac{1}{C-P}\right]$$

and the true resistance

$$R_\infty = K\alpha$$

Now suppose there has been an error X in locating the true centre from which the measurements should be made. Then P must be replaced by $(P + X)$ and C by $(C + X)$ so

$$\left[\alpha - \frac{1}{P+X} - \frac{1}{C+X} + \frac{1}{C-P}\right]$$

Let $P = \lambda C$, then

$$R = K\alpha + \frac{K}{C}\left[-\frac{1}{\lambda\left(1+\frac{X}{\lambda C}\right)} - \frac{1}{1+\frac{X}{C}} + \frac{1}{1-\lambda}\right]$$

For small values of X/C:

$$\frac{1}{1+\frac{X}{\lambda C}} = 1 - \frac{X}{\lambda C}$$

$$\frac{1}{1+\frac{X}{C}} = 1 - \frac{X}{C}$$

Write $X/C = \delta$, then

$$R = K\alpha + \frac{K}{C}\left[-\frac{1-\delta/\lambda}{\lambda} - (1-\delta) + \frac{1}{1-\lambda}\right]$$

$$= K\alpha + \frac{K}{C}\left[\delta\left(\frac{1+\lambda^2}{\lambda^2}\right) + \frac{\lambda^2+\lambda-1}{(1-\lambda)}\right]$$

$$= K\alpha + \frac{K}{C}[f\delta + g] \qquad (10.15)$$

where

$$f = \frac{1+\lambda^2}{\lambda^2}, \qquad g = \frac{\lambda^2+\lambda-1}{(1-\lambda)}$$

In equation (10.15), there are three unknown quantities, $K\alpha$ which is the true resistance, K/C and δ.

Suppose three values of λ are chosen, λ_1, λ_2 and λ_3 resulting in corresponding values of f and g, and the measured resistances are R_1, R_2 and R_3. Then

$$R_1 = K\alpha + \frac{K}{C}[f_1\delta + g_1] \tag{10.16}$$

$$R_2 = K\alpha + \frac{K}{C}[f_2\delta + g_2] \tag{10.17}$$

$$R_3 = K\alpha + \frac{K}{C}[f_3\delta + g_3] \tag{10.18}$$

From (10.16) and (10.17)

$$R_1 - R_2 = \frac{K}{C}[(f_1 - f_2)\delta + (g_1 - g_2)] \tag{10.19}$$

From (10.17) and (10.18)

$$R_2 - R_3 = \frac{K}{C}[(f_2 - f_3)\delta + (g_2 - g_3)] \tag{10.20}$$

Dividing (10.19) by (10.20) gives

$$\frac{R_1 - R_2}{R_2 - R_3} = \frac{(f_1 - f_2)\delta + (g_1 - g_2)}{(f_2 - f_3)\delta + (g_2 - g_3)}$$

Solving this for δ leads to

$$\delta = \frac{R_1(g_2 - g_3) + R_2(g_3 - g_1) + R_3(g_1 - g_2)}{-R_1(f_2 - f_3) - R_2(f_3 - f_1) - R_3(f_1 - f_2)} \tag{10.21}$$

Substituting this value of δ in (10.19) leads to

$$\frac{K}{C} = \frac{-R_1(f_2 - f_3) - R_2(f_3 - f_1) - R_3(f_1 - f_2)}{f_1(g_2 - g_3) + f_2(g_3 - g_1) + f_3(g_1 - g_2)} \tag{10.22}$$

Substituting from (10.21) and (10.22) in (10.16) leads to

$$K\alpha = R_\infty = \frac{R_1(f_2g_3 - f_3g_2) - R_2(f_1g_3 - f_3g_1) - R_3(f_2g_1 - f_1g_2)}{f_1(g_2 - g_3) + f_2(g_3 - g_1) + f_3(g_1 - g_2)} \tag{10.23}$$

It now remains to choose suitable values of λ. These should not be too small for two reasons. One is that the approximations used in the

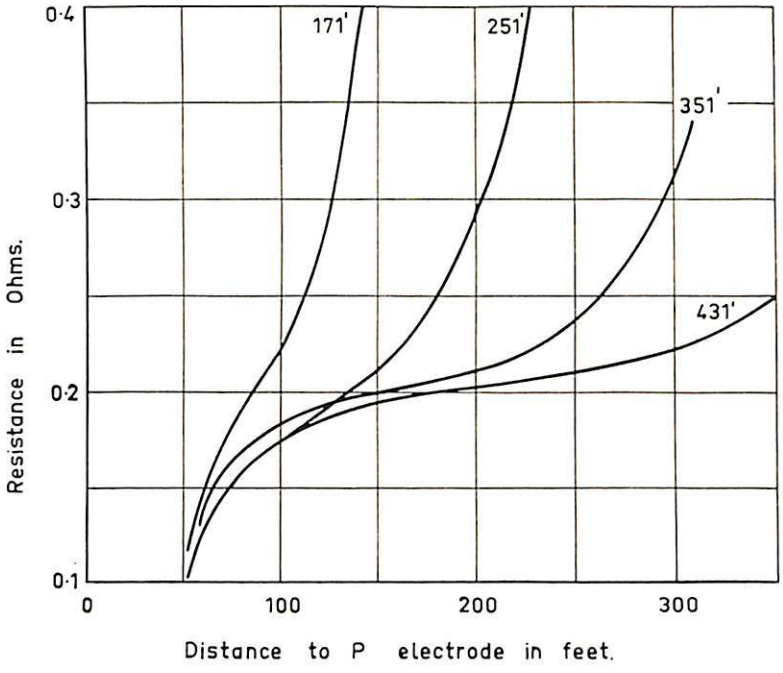


Fig. 10.14. *Resistance curves of substation.*

calculations assume that λ is not small and the other is that the agreement between the earth resistance curve of any electrode and that of its equivalent hemisphere becomes better the greater the distance from the electrode. On the other hand λ should not be too large, as the current electrode will have too great an influence on the results if it is. The most suitable values are suggested as:

$$\lambda_1 = 0{\cdot}4 \qquad \lambda_2 = 0{\cdot}6 \qquad \lambda_3 = 0{\cdot}8$$

This gives the following figures

$$f_1 = 7{\cdot}2500 \qquad g_1 = -1{\cdot}8333$$

$$f_2 = 3{\cdot}7778 \qquad g_2 = -0{\cdot}1667$$

$$f_3 = 2{\cdot}5625 \qquad g_3 = \quad 2{\cdot}7500$$

Substituting these in equation (10.23) gives

$$R_\infty = -1{\cdot}3350R_1 + 3{\cdot}0407R_2 - 0{\cdot}7057R_3 \qquad (10.24)$$

If this method is applied to the three largest spacings in the curves of Fig. 10.14 the figures obtained are as follows:

Current electrode at 451 ft

$\lambda = 0{\cdot}4$	$P = 180$	$R_1 = 0{\cdot}198$	Calculated $R = 0{\cdot}206\ \Omega$
$\lambda = 0{\cdot}6$	$P = 271$	$R_2 = 0{\cdot}213$	
$\lambda = 0{\cdot}8$	$P = 361$	$R_3 = 0{\cdot}266$	

Current electrode at 351 ft

$\lambda = 0{\cdot}4$	$P = 140$	$R_1 = 0{\cdot}197$	Calculated $R = 0{\cdot}204\ \Omega$
$\lambda = 0{\cdot}6$	$P = 211$	$R_2 = 0{\cdot}213$	
$\lambda = 0{\cdot}8$	$P = 281$	$R_3 = 0{\cdot}272$	

Current electrode at 251 ft

$\lambda = 0{\cdot}4$	$P = 100$	$R_1 = 0{\cdot}176$	Calculated $R = 0{\cdot}212\ \Omega$
$\lambda = 0{\cdot}6$	$P = 151$	$R_2 = 0{\cdot}210$	
$\lambda = 0{\cdot}8$	$P = 201$	$R_3 = 0{\cdot}292$	

The smaller electrode spacings were not used as it was felt that $X/\lambda C$ was becoming too large for the approximations in the calculations to be valid. The average of the three calculated values is 0·207 ohms with a maximum variation of 2·5 per cent. This average value is about 6 per cent less than that given in Table 10.10 but the individual values leading to the average are more consistent.

It is evident that more practical experience is required with this method to determine its utility.

11

INSTRUMENTS AND EQUIPMENT FOR EARTH-RESISTANCE MEASUREMENTS

In Chapter 10 the method of measuring an earth electrode resistance is described and in Chapter 2, the method of measuring soil resistivity. It is now necessary to consider the instruments and equipment necessary to enable these measurements to be carried out. In both cases the resistance measured is that of a body of earth and some of the characteristics of this have been discussed in preceding chapters.

When considering a suitable instrument for making these measurements, however, there are certain other factors to be considered. Conduction through the soil is, as already pointed out, almost entirely electrolytic, and so the passage of an electrical current through the soil will set up back e.m.f's. These can cause currents to flow through the measuring circuits of the instruments and, unless precautions are taken, can produce errors in the measurement.

Another point which must be considered is the presence of stray currents in the soil. With the large electricity supply systems and electrical traction systems of the present day, a large amount of stray current is to be found in the soil. This is particularly true in the neighbourhood of sub-stations and so becomes an extremely important factor, when the earth resistance of such a sub-station is being measured. These stray currents may be of power frequency, 50 cycles per second in this country, or may be third harmonic currents. In any case steps must be taken to eliminate them from the instrument.

Any instrument must be self-contained, it must possess its own source of power, battery or hand generator, and must be readily portable. Measurements usually have to be made in places where no power supply is available and, particularly in resistivity surveying, instruments may have to be carried long distances over difficult terrain. The auxiliary equipment in the form of electrodes, connecting cables and so on, must be robust but, at the same time, as light and convenient as possible.

The Three Components of an Instrument

Any instrument designed for this purpose will consist essentially of three components:

(1) Power source
(2) Device for eliminating electrolytic and stray currents
(3) The measuring system.

A number of modern instruments are available in which all three components take different forms.

The Power Source

In making a measurement of an earth electrode resistance, the testing current has to pass through the resistance of the earth electrode itself and also through the auxiliary current electrode. In making a resistivity measurement the testing current has to pass through two current electrodes. Thus, the current has to pass through a circuit, the resistance of which may be several hundreds of ohms and may even, in certain cases, be thousands of ohms. So, unless very delicate measuring equipment is to be used, the voltage used has to be fairly high. In actual instruments it varies from 50 to 500 volts and this must be produced by a self-contained unit in the instrument itself, or if, as in certain special cases, it is a separate unit, this must be small and light, and easily connected to the measuring circuit.

Owing to the possibility of electrolysis it is not advisable to use direct current although this could have definite advantages in other ways. Alternating current or periodically reversed direct current are universally employed. Consideration must then be given to the frequency to be used. Electrolysis does occur even with alternating current of power frequency so that it would appear that a higher frequency is to be preferred. Care is necessary here, however, as with higher frequencies it is possible to have mutual induction effects between the testing cables and between the cables and earth. Since soil is a comparatively high resistance conductor there is also the possibility of skin effect in the soil. The best frequency is probably one differing from the power frequency but not too high, say 60 to 70 cycles per second. It is necessary to depart from the power frequency itself, as this makes the elimination of power frequency stray currents easier.

The requirements of the power source may therefore be summed up as a voltage to suit the measuring circuit, 50 to 500 volts, of a frequency of 60 to 70 cycles per second and capable of delivering sufficient current for the measuring circuit required.

In many instruments the power source is a small hand-driven generator and, in this case, it is fairly easy to develop the required voltage and there is an advantage in that the frequency also can be varied by altering the handle speed. An alternative used by some manufacturers is a transistor converter operated from a battery. This can be used to develop the required voltage but the frequency is fixed and this may be a disadvantage.

Device for Eliminating Electrolytic and Stray Currents

This is a very important and characteristic part of any earth testing instrument. Any stray current, i.e., any current other than that produced by the instrument must be prevented from affecting the measuring circuit. This can be done either by arranging a circuit to by-pass the measuring circuit, or to chop the stray current up so that it can have no effect. The detecting or measuring instruments are normally of the moving-coil system, so that the current has to be rectified before passing through the instrument. In the case where the power source is a battery-operated transistor converter, the rectifier will be a static one, and arrangements must be made in this rectifier unit to reject any current not of the same frequency as the testing current.

When the power source is a hand-driven generator, a mechanical rectifier is mounted on the shaft. This will rectify synchronously the current from the set itself. By varying the handle speed or frequency, the stray currents will be rectified asynchronously and the nett current through the measuring circuit will be zero.

In some instruments the power source is a battery-driven vibrator and transformer. In this case rectifier contacts are mounted in the vibrator and act in the same way as the mechanical rectifier used with the hand generator.

The Measuring System

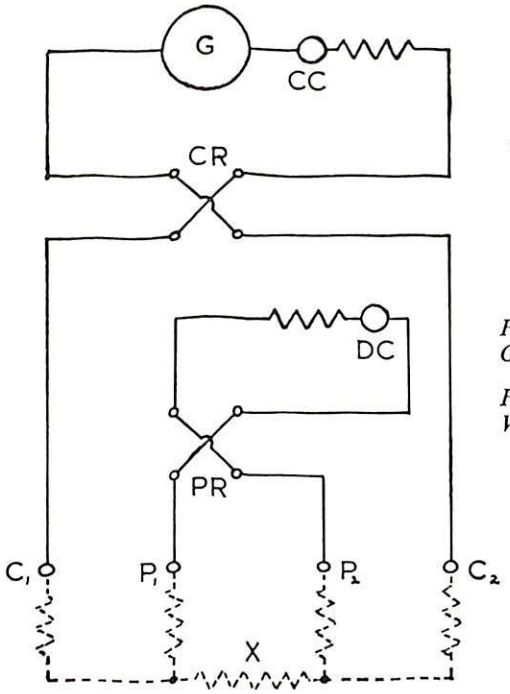
In modern instruments there are three forms of measuring circuit used. These may be defined as:

(1) The voltmeter-ammeter or ohmmeter system
(2) The resistance comparator system
(3) The null balance system.

Each of these systems has advantages and disadvantages.

The Ohmmeter System

A diagram of a typical instrument using the ohmmeter system is given in Fig. 11.1. The power source is a hand-driven d.c. generator



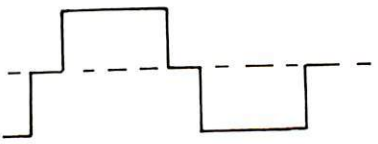


Fig. 11.1. (*left*).
Ohmmeter type earth tester.

Fig. 11.2. (*top*).
Waveform of ohmmeter instrument.

G, and current from this generator passes through the control coil CC of a moving coil ohmmeter and from there to a current reverser CR before being passed through the soil via the terminals C_1 and C_2.

The current reverser CR is mounted on the shaft of the generator and periodically reverses the current, so that flowing through the soil has the wave-form given in Fig. 11.2. The passage of this current through the resistance under test, shown dotted in Fig. 11.1, produces a voltage drop across it having the same wave-form. This voltage drop in turn sends a current via the terminals P_1 and P_2 to a circuit consisting of a potential rectifier PR and the deflecting coil of the moving coils ohmmeter DC. The potential rectifier is mounted also on the shaft of the generator and runs in synchronism with the current reverser CR, so that the current through DC is a rectified version of Fig. 11.2. Now, if I is the main current, the voltage drop across X is IX and the current through DC is proportional to this. The position taken by the moving system of a moving coils ohmmeter is dependent on the ratio of the currents in the two coils and so is dependent on X. The instrument can thus be calibrated to read X directly.

Any direct current produced by electrolytic action is converted by the potential rectifier *PR* into an alternating current and so can produce no effect on the moving coils ohmmeter. Stray alternating currents are chopped by the rectifier and unless the generated frequency happens to be the same as that of the stray currents, can have no effect on the instrument reading. If the frequencies do coincide, this is indicated by wandering of the instrument pointer and this is readily eliminated by running the hand-driven generator at a faster speed until the reading becomes steady.

The resistances of the current electrodes are included in the main current circuit and so affect the value of this current. Since this, in turn,

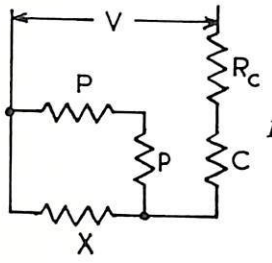


Fig. 11.3. *Simplified circuit.*

affects the voltage drop across the resistance X, the value of the current electrode resistances have no effect on the instrument readings. The resistances of the potential electrodes are in series with the internal resistance of the instrument and, if the latter is made high enough, the effect of the values of potential electrode resistance normally encountered can be made very small.

There are, however, limits to what can be done in this connection. It is possible to apply a correction to the readings if the potential electrode resistances are high. To examine this, consider the simplified circuit of Fig. 11.3.

In this diagram:

V = voltage of the generator
R_c = internal resistance of the control circuit
C = total resistance of the current electrodes
P = internal resistance of the deflecting circuit
p = total resistance of the potential electrodes
X = the resistance under test.

Now if I is the value of current passing through R_c and C, it is evident from the law of parallel circuits, that the current passing through P is $XI/(X + P + p)$, and so the ratio of the currents in the two coils is

$$\frac{X}{X + P + p}$$

The reading on the instrument is determined by this ratio and it is evident, that if p is not constant, then errors will be introduced. When calibrating the potential electrode resistance is given a value p_1 and the reading on the scale will be X, when the current ratio is

$$\frac{X}{X + P + p_1}$$

For any value of p therefore the actual reading on the instrument will be

$$\frac{X(X + P + p_1)}{(X + P + p)}$$

or since X is usually small compared with P the reading will be

$$R = \frac{X(P + p_1)}{(P + p)} \tag{11.1}$$

In order to apply this correction it is necessary to measure p. This can be done by altering the connections at the instrument terminals as indicated in Fig. 11.4. In this diagram

(a) shows the connections for measuring the resistance of an earth electrode. The terminals P_1 and C_1 are joined together and to the earth electrode, P_2 is connected to the potential electrode and C_2 to the current electrode;

(b) shows the connections for measuring the potential electrode resistance. The potential electrode is connected to the linked terminals P_1 and C_1 and the earth electrode is connected to the terminal P_2, the terminal C_2 remaining connected to the current electrode. It has been shown in Chapter 10 that this method of connection can give errors, but since this would be an error in a correction, it is not serious;

(c) shows the usual connections for soil resistivity, and

(d) shows the connections for measuring the total resistance of the potential electrodes. The terminals P_1 and C_1 are linked

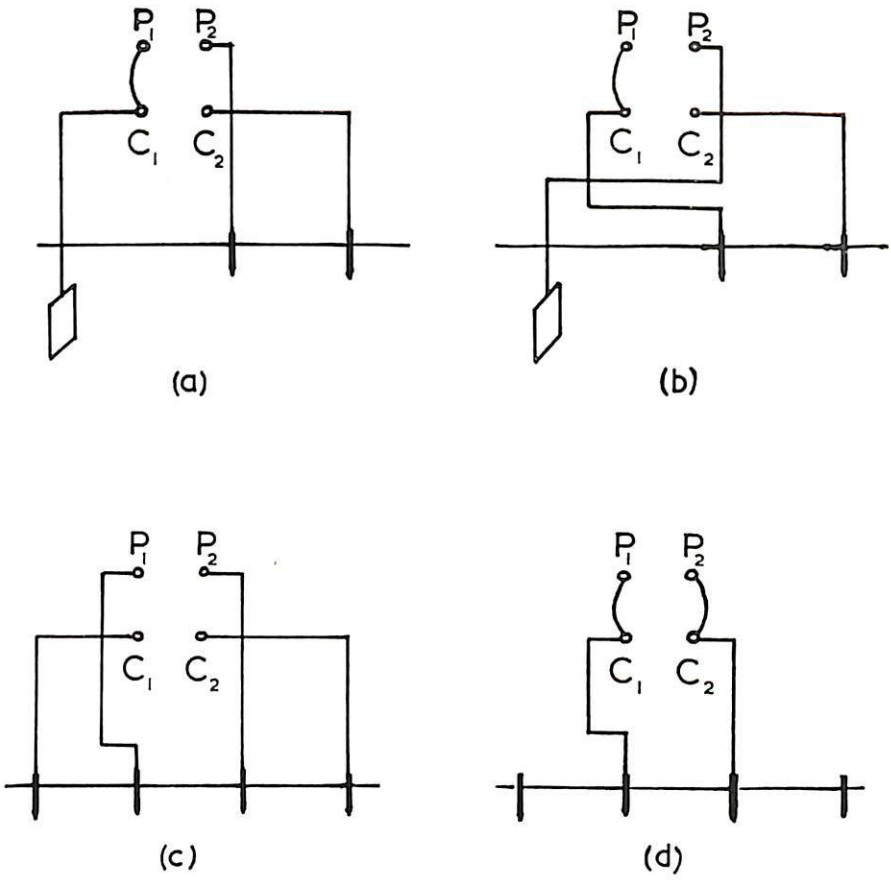


Fig. 11.4. *Arrangement of instrument connections.*

together and connected to one potential electrode, and terminals P_2 and C_2 are linked together and connected to the second potential electrode.

Having obtained the value of p, equation (11.1) can be used to calculate the value of X, for a measured value of R.

Two or more ranges can be provided by altering the internal resistances of the control and deflecting circuits. When such an instrument is used and measurements can be made on two successive ranges, the true values can be obtained without measuring the potential electrode resistance.

Suppose
R_1 = reading
P_1 = internal deflecting circuit resistance
p_1 = calibration value of potential electrode resistance
} on lower range

R_2 = reading
P_2 = internal deflecting circuit resistance
p_2 = calibration value of potential electrode resistance
} on higher range

X = true value of earth resistance
P = actual potential electrode resistance

Then

$$X = \left(\frac{P_1 + P}{P_1 + p}\right)R_1$$
$$X = \left(\frac{P_2 + P}{P_2 + p}\right)R_2 \qquad (11.2)$$

By eliminating p between these two equations

$$X = \frac{R_1R_2(P_2 - P_1)}{R_1(P_2 + p) - R_2(P_1 + p)} \qquad (11.3)$$

From this equation and knowing the internal resistances of the two ranges and the calibration particulars it is possible to calculate the true resistance X. For one form of this instrument, a nomogram can be calculated based on this equation and this is given in Fig. 11.5.

The Resistance-comparator System

A diagram of this type of instrument is given in Fig. 11.6. V is an alternating voltage from a suitable source and produces a current I flowing through the primary of the current transformer T, the current electrodes C_1 and C_2 and the earth resistance X. If the current transformer has a ratio of unity, then a current I passes also through the standard resistance R. The voltage drop across $R = IR$ is added to the voltage drop across $X = IX$, and the total voltage drop is $I(R + X)$. This produces a current through the indicating instrument G, equal to

$$I(R + X)/(P_1 + P_2 + R_1),$$

where R_1 is an adjustable resistance and includes the resistance of G.

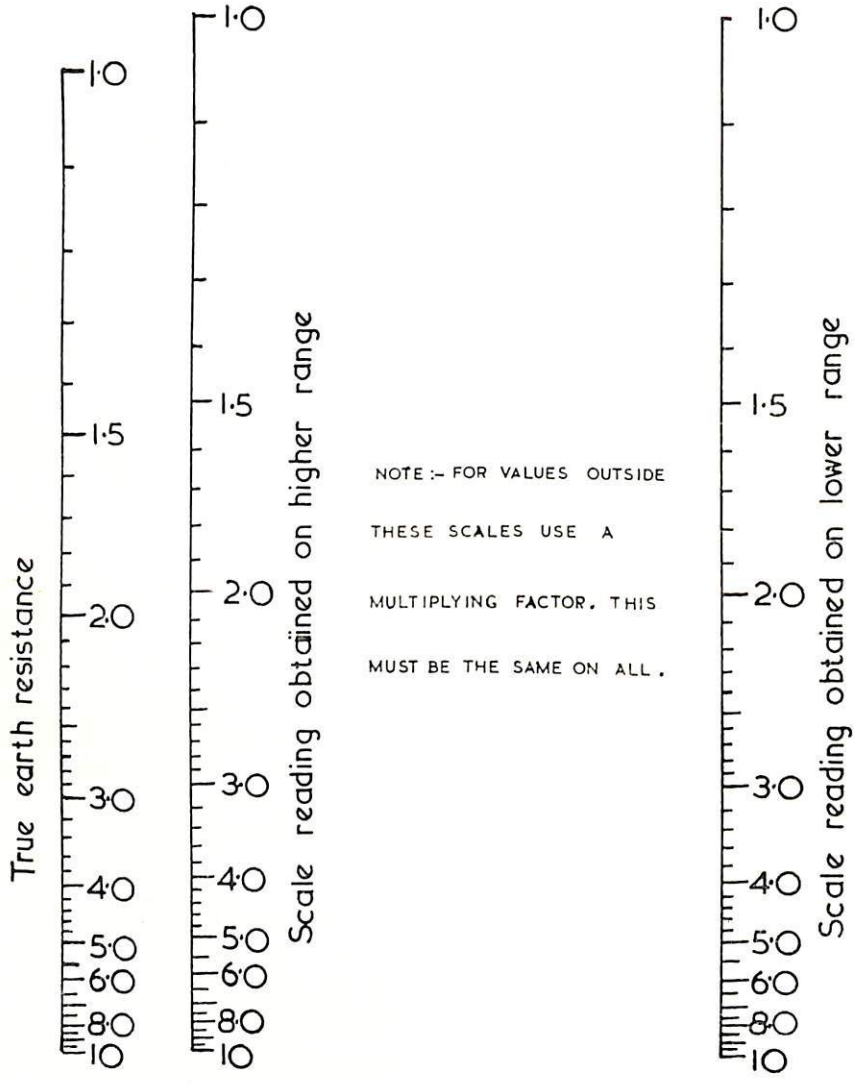


Fig. 11.5. *Nomogram for use with one form of ohmmeter earth tester* (2-*range series* 2 *Megger Earth Tester*) *to give true earth resistance when readings can be obtained on both ranges.*

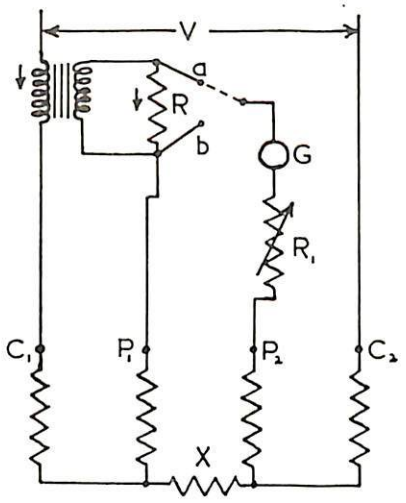


Fig. 11.6. *Principle of comparator system.*

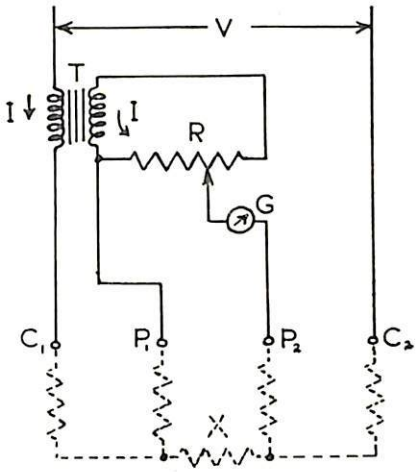


Fig. 11.7. *Principle of null balance system.*

The procedure now is to adjust R_1 until the reading on the indicator is full scale corresponding to a current I_0. This means

$$\frac{I(R+X)}{P_1+P_2+X} = I_0$$

If now the switch is moved to position (b), the volt drop applied to the circuit containing G is IX. The current is then

$$\frac{IX}{P_1+P_2+X} = i$$

Dividing one equation by the other then

$$\frac{i}{I} = \frac{X}{R+X} \tag{11.4}$$

This means that the fraction of the full scale deflection is dependent only on X, and so the scale of the instrument can be calibrated to read X directly. The adjustment of R_1 makes full correction for the resistances of both current and potential electrodes.

In the practical realisation of this instrument, the source is a hand-driven generator and this carries a synchronous rectifier on the shaft. The indicator G is a moving coil instrument and the synchronous rectifier will thus serve to eliminate any stray currents in the manner already described.

Multi ranges are provided by tappings on the transformer and by altering the value of the resistance R.

It will be noted that the scale shape will be somewhat limited if controlled only by equation (11.4), and in practice this is modified by the use of a shunt across the indicating instrument.

Null Balance System

The basic diagram of the null balance system is given in Fig. 11.7. Current from an a.c. power source V passes a current I through the primary of a current transformer T and thence through the earth via the terminals C_1 and C_2. The transformer T, assumed at the moment to have a 1:1 ratio has a potentiometer resistance R connected to its secondary, so that a current I also flows through this resistance. The procedure then is to balance the voltage drop across the tapped off portion of R, the balance being indicated by zero current in the galvanometer G. Since R and X are carrying equal currents, it is evident that at balance $R = X$. The potentiometer resistance R can thus be scaled to read directly the value of the resistance X.

The resistances of the various electrodes can theoretically have no effect on the balance condition. The resistances of the current electrodes are included in the main current circuit and thus are factors affecting the value of the current I. Since, however, R and X are carrying equal currents I, the balance is unaffected by changes in I. The resistances of the potential electrodes are in series with the galvanometer G, and since at balance no current is flowing through this circuit, these resistances can have no effect at least theoretically, on the balance condition. It may happen that with high electrode resistances, the sensitivity of the balance may be affected, in that the amount of current which can flow for a given degree of out of balance may be reduced.

The adjustable potentiometer may either be a continuously adjustable rheostat, or may be arranged like a resistance box, with the figures shown as digits. Multi-range instruments can be provided in various ways, among these being a tapped transformer so that on some ranges the transformer ratio is no longer unity. Precautions also have to be taken against overloading the galvanometer when the adjustment is some way off balance, and shunts are often provided for this purpose.

Combined Systems

There is one instrument in which two of the systems of measurements already described are combined. This is the Geophysical "Megger" Earth Tester, made by Evershed and Vignoles Limited of London,

and it combines the ohmmeter and null balance systems. The basic diagram of this instrument is given in Fig. 11.8.

The source of power is again a hand-driven generator, but in this case there are two independent windings T_1 and T_2. The main testing current is provided by the winding T_1, and passes through the control coil CC and thence through the earth via the terminals C_1 and C_2.

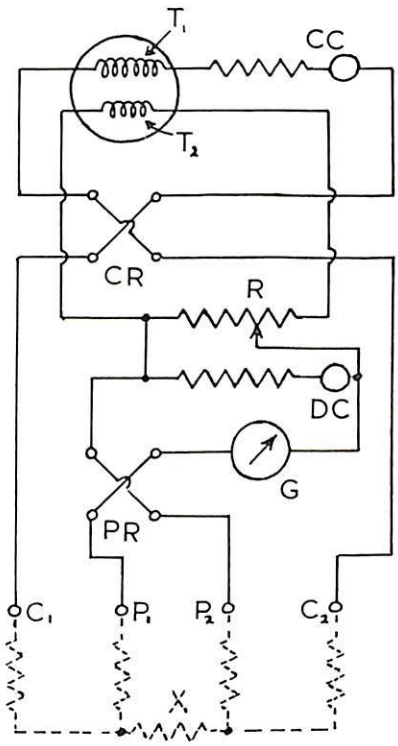


Fig. 11.8. *Combination of ohmmeter and null balance.*

The voltage drop across X is balanced against a voltage drop across an adjustable resistance R fed from the second winding T_2 the balance being indicated by the galvanometer G. The deflecting circuit of the ohmmeter is connected also across the resistance R, and so the voltage applied to the deflecting circuit is independent of the potential electrode resistances. The ohmmeter is again calibrated directly in terms of the resistance. The advantage of this system is that lower ranges are possible than with the simple ohmmeter because a lower resistance and so more sensitive deflecting circuit is possible. Multi ranges can be

provided by altering the circuit resistance and connecting suitable shunts across R. The ranges actually provided are:

0–0·3, 0–1, 0–3, 0–10 and 0–30 ohms.

Typical Instruments

A number of instruments commercially available are illustrated in Plate 1. A brief description of these instruments is as follows:

(*a*) *"Megger" Earth Tester, Series* 2, *made by Evershed and Vignoles Limited of London.* A number of two-range instruments are available and one has ranges 0–30 and 0–300 ohms. The calibration values of the potential electrode resistance are 600 ohms for the 30 ohms range and 3000 ohms for the 300 ohms range. The nomogram of Fig. 11.5 applies directly to this instrument.

(*b*) *"Megger" Earth Tester, Null balance system, made by Evershed and Vignoles Limited.* This instrument has three ranges 0–10, 0–100 and 0–1000 ohms. It will deal adequately with electrode resistances up to 1500 ohms each.

(*c*) *Earth Resistance Meter, Model* 666 *by Norma.* This works on the null balance system and has ranges 0–5, 0–50, 0–500 and 0–5000 ohms. It is claimed that for an accuracy of 5 per cent of full scale, the auxiliary electrode resistances can reach the following values: 5 ohms–5000 ohms, 50 ohms–20,000 ohms, 500 ohms–50,000 ohms, 5000 ohms–200,000 ohms. Frequency 75 cycles per second. Power source, transistor converter operated from flash lamp batteries.

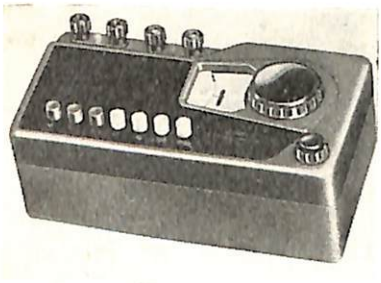
(*d*) *Tellohm Soil Resistance Meter made by Nash and Thompson Limited.* Two models are available each having ranges of 0–0·3, 0–1, 0–3, 0–10, 0–100, 0–1000 and 0–10,000 ohms.

One of these is intended for geophysical prospecting and it is stated that the resistance of the auxiliary electrodes should be 300 ohms for highest sensitivity. The other model is for corrosion measurements and the auxiliary electrode resistance is 100 ohms for maximum sensitivity. This instrument works on the null balance system and the power source is a vibrator unit operated from dry cells. The operating frequency is nominally 110 cycles per second.

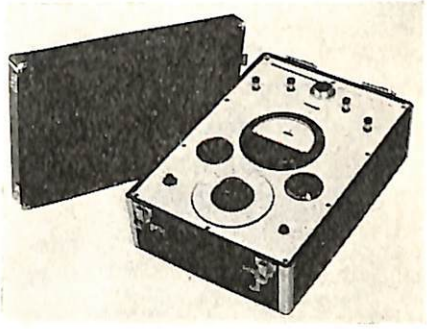
(*e*) *Earth Tester by Siemens-Halske.* Two models are available, one having ranges of 0–10, 0–100 and 0–1000 ohms, and the other 0–1, 0–10, 0–100 and 0–1000 ohms. An accuracy of $\pm$ 5 per cent is claimed with electrode resistances of 3000 ohms on the 1 ohm range, 50,000 ohms on the 10 ohm range and 50,000 ohms on the 100 ohm range. This instrument works on the null balance system and the supply is obtained from a hand-driven generator.

(*a*) *Series II Megger Earth Tester.*

(*b*) *Megger Earth Tester Null Balance.*

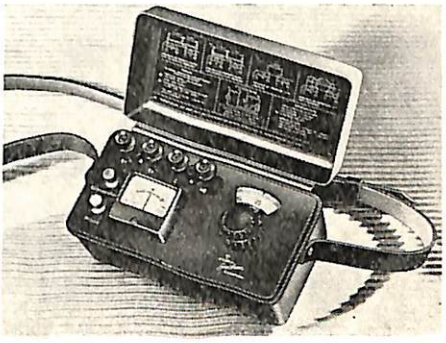


(*c*) *Norma.*



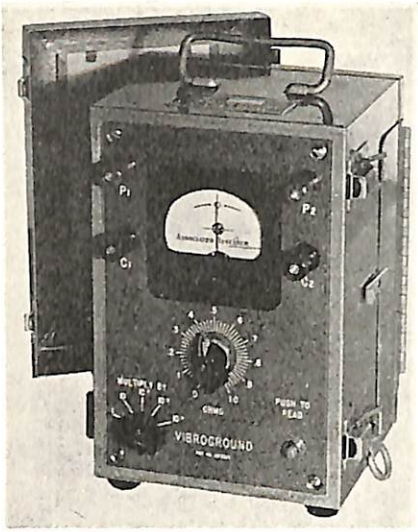
(*d*) *Tellohm.*

(*e*) *Siemens-Halske.*

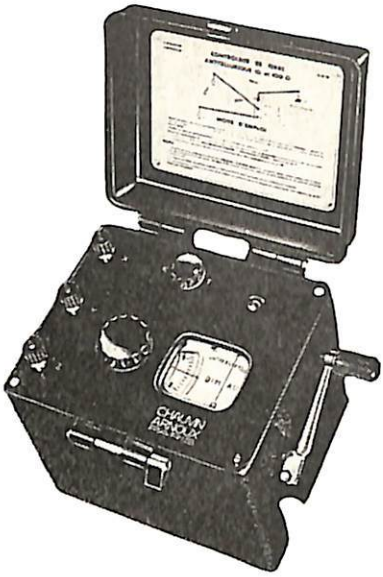


(*f*) *Geohm by Gossen.*

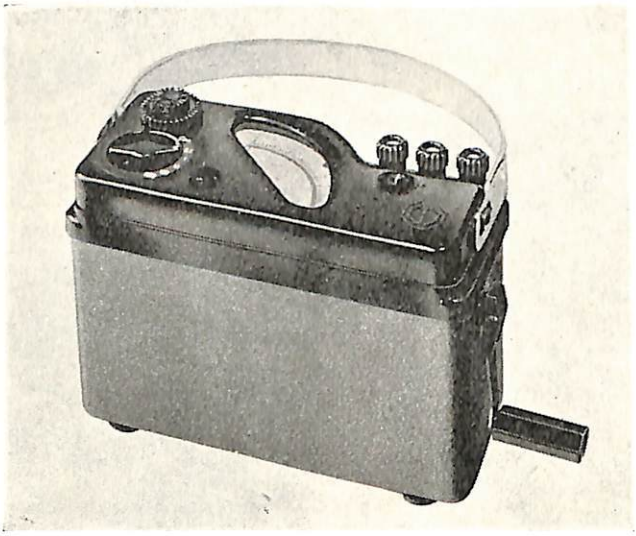
PLATE 1.



(*g*) *Vibroground.*



(*i*) *Chauvin Arnoux.*



(*h*) *Metrawatt.*

PLATE 1 (*continued*).

(*f*) *Geohm Earth Tester made by Gossen.* This instrument has ranges of 0–5, 0–50, 0–500 and 0–5000 ohms. The source is a battery-driven vibrator, the frequency being about 108 cycles per second. A means is provided for checking whether the auxiliary electrode resistance is excessive.

(*g*) *Vibroground by Associated Research Inc. of Chicago.* There are two industrial models, one having ranges of 0–1, 0–10, 0–100 and 0–1000 ohms and the other having ranges of 0–1, 0–10, 0–100, 0–1000 and 0–10,000 ohms. The accuracy claimed is within 2 per cent of the full scale reading on any range. The power source is a battery-operated vibrator. Some special models are available for certain specialised methods of resistivity surveying.

(*h*) *Earth Tester by Metrawatt.* This instrument has ranges of 0–2, 0–10, 0–100, 0–1000 and 0–10,000 ohms. An accuracy of $\pm 1{\cdot}5$ per cent of scale length is claimed and also that satisfactory results are obtained with auxiliary electrode resistance of 50 to 200 ohms on the lowest range, increasing to 60,000 to 100,000 ohms on the highest range. The power source is a hand-driven generator.

(*i*) *Earth Tester by Chauvin Arnoux.* This has two ranges of 0–10 and 0–100 ohms. It is claimed that the auxiliary electrode resistance can reach 10,000 ohms without affecting the accuracy of the measurement. The power source is a hand-driven generator, the frequency being approximately 150 cycles per second.

This does not profess to be a complete list of all instruments available but is representative. There are a number of special equipments made and sold by companies specialising in geoelectrical surveying. These are in general based on the voltmeter-ammeter method, the equipment including amplifiers and other electronic equipment for measuring the current and the voltage drop.

The Probable Values of Auxiliary Electrode Resistances

It will be noted in the descriptions of the various instruments that the range of values of auxiliary electrode resistances for which they are designed shows very considerable variation. This is indicative of the lack of definite information on this particular subject, and no comprehensive investigation has been carried out on this subject. Because of this the author and his firm decided to make an examination of the possible values to be met in this country.

This work, which is being done with the aid of the Electricity Boards, is not yet complete but an interim summary of the results to date are given in Table 11.1. The electrodes used were $\frac{1}{2}$ in. dia. mild steel rods.

TABLE 11.1

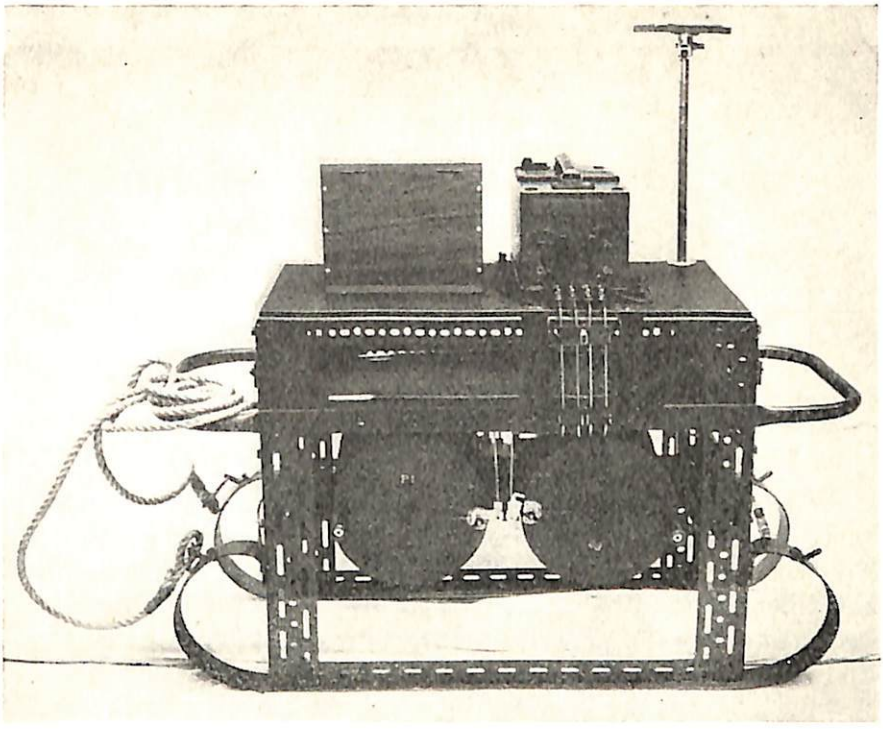
Length of rod (ft)	*Average resistance (Ω)*	*Probability that resistance will exceed 1500 Ω*
1	437	11·2 per cent
2	277	negligible
3	209	negligible

It will thus be seen that, so far as this country is concerned, the probable values of auxiliary electrode resistances are less than those quoted. There are, of course, odd cases where the resistance is much higher and it may be that the situation is rather different in other countries.

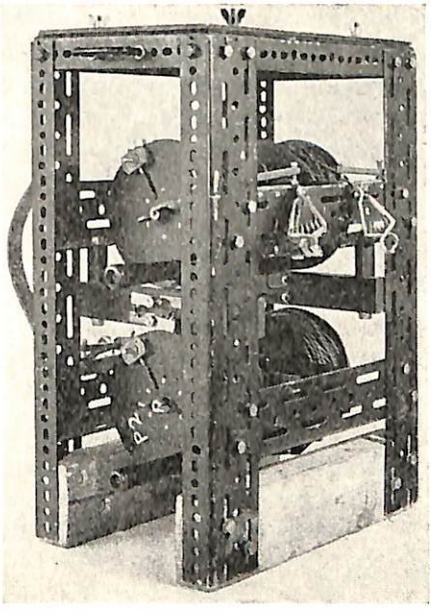
Auxiliary Equipment

In addition to the measuring instrument, the auxiliary equipment which is required to carry out any earth resistance measurement is very important. Particularly in resistivity surveying, the instrument and the necessary electrodes and connecting cables often have to be carried very long distances and often over difficult country. The design of these accessories does require some careful consideration. The electrodes can be very simple, short lengths of steel rod, say 15 in to 18 in are usually adequate, and these need not be more than $\frac{1}{2}$ in. diameter. A suitable hammer must be provided to drive these rods into soil which is too hard for them to be driven by hand.

The connecting wires or cables should be rubber covered and very strong and tough. The size of the conductor is determined by mechanical rather than electrical consideration since the resistance of these cables will undoubtedly be small compared with the resistance of the electrodes to which they are connected. A 4-mm ignition cable, rubber covered, has been found to be very suitable. Considerable lengths of such cables may be necessary in some cases to reach from the instrument to the electrodes. In general, such cables must be wound on suitable reels so that they can easily be wound out and in; there is nothing more exasperating and time wasting than a mass of tangled wire. A number of manufacturers who make instruments supply kits of spikes and connecting leads and a number of users have devised equipment to enable them to transport and use it easily.



(a) *Large Trolley.*



(b) *Small Trolley.*

PLATE 2.

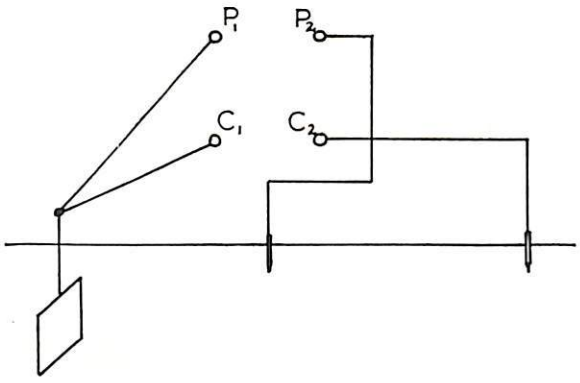


Fig. 11.9.
Elimination of lead resistance.

An equipment specially designed for use in a large-scale resistivity survey recently carried out in this country is illustrated in Plate 2(*a*). This consists of a trolley made of aluminium angle mounted on steel skis to enable it to be moved over all types of ground. A flat table is provided on the top on which the measuring instrument can be placed, and under this four drums carrying the required connecting cables are mounted. Means are provided by which the inner ends of the cables are connected to the instrument. Each drum carries 300 ft of wire, which was more than adequate for this particular survey.

Another similar but smaller trolley is illustrated in Plate 2(*b*). This is not mounted on skis, but is provided with carrying handles so that it can easily be carried. The drums in this case each carry 200 ft of cable.

Elimination of Lead Resistance

Generally speaking lead resistance is not important. There is, however, one case where it may be important.

Normally, when measuring the resistance of an earth electrode, one current and one potential terminal are joined together and connected to the earth electrode. In this case the resistance of the lead is included in the measurement and, if the earth electrode resistance is low, the lead resistance may be an appreciable part of the whole. It can either be measured or separate leads run from the potential and current terminals as in Fig. 11.9. In the latter case the resistance of the lead is not included in the measurement. In those instruments which have only three terminals this cannot be done.

12

THE MEASUREMENT OF EARTH LOOP RESISTANCE OR IMPEDANCE

One of the main purposes of connecting to earth is to ensure safety to a consumer of electricity in the event of a fault developing in any of his equipment. For this to be so it is essential that sufficient current shall flow in the event of such a fault, that the protective gear, fuses or circuit breakers, should operate and disconnect the faulty circuit.

The resistance or impedance of the fault path should therefore be measured to ensure that the required value of fault current will flow. This circuit usually contains earth electrodes and so this measurement must be included in any consideration of earth resistance measurement. There has been considerable discussion on this subject and it is still one in which there is not complete agreement, the disagreement being, not on the need for such a test, but on the manner in which it should be carried out.

Regulations

The need for such a test is emphasised by the fact that there are certain clauses in the thirteenth edition of the Regulations for the Electrical Equipment of Buildings issued by the Institution of Electrical Engineers. The following are the relevant clauses:

406. The earthing arrangement of the consumer's installation shall be such that on the occurrence of a fault of negligible impedance from a phase or non-earthed conductor to adjacent exposed metal, a current corresponding to three times the rating of the fuse or one and a half times the setting of the overload circuit breaker can flow, so that the faulty circuit is made dead. Alternatively, and in every instance where this requirement cannot be met, an earth-leakage circuit breaker shall be installed in accordance with Regulation 410. (In a note following this regulation it is stated that it is desirable that the maximum sustained voltage developed between exposed metal required to be earthed and the consumer's earth terminal under fault conditions should not exceed 40 volts r.m.s.).

507. Where fuses or overload circuit breakers are used as a protection against dangerous earth leakage currents, each completed installation

and if possible, each final sub-circuit, shall be tested to ensure that the impedance of the earth fault loop will permit compliance with the requirements of Regulation 406.

Note.—The earth fault loop (line-earth loop) comprises the line conductor from the point of fault back to the supply transformer, the path through the transformer winding, the earthed neutral point of the transformer, the path from that point to the consumer's earthing lead, the earthing lead, and the earth continuity conductor back to the point of fault.

The method used for testing may give the loop impedance in ohms; alternatively, it may give a direct indication of the maximum permissible setting of the fuse or circuit breaker, or an indication whether a fuse or circuit breaker of a given rating will operate satisfactorily under fault conditions.

(*a*) *Tests in Line-Earth Loop*

These tests may be achieved by an instrument which determines the current flowing when a known resistance (usually of the order of one ohm) is connected between line and earth. In using such an instrument, care should be taken that no ill effects can arise if the earthing circuit is defective.

(*b*) *Tests in Neutral-Earth Loop*

Except where the system is earthed by protective multiple earthing, (P.M.E.) it is permissible to test the neutral-earth loop instead of the line-earth loop. This may be achieved by a method which injects current into the former loop either by means of a transformer fed from the mains, or from a d.c. source the polarity of which is rapidly and continuously reversed. The instrument employed must be so designed or used that its indications are not affected by neutral currents flowing in the system. In using such an instrument care should be taken to ensure that no ill effects can arise if the circuit tested is defective. Measurements on systems fed from small transformers may require compensation to include an allowance for the impedance of the windings of transformers, balancers, etc.

The tests are preferably made by a.c. of mains frequency, at currents of the order of 1·5 times the rated current of the circuit although currents of 25 amperes need not be exceeded. If a test with rapidly reversed d.c. is made on an a.c. installation, it should be ensured by inspection that no inductor (choke) is incorporated in the earth continuity conductor. When a measurement on an a.c. system is made with less than 10 amperes a.c. or with rapidly reversed d.c., and the earth continuity conductor is wholly or mainly of steel conduit or pipe, the effective value

TABLE 12.1. MAXIMUM VALUES OF LOOP IMPEDANCE FOR FUSED CIRCUITS

Fuse rating (amps)	*Required current (amps)*	*Maximum impedance in ohms when voltage is*					
		200	210	220	230	240	250
5	15	13·33	14·00	14·67	15·33	16·00	16·67
7	21	9·52	10·00	10·48	10·94	11·42	11·91
13	39	5·13	5·38	5·64	5·90	6·16	6·41
15	45	4·44	4·67	4·88	5·11	5·33	5·56
30	90	2·22	2·34	2·44	2·56	2·66	2·78
60	180	1·11	1·17	1·22	1·28	1·33	1·39

Fuse ratings of 5, 7 and 13 amps are included in this Table because of the increasing use of ring main systems, in which the appliances are connected by fused plugs, the fuses having these ratings.

The corresponding values for circuit breakers are given in Table 12.2.

shall be taken as twice the measured value, less the value measured at the consumer's earth terminal. In all other cases the effective value should be the measured value, subject to any necessary correction for impedance of supply transformers, balancers, etc., as mentioned above.

Maximum Values of Loop Impedance

Regulation 406 requires that three times the rated current must flow in the event of a fault on a system protected by fuses. This gives the maximum values of loop impedance for various voltages as set out in Table 12.1.

Basis of Line-Earth Testers

The basic circuit for most forms of line-earth tester is drawn in Fig. 12.1. Generally a resistance R is connected between the line conductor and the earth connection at the point where the test is to be made. This means that this resistance is placed in series with the line earth loop, and so the voltage across R will be affected by the value of the loop impedance. This voltage may be measured and so a direct indication of the loop may be obtained or it may be used to trigger off indicating circuits, which will show whether the loop value is above or below the requisite maximum value.

TABLE 12.2. MAXIMUM VALUES OF LOOP IMPEDANCE FOR CIRCUITS PROTECTED BY CIRCUIT BREAKERS

Ratings (amps)	*Required current (amps)*	*Maximum impedance in ohms when voltage is*					
		200	210	220	230	240	250
15	22·5	8·88	9·34	9·76	10·22	10·66	11·11
30	45·0	4·44	4·67	4·88	5·11	5·33	5·56
60	90·0	2·22	2·34	2·44	2·56	2·66	2·78
120	180·0	1·11	1·17	1·22	1·28	1·33	1·39
200	300·0	0·67	0·70	0·73	0·77	0·80	0·83

It will be realised that R has to carry the required testing current, which may be about 25 amps, and with a low loop impedance on a 240-volt circuit, the power absorbed by this resistor is 6 kilowatts. For this reason, and others, arrangements are usually made so that the current flows only for a very short time, usually a small fraction of a second.

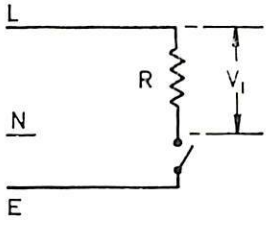


Fig. 12.1.
Basic circuit for line-earth tester.

Practical Form of Line-Earth Loop Tester

In one practical form of this instrument the voltage across the resistor R is measured by what is best described as a ballistic voltmeter. The current is switched through the resistor R for 0·2 second and so the indicating instrument must respond within this time and give a deflection which is dependent only on the voltage. After a number of trials the instrument finally used was a modification of the induction-type instrument used as a house service meter.

If the usual current coil of such a meter is rewound as another voltage winding and the measured voltage is applied to this as well as to the usual voltage winding, it is easy to show that torque on the moving system is proportional to the square of the voltage. The

voltage is applied for 0.2 second and, if the movement is well damped, the throw will be dependent on the square of the voltage and thus on the resistance of the loop. The scale of the instrument is so calibrated.

The complete diagram of this instrument is given in Fig. 12.2. The application of the mains voltage is controlled by a circuit breaker and the time this is closed is in turn controlled by a timing motor. The operation of the instrument is as follows.

When the push button *S6* is pressed the timing motor *M* is connected between Line and Neutral. The motor drives the cam *T* which operates two sets of contacts, actually micro-switches (*B*) and (*C*). The contacts *B* complete the circuit to the operating coil of the contactor *D*, and are so arranged that the contactor is closed for 0·2 second. The contacts of *D* put the resistor R_1 between Line and Earth, and in parallel with

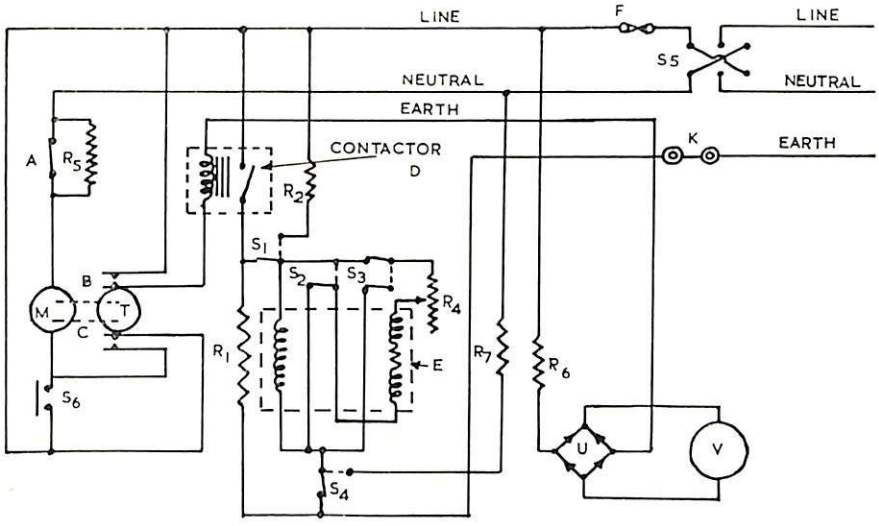


Fig. 12.2. *Line earth loop tester.*

the resistor are the two coils of the ballistic voltmeter *E*. This voltmeter will then deflect to a point depending on the loop resistance. The second pair of cam operated contacts *C* are in parallel with the push-button, and keep the motor running when the push button is released, until the cam *T* has made one complete revolution, when the contacts open and the motor stops.

The voltage across the resistor R_1 is dependent not only on the loop resistance but also on the voltage of the supply. The instrument can thus be calibrated for one value of supply voltage. In order to compensate for other values of this voltage, a tapped resistor R_4 is provided

in series with one of the instrument coils. A rectifier-operated voltmeter V is included in the instrument, and the procedure is to measure the voltage and to set the tapping on R_4 to the corresponding point.

Since the voltmeter acts ballistically the pointer will remain at the reading and will need to be reset for the next measurement. A reset button is provided which changes over the switches $S1$, $S2$, $S3$ and $S4$ to the dotted positions; the coils of the instrument are connected across the supply with one reversed, so that the pointer is driven backwards to the starting position.

A further feature of the instrument is a neon lamp indicator which shows whether the polarity of the supply is correct, and a reversing switch $S5$ is provided for use if the polarity is incorrect.

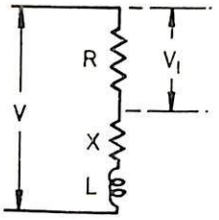


Fig. 12.3.
Resistance and inductance of the earth loop.

Resistance or Impedance

The point now arises as to the nature of the reading of the instrument when the earth loop includes inductance. The Regulation calls for a measurement of impedance, and it can be shown that the reading obtained is greater than the resistance but less than the impedance.

Consider the circuit of Fig. 12.3 in which X and L are the resistance and inductance of the earth loop. The current flowing is $V(R + X + j\omega L)$ and the voltage drop V_1 is

$$V_1 = \frac{VR}{R + X + j\omega L}$$

The instrument responds to the r.m.s. value which is

$$\frac{VR}{[(R + X)^2 + \omega^2 L^2]^{1/2}} \tag{12.1}$$

When the loop contains resistance X_1 only, the voltage becomes

$$\frac{VR}{(R + X_1)} \tag{12.2}$$

The reading obtained when there is inductance will be X_1 so that the voltages in (12.1) and (12.2) must be equal. This means

$$R + X_1 = [(R + X)^2 + \omega^2 L^2]^{1/2}$$

or

$$X_1 = [(R + X)^2 + \omega^2 L^2]^{1/2} - R$$

or

$$\frac{X_1}{X} = \left[\left(\frac{R}{X} + 1\right) + \frac{\omega^2 L^2}{X^2}\right]^{1/2} - \frac{R}{X} \tag{12.3}$$

If Z = impedance, then

$$Z = (X^2 + \omega^2 L^2)^{1/2}$$

or

$$\frac{Z}{X} = \left(1 + \frac{\omega^2 L^2}{X^2}\right)^{1/2}$$

So

$$\frac{X_1}{Z} = \frac{X_1}{X} \cdot \frac{X}{Z} = \frac{\left[\left(\frac{R}{X} + 1\right)^2 + \frac{\omega^2 L^2}{X^2}\right]^{1/2} - \frac{R}{X}}{\left(1 + \frac{\omega^2 L^2}{X^2}\right)^{1/2}} \tag{12.4}$$

From equation (12.3) and (12.4) the errors referred to resistance or impedance are given in the curves of Fig. 12.4 and 12.5. These are fairly theoretical calculations, and from these curves it is possible to state the limiting values of $\omega L/X$ for a given percentage error.

The figures for say 5 per cent error are given in Table 12.3.

TABLE 12.3.

5 per cent error in resistance		*5 per cent error in impedance*	
R/X	$\omega L/X$	R/X	$\omega L/X$
0·1	0·334	0·1	any
0·5	0·381	0·5	0·660
1	0·449	1	0·500
5	0·780	5	0·363
10	1·05	10	0·343
50	2·25	50	0·331
100	3·17	100	—
200	4·46	200	0·323

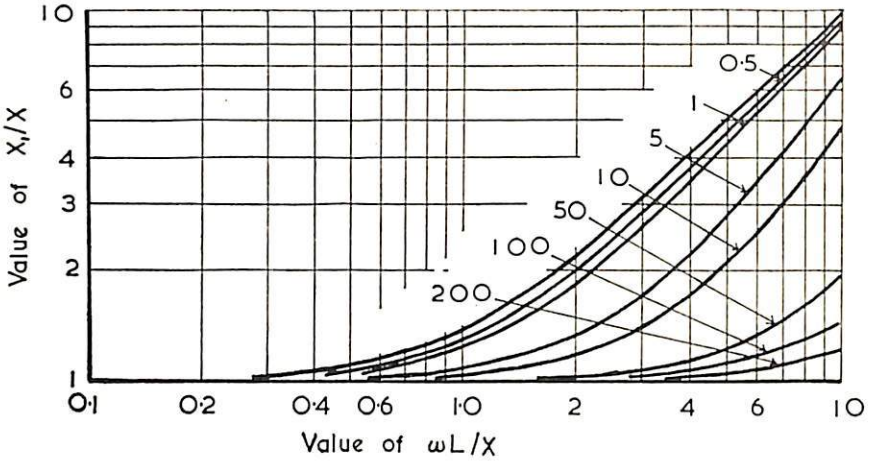


Fig. 12.4. *Errors due to inductance in resistance measurement.*

Practical Values of Inductance

An inductive reactance in the earth loop is nearly always due to the use of steel conduit and the important case is that where the steel conduit is the earth continuity conductor. The inductive reactance of steel conduit can be calculated and this has been done by the Electrical Research Association and the results are given in their report V/T111, "The Impedance of Steel Conduit used as an Earth Continuity

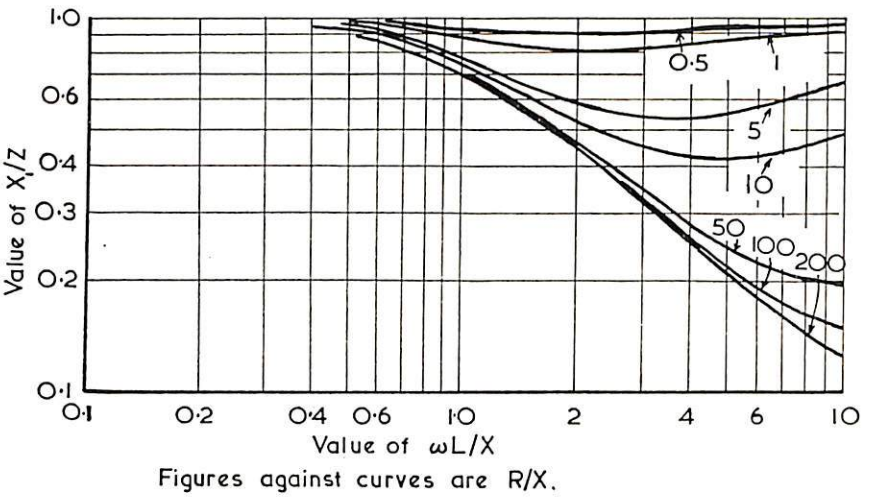


Fig. 12.5. *Errors due to inductance in impedance measurement.*

Conductor". The figures and curves mentioned later are based on the calculations given in this E.R.A. report. The first step is to calculate the impedance of various lengths of steel conduit of various sizes and both heavy and light gauge. The results of these calculations are given in the curves of Fig. 12.6, which gives the values at 50 cycles of $\frac{1}{2}$ in., $\frac{3}{4}$ in., 1 in. and $1\frac{1}{2}$ in. diameter conduit, both heavy and light gauge.

One point which emerges immediately is this. The I.E.E. Wiring Rules state that the impedance of the earth continuity conductor must not exceed 1 ohm and so, in those cases where the steel conduit is used in this capacity, there is a limit to the length which may be used. These limiting lengths are given in Table 12.4.

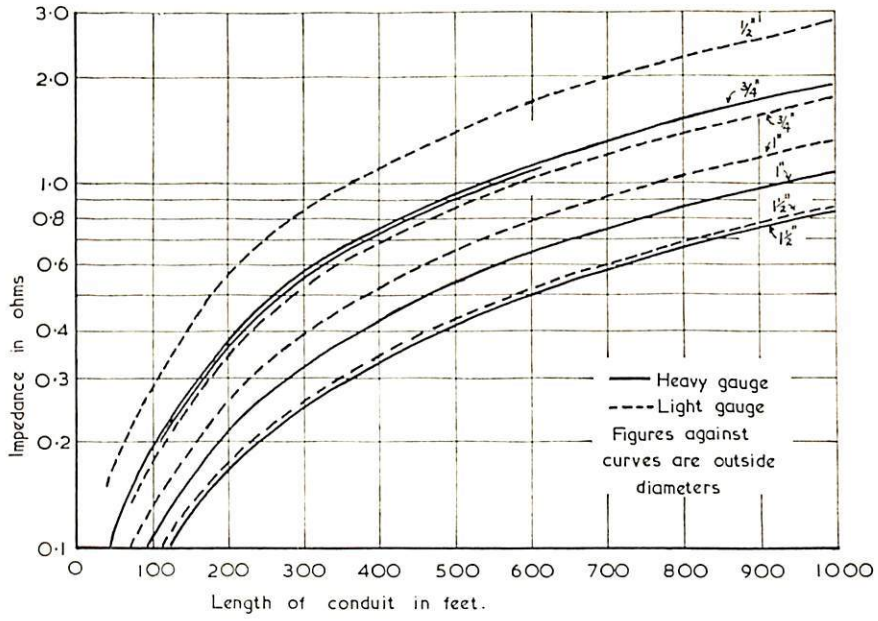


Fig. 12.6. *Impedance of steel conduit.*

Another point to note is that this impedance is made up of the a.c. resistance of the conduit and the inductive reactance, and that both these quantities can be up to four times the d.c. resistance. The two quantities are generally fairly close in value, so that the angle of the impedance is generally about 45°.

If it is assumed that the whole loop to be tested consists of steel conduit, then this is the worst case and will produce the maximum error. In many practical cases there will be an additional resistance in

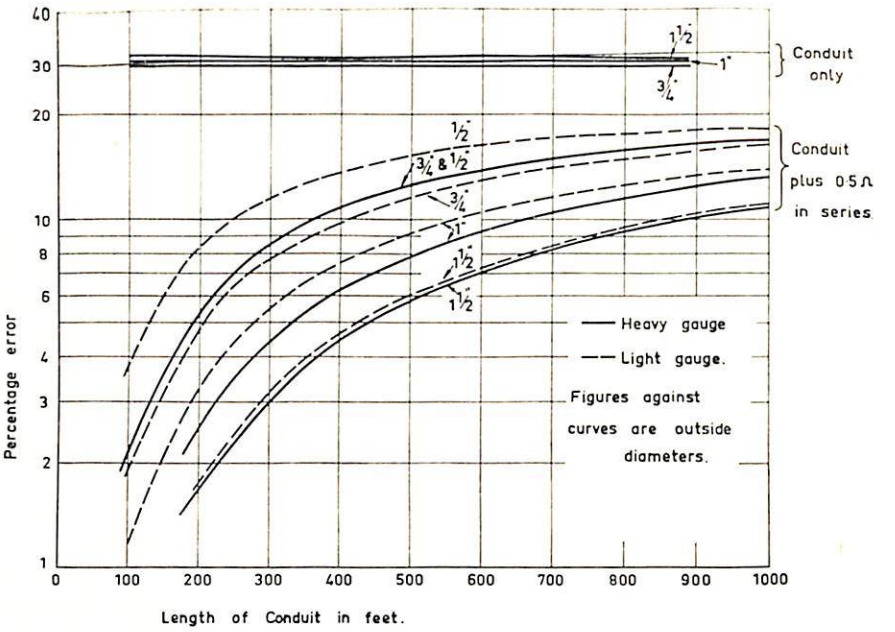


Fig. 12.7. *Earth loop tester—errors due to impedance of steel conduit.*

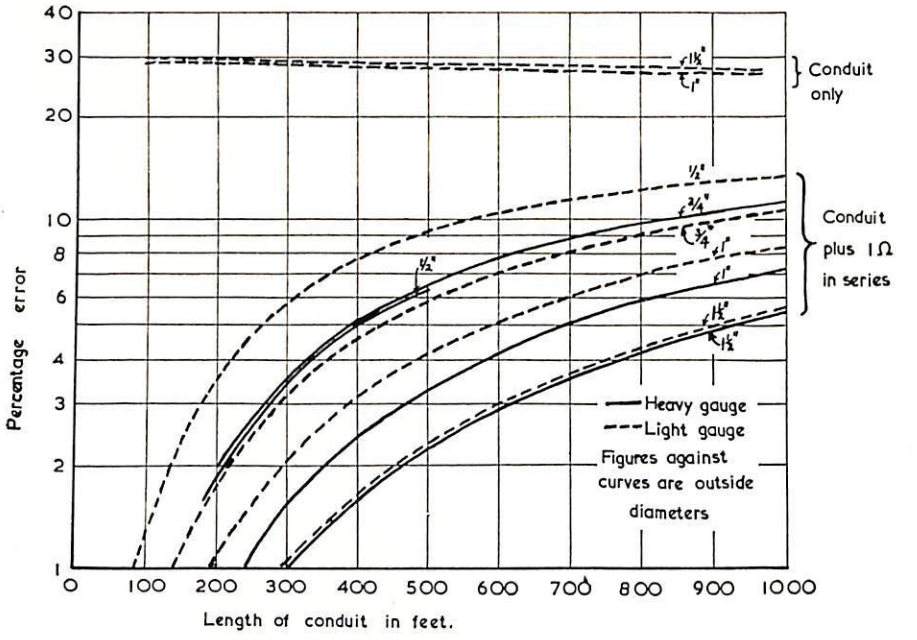


Fig. 12.8. *Earth loop tester—errors due to impedance of steel conduit.*

TABLE 12.4. LENGTH OF CONDUIT HAVING AN IMPEDANCE OF 1 OHM AT 50 CYCLES

Diameter of conduit (in)	*Gauge*	*Length in feet*
1½	Heavy	>1000
1½	Light	>1000
1	Heavy	940
1	Light	760
¾	Heavy	530
¾	Light	580
½	Heavy	550
½	Light	355

series with the conduit, this additional resistance possibly being provided by joints in the conduit itself, and by other parts of the loop such as earth connections. The errors resulting are given in the curves of Figs. 12.7 and 12.8. These are for an instrument having an internal resistance of 10 ohms. In the first of these the values are given for heavy gauge conduit alone, and for all the sizes of conduit considered, with an

TABLE 12.5. MAXIMUM LENGTH OF CONDUIT, WITH SERIES RESISTANCE, FOR ERROR NOT EXCEEDING 5 PER CENT

Series Resistance	*Diameter (in)*	*Gauge*	*Length Feet*
0·5	1½	Heavy	437
0·5	1½	Light	425
0·5	1	Heavy	332
0·5	1	Light	278
0·5	¾	Heavy	189
0·5	¾	Light	204
0·5	½	Heavy	189
0·5	½	Light	127
1·0	1½	Heavy	926
1·0	1½	Light	904
1·0	1	Heavy	693
1·0	1	Light	588
1·0	¾	Heavy	393
1·0	¾	Light	434
1·0	½	Heavy	400
1·0	½	Light	265

additional resistance of 0·5 ohm. In the second the values are given for light gauge conduit alone and for all the sizes of conduit considered, with an additional resistance of 1 ohm.

It should be noted that the error with conduit alone is for all practical purposes independent of the size, gauge and length of the conduit and the average error is about 29 per cent. This is the maximum error which can occur. With additional series resistance the errors are considerably reduced and Table 12.5 gives the maximum length of conduit with the two values of series resistance if the error is not to exceed 5 per cent.

If an error of 10 per cent is permissible, then the lengths of conduit which can be included in the circuit can be increased considerably.

So, to sum up, there is a limit to the length of steel conduit which can be used as an earth continuity conductor, the maximum error of 29 per cent occurs when steel conduit forms the whole of the earth loop, and additional resistance in the loop will reduce the error considerably.

Line-Earth Loop Testers of the GO-NO GO type

It is considered by a number of people that it is sufficient to check whether the circuit is of such an impedance that it will pass the required current in the event of a fault. This has led to the development of testing sets which simply give an indication whether the impedance is above or below the required value.

As in the indicating type of instrument, a resistor is connected between the line conductor and earth for a very short time and the voltage across this resistor which, as already explained, is dependent on the loop impedance, is used to trigger an indicating device, which shows whether it is below or above a certain value. This particular value is the voltage drop which would occur if the loop had an impedance such that the required current, three times the fuse rating, would flow in the event of a fault.

One such instrument has been devised by Roscoe. On pressing the operating button, the testing current flows through a miniature double-pole circuit breaker with the two poles connected in series, a selector switch and a selected non-inductive resistor. The circuit breaker breaks the circuit thus interrupting the test current in about $1\frac{1}{2}$ cycles. The voltage drop across the resistor is applied through the medium of a potential divider to the grid of a cold cathode valve which strikes if the voltage reaches a certain value corresponding to the passage of a current of three times the selected fuse rating and continues to glow until the reset switch is operated by hand. The value of resistor chosen

by the selector switch depends on the fuse rating in question. The instrument is fitted with various devices, such as polarity checking, and a means of determining whether the earth loop is complete, so that a test can be made safely.

Another instrument has been devised by Baggot. In this, again a resistor is connected between the line and earth connections and the voltage across this is applied to a gas filled trigger tube. If this voltage is high enough the tube fires and energises an indicating circuit, a neon lamp, thus showing that the impedance of the loop is below the required value. Since the current flowing may be high it is necessary to limit the time the current flows and this is achieved by the use of switched diodes or silicon controlled rectifiers. The circuits are so arranged that the current does not last for more than one cycle of the a.c. supply.

Since in all these instruments a non-inductive resistor is switched in series with the earth loop, they are subject to the same errors as in the

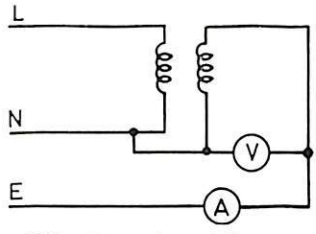


Fig. 12.9. *Neutral earth loop tester.*

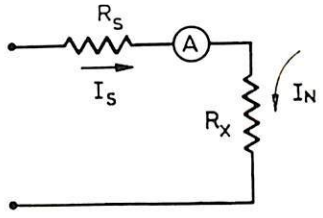


Fig. 12.10. *Ammeter-voltmeter method.*

indicating instrument, when inductance is present in the earth loop. The considerations relating to resistance, impedance and inductance given earlier in this chapter apply equally to these instruments.

Neutral Earth Loop Tests

The original form of the Regulations called for a test of the neutral earth loop, and this was later supplemented by the line-earth loop test, when the disadvantages of the former were fully recognised. In the most usual forms the test consists of injecting current into the neutral earth loop, from the secondary of a transformer, the primary being connected between line and neutral. This is illustrated in Fig. 12.9, the necessary operating switches being omitted. The current flowing in the loop is measured by the ammeter *A*, and the applied voltage is measured by the voltmeter *V*. The ratio voltage/current gives the impedance of the neutral earth loop.

Now when a fault to earth occurs on a consumer's premises, the fault current flows through the following path:

Secondary winding of the supply transformer,
Line conductor,
Fault,
Consumer's earth continuity conductor,
Consumer's earth,
Body of earth between consumer's earth and substation earth,
Substation earth.

In the circuit of Fig. 12.9 the test current flows through the following path:

Neutral conductor,
Consumer's earth-continuity conductor,
Consumer's earth,
Body of earth between consumer's earth and substation earth,
Substation earth.

In comparing these two circuits, it is clear that there are two main differences. In the neutral loop test, the test path includes the neutral conductor while the fault path includes the line conductor. This does not matter if these conductors have the same cross-section and hence the same impedance, but this is not always the case. Again the test path does not include the secondary of the supply transformer and the amount this contributes to the impedance of the fault path is unknown. For this type of test, the substitution of the neutral conductor for the line conductor and the omission of the transformer winding must be accepted.

Effect of Existing Currents in the Neutral

The fact that the neutral conductor is included in the loop introduces a further difficulty. The neutral conductor is probably shared by a number of consumers, and as a result may be carrying a considerable current. This means that there will exist between the connection to the neutral conductor and that to the earth-continuity conductor a voltage due to the passage of this current. Some typical measurements of this voltage drop gave values up to 4 volts. This does not seem very high, but it has to be compared with the injected voltage 10 to 20 volts and it thus becomes important. This voltage drop, incidentally, is continually varying both in magnitude and phase. The fact that this voltage drop can produce errors is evidently recognised by the manufacturers of some of the instruments available, since it is quite usual for the instructions to include a statement that the polarity of the testing current

should be reversed, two readings taken and the mean accepted as the true value. This does not, however, give the correct value.

The voltmeter-ammeter method can be shown simply as in Fig. 12.10, in which R_x represents the resistance of the earth loop, and R_s the internal resistance of the testing set. The latter applies a voltage V_s to the circuit, producing a current I_s which is measured by the ammeter A, while the loop resistance R_x is already carrying a current I_N. Now it follows that

$$I_s = \frac{V_s - I_N R_x}{R_s + R_x}$$

In most of the instruments using this principle the voltmeter is omitted and the ammeter is scaled directly in ohms, an inverse scale. For calibration purposes then, with $I_N = 0$ the condition is $k/I_s = R_x$ where k is the calibration constant i.e.

$$k = \frac{V_s R_x}{R_s + R_x}$$

The reading y, obtained in the general case will then be

$$y = \frac{V_s R_x}{(R_s + R_x)} \cdot \frac{1}{I_s}$$

Now consider the fact that the current I_N already flowing in the loop may have any phase relative to the testing voltage and so may be of the form $\alpha + j\beta$. The current I_s is then in the form

$$I_s = \frac{(V_s - \alpha R_x) - j\beta R_x}{R_s + R_x}$$

and, since the instrument responds to the r.m.s. value of the current, the current producing the reading is

$$\frac{[(V_s - \alpha R_x)^2 + \beta^2 R_x{}^2]^{1/2}}{R_s + R_x}$$

and the actual scale reading will be

$$y = \frac{V_s R_x}{(R_s + R_x)} \cdot \frac{(R_s + R_x)}{[(V_s - \alpha R_x)^2 + \beta^2 R_x{}^2]^{1/2}} = \frac{V_s R_x}{[(V_s - \alpha R_x)^2 + \beta^2 R_x{}^2]^{1/2}}$$

This can be written in the form

$$\frac{y}{R_x} = \frac{V_s}{[V_s{}^2 - 2\alpha V_s R_x + \alpha^2 R_x{}^2 + \beta^2 R_x{}^2]^{1/2}} \qquad (12.6)$$

Now if V_n is the voltage drop in the neutral due to the current I_n,

then $V_n^2 = (\alpha^2 + \beta^2)R_x^2$: furthermore, if ϕ is the phase difference between this voltage drop and the testing voltage V_s, then

$$\alpha R_x = V_n \cos \phi.$$

Then

$$\frac{y}{R_x} = \frac{V_s}{[V_s^2 - 2V_sV_n \cos \phi + V_n^2]^{1/2}} = \frac{1}{\left[1 - \frac{2V_n}{V_s} \cos \phi + \left(\frac{V_N}{V_s}\right)^2\right]^{1/2}}$$

Let $V_n/V_s = n$, then

$$\frac{y}{R_x} = \frac{1}{[1 - 2n \cos \phi + n^2]^{1/2}} \tag{12.7}$$

If now a second reading is taken with the polarity of the testing voltage reversed, it can be shown that this ratio becomes

$$\frac{y}{R_x} = \frac{1}{[1 + 2n \cos \phi + n^2]^{1/2}} \tag{12.8}$$

It is stated that the mean of these two values should be taken as the loop resistance. It is evident that this is not so unless $n = 0$, and a number of calculations based on equations (12.7) and (12.8) giving the error with various values of n and ϕ, have been made, and the results are given in the curves of Fig. 12.11. It will be seen that considerable errors can arise even with quite low voltage drops in the neutral.

Neutral Loop Tests with Light Current Instruments

The Regulation states that the neutral test should be made with a current of the order of 25 amps, and most of the instruments specially designed for this test are arranged to deliver currents of this order. There has been considerable investigation into the results of testing with currents of different magnitudes. In most earth-continuity conductors there are usually clamped joints, such as a clamp round a water pipe, and semi-insulating films can develop in such clamps. These films require a definite voltage to break them down and make the joint fully conducting. The argument has been put forward that a comparatively heavy current should be used in order to break down the film. This is not quite the correct way to consider the problem.

If a testing set can deliver a high current, it must have a low output impedance and so, if the external circuit offers at first a high impedance, such as that due to a clamped joint, then practically the whole of the voltage of the testing set is applied to the external circuit, with the

result that any film is broken and the ultimate impedance indicated is low. On the other hand, any testing set which can deliver only a small current will have a high-output impedance and so the voltage applied to the external circuit will be low and insufficient to break down a film. The result is that the impedance indicated will be high.

That this is a practical condition which can be met is illustrated by tests made at a house in Kent. The first tests of the neutral loop were made with instruments delivering currents of the order of 300 milli-amperes and the value indicated was 26 ohms. This same value was obtained with two instruments and by two independent observers. The

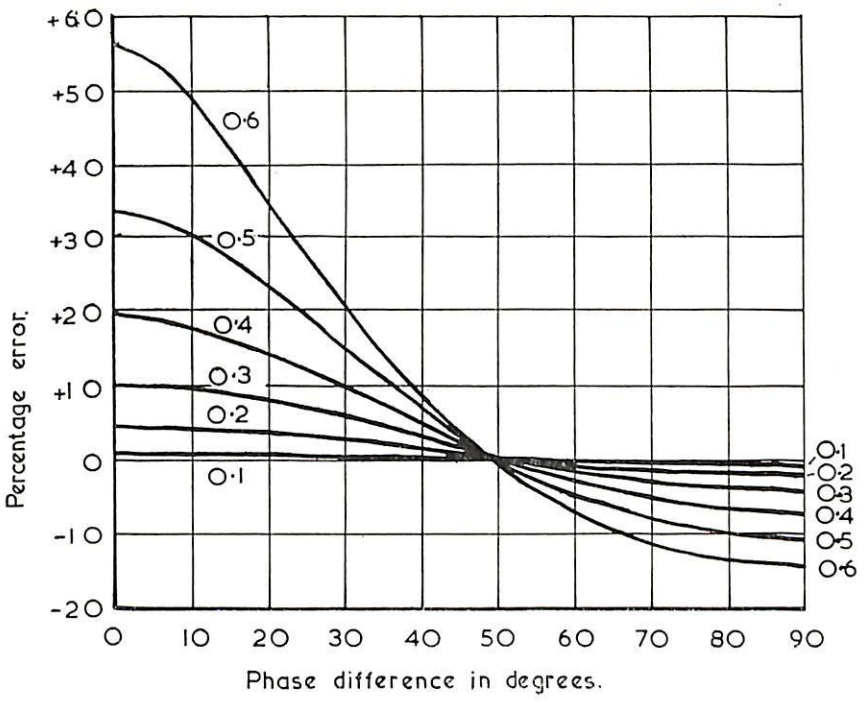


Fig. 12.11. *Errors due to voltage drop in neutral.*

test was then repeated with instruments giving currents of the order of 10 to 20 amps and the result was 1·45 ohms. A repeat test with the low current instruments then gave a figure of 1·40 ohms. Examination of the earth-continuity conductor showed that this was taken to a clamp round a water pipe and subsequently it was found that a film had formed between the clamp and the pipe. Incidentally the voltage drop in the neutral in this case was found to be 12 to 14 volts.

Another argument sometimes brought forward in support of the

higher current is that if the earth continuity conductor is not carefully made, it may happen that at some point it is completed by a single strand of wire clamped under a screw and that this heavier current will fuse this strand, and the instrument will then indicate an open-circuit. Experiments show that little reliance can be placed on this, as the currents delivered by these instruments are insufficient to fuse single strands of the wires usually employed for these conductors.

The best value for the testing current is thus a very debatable quantity, and good arguments can be put forward in favour both of light and heavy currents. It is probable that the really best arrangement would be to test at both light and heavy, but this involves complications in the instruments necessary.

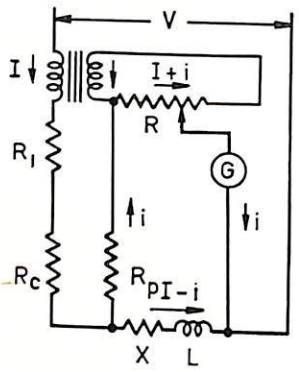


Fig. 12.12.
Circuit for null-balance type of earth tester.

The earth resistance measuring instruments described in Chapter 11 all deliver currents of the order of milliamperes, but they can be used for neutral loop measurements and have the distinct advantage that they are designed to deal with extraneous currents in the resistance tested, and consequently will not be affected by the voltage drop already existing in the neutral conductor. Modifications of some of these instruments are available, in which the additional desirable features of neutral loop testers are incorporated. It is essential that connection should be made to the neutral and not to the line conductor. These instruments are fitted with a neon warning lamp, and a push button so that the measuring circuits of the instrument are not connected to the loop until the connection has been checked.

Effect of Inductance on the Readings of Light-current Instruments

It has to be accepted that, with most of the earth testing instruments, the result obtained is not the impedance of the loop but is more nearly

the resistance. As an example consider the null balance type of earth tester shown in Fig. 12.12. In this diagram:

V = voltage of the power source
R_1 = internal resistance of the current circuit
R_c = total resistance of the current electrodes
R_p = total resistance of the potential electrodes including the galvanometer circuit
R = value of the balancing resistance
X = resistance of the test circuit
L = inductance of the test circuit.

Electrode resistances are included in these general considerations, although in the neutral loop tests, the electrode resistances are zero.

Let I be the current flowing through the main circuit and i the current through the galvanometer circuit. Then the current conditions are as shown in Fig. 12.12. Then from this diagram:

$$V = I(R_1 + R_c) + (X + j\omega L)(I - i) = I(R_1 + R_c + X + j\omega L)$$

$$- i(X + j\omega L)(X + j\omega L)(I - i) = R_p i + R(I - i)$$

Solving these equations for i leads to

$$i = \frac{V\{(X - R) + j\omega L\}}{[R_p(R_1 + R_c + X) + (R_1 + R_c)(X - R) + j\omega L(R_1 + R_c + R_p)]}$$

$$= \frac{V(\alpha + j\beta)}{\gamma + j\delta} \quad (12.9)$$

Note that if $L = 0$, the balance condition $i = 0$, means that $X = R$ which is the normal condition. Now if V is of the form $V \sin \omega t$, the current i is of the form $i = I_m \sin(\omega t + \phi)$, and it is rectified in synchronism with the voltage V. Thus the nett rectified current is

$$\frac{1}{\pi}\int_0^{\pi} I_m \sin(\omega t + \phi) = -\frac{I_m}{\omega\pi}[-\cos(\omega t + \phi)]_0^{\pi} = \frac{2I_m \cos\phi}{\omega\pi} \quad (12.10)$$

Balance is obtained when this nett rectified current is zero and the condition for this, from equation (12.10) is $\cos\phi = 0$.

From equation (12.9)

$$i = \frac{V(\alpha + j\beta)}{\gamma + j\delta} = \frac{V\{\alpha\gamma + \beta\delta + j(\beta\gamma - \alpha\delta)\}}{\gamma^2 + \delta^2} \quad (12.11)$$

The conditions $\cos\phi = 0$ means that $\alpha\gamma + \beta\delta = 0$ and so

$$(X - R)[R_p(R_1 + R_c + X) + (R_1 + R_c)(X - R)] + \omega^2 L^2(R_1 + R_c + R_p) = 0$$

or

$$(X - R)^2(R_1 + R_c) + (X - R)R_p(R_1 + R_c + X) + \omega^2 L^2(R_1 + R_c + R_p) = 0$$

Solving this equation for $X - R$, putting $R_1 + R_c = R_0$ and assuming that X can be neglected in comparison with R_0, leads to

$$R - X = \frac{R_p - \sqrt{R_p^2 - 4\omega^2 L^2(1 + R_p/R_0)}}{2} \tag{12.12}$$

The quantity $(X - R)$ is the absolute value of the out of balance resistance and it is interesting to note that this is dependent on the resistances of the instrument circuits as well as the inductance, and the percentage error becomes less for a given inductance as the value of X is increased.

In the neutral loop tests, the electrode resistances are zero, so that R_0 becomes the internal resistance of the current circuit, and R_P becomes the internal resistance of the galvanometer circuit. Equation (12.12) also shows that balance cannot be obtained if

$$4\omega^2 L^2\left(1 + \frac{R_P}{R_0}\right) > R_P^2$$

i.e.

$$\omega L > \frac{R_P}{2\sqrt{(1 + R_P/R_0)}}$$

In general, the galvanometer resistance will be low compared with R_0 so that approximately

$$R - X = \frac{R_P - \sqrt{(R_P^2 - 4\omega^2 L^2)}}{2}$$

$$= \frac{R_P}{2}\left[1 - \left\{1 - \frac{4\omega^2 L^2}{R_P^2}\right\}^{1/2}\right] \tag{12.13}$$

Since, in general, $4\omega^2 L^2$ is small compared with R_P^2, by the binomial theorem

$$R - X = \frac{R_P}{2}\left[1 - \left(1 - \frac{2\omega^2 L^2}{R_P^2}\right)\right] = \frac{\omega^2 L^2}{R_P} \tag{12.14}$$

It follows from this that the higher the value of R_P, i.e. the galvanometer resistance, the lower is the error due to any inductance L. It is also obvious that the reading obtained is close to the resistance and not the impedance. If the resistance is 0·5 ohm, and the galvanometer resistance is 100 ohms, and an error of 5 per cent is permissible, then

$$\frac{5}{100}\,0{\cdot}5 = \frac{\omega^2 L^2}{100}$$

i.e.

$$\omega^2 L^2 = 2{\cdot}5 \quad \text{or} \quad \omega L = 1.58$$

This is a much higher value than is likely to be found associated with a resistance of 0·5 ohms. In general it would appear that with the values of inductance likely to occur in practice, the null balance earth tester measures the resistance of the circuit and not the impedance.

In the case of the instruments using periodically reversed direct current it can be shown theoretically and has been proved experimentally that very high values of inductance are necessary to produce errors of any consequence, so that again these instruments will measure resistance and not impedance.

Further Note on Neutral Loop Tests

The Regulation definitely states that the neutral loop test cannot be used in circuits where protective multiple earthing (P.M.E.) is employed. Also it may happen that the neutral is accidentally earthed at points along its length and when this occurs, a neutral loop test may give a false result.

Summary

There are two tests which can be used, the line-earth test and the neutral earth test. In the former the correct path is tested but the instruments do not give the impedance of the fault path, but a value which is closer to the resistance. In the latter, the correct path is not tested and it is assumed that the neutral conductor has the same impedance as the line conductor. With the current injection type of instrument, the value measured is the impedance, but considerable errors may be introduced if there is already current flowing in the neutral. With the earth tester type of instrument the value obtained is not affected by the current in the neutral, but is closer to the resistance of the loop rather than the impedance.

It would thus appear that at the moment there is no single instrument which will give the true impedance of the fault path.

APPENDIX 1

VALUES OF ρ_a/ρ_1 FOR NEGATIVE VALUES OF k

	$k=$									
h/a	−0·1	−0·2	−0·3	−0·4	−0·5	−0·6	−0·7	−0·8	−0·9	−1·0
0	0·8182	6667	5385	4286	3333	2500	1765	1111	0526	0
0·1	0·8229	6733	5454	4350	3389	2544	1796	1131	0536	0
0·2	0·8366	6941	5691	4587	3607	2733	1950	1246	0609	0033
0·3	0·8567	7277	6113	5058	4097	3221	2418	1682	1003	0378
0·4	0·8787	7672	6642	5690	4805	3982	3215	2497	1825	1194
0·5	0·8997	8059	7182	6358	5582	4850	4158	3502	2881	2290
0·6	0·9179	8404	7671	6976	6315	5686	5086	4512	3963	3438
0·7	0·9331	8695	8089	7510	6957	6426	5916	5426	4954	4499
0·8	0·9455	8933	8434	7955	7514	7049	6622	6208	5809	5422
0·9	0·9554	9121	8713	8317	7934	7564	7213	6859	6522	6196
1·0	0·9633	9279	8938	8609	8290	7981	7682	7391	7108	6833
1·1	0·9691	9403	9119	8845	8579	8320	8068	7824	7586	7352
1·2	0·9746	9501	9264	9034	8811	8593	8382	8176	7975	7779
1·3	0·9787	9581	9381	9187	8999	8815	8636	8465	8291	8125
1·4	0·9820	9646	9477	9312	9152	8996	8844	8695	8550	8408
1·5	0·9847	9698	9554	9414	9277	9144	9014	8886	8762	8640

VALUES OF σ_a/σ_1 FOR POSITIVE VALUES OF k

	$k=$									
h/a	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
0	0·8182	6667	5385	4286	3333	2500	1765	1111	0526	0
0·1	0·8248	6798	5581	4549	3666	2907	2251	1682	1184	0722
0·2	0·8412	7097	5994	5057	4252	3555	2944	2403	1914	1443
0·3	0·8617	7458	6471	5621	4881	4228	3647	3123	2638	2164
0·4	0·8830	7825	6952	6185	5504	4894	4341	3832	3356	2884
0·5	0·9026	8168	7405	6719	6099	5533	5012	4524	4061	3597
0·6	0·9197	8472	7812	7207	6650	6131	5646	5185	4741	4293
0·7	0·9341	8723	8167	7639	7143	6675	6230	5802	5383	4957
0·8	0·9459	8950	8468	8011	7564	7158	6755	6362	5975	5576
0·9	0·9555	9132	8721	8327	7947	7578	7218	6864	6509	6141
1·0	0·9632	9275	8929	8592	8262	7939	7620	7302	6982	6647
1·1	0·9699	9394	9101	8812	8527	8245	7964	7682	7395	7093
1·2	0·9744	9491	9241	8994	8748	8503	8256	8007	7752	7482
1·3	0·9784	9570	9357	9145	8933	8718	8503	8287	8058	7818
1·4	0·9817	9635	9453	9270	9086	8900	8712	8519	8319	8106
1·5	0·9844	9688	9531	9374	9214	9052	8887	8718	8542	8353
1·6	0·9867	9732	9597	9460	9321	9180	9035	8886	8731	8561
1·7	0·9885	9769	9651	9532	9411	9287	9160	9029	8891	8743
1·8	0·9900	9799	9697	9593	9486	9378	9266	9150	9029	8897
1·9	0·9913	9825	9735	9644	9550	9455	9356	9254	9146	9029
2·0	0·9924	9846	9768	9687	9605	9520	9433	9342	9246	9142

APPENDIX 2

VALUES OF σ_{na}/σ_a FOR POSITIVE VALUES OF k, WHEN $n = 1{\cdot}5$

	$k=$									
h/a	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
0	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000
0·1	0·9973	0·9936	0·9882	0·9807	0·9697	0·9533	0·9279	0·8869	0·8149	0·6667
0·2	9861	9700	9513	9295	9039	8740	8386	7962	7434	6667
0·3	9761	9516	9263	8996	8713	8407	8072	7696	7255	6667
0·4	9678	9373	9075	8780	8484	8180	7863	7524	7144	6670
0·5	9626	9282	8958	8647	8345	8046	7742	7428	7089	6683
0·6	9600	9236	8899	8581	8277	7981	7689	7391	7078	6718
0·7	9596	9227	8887	8569	8267	7976	7691	7406	7111	6781
0·8	9606	9245	8911	8598	8313	8016	7739	7463	7181	6872
0·9	9626	9277	8958	8655	8367	8090	7821	7553	7284	6990
1·0	9652	9326	9021	8731	8455	8188	7928	7671	7410	7131
1·1	9674	9377	9090	8817	8549	8299	8051	7804	7554	7287
1·2	9707	9430	9163	8907	8647	8419	8182	7946	7708	7453
1·3	9735	9483	9235	8997	8760	8539	8315	8090	7865	7623
1·4	9760	9531	9304	9084	8870	8657	8447	8236	8021	7792
1·5	9784	9573	9368	9166	8967	8770	8574	8376	8174	7958

VALUES OF σ_{na}/σ_a FOR POSITIVE VALUES OF k, WHEN $n = 2{\cdot}0$

	$k =$									
h/a	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
0	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000
0·1	0·9960	0·9904	0·9826	0·9711	0·9546	0·9299	0·8919	0·8303	0·7224	5000
0·2	9806	9578	9310	8995	8622	8178	7647	7000	6184	5000
0·3	9657	9296	8916	8511	8075	7602	7085	6511	5854	5000
0·4	9526	9070	8622	8176	7726	7264	6783	6270	5703	5002
0·5	9429	8905	8412	7941	7483	7031	6576	6108	5607	5013
0·6	9369	8803	8284	7799	7340	6896	6460	6022	5565	5041
0·7	9340	8752	8220	7730	7259	6836	6414	5997	5570	5092
0·8	9335	8743	8210	7720	7277	6837	6426	6023	5616	5172
0·9	9347	8761	8237	7754	7306	6885	6483	6092	5701	5280
1·0	9372	8807	8293	7821	7382	6970	6577	6196	5816	5412
1·1	9398	8862	8367	7909	7483	7081	6698	6326	5956	5566
1·2	9439	8926	8453	8014	7601	7211	6838	6476	6116	5738
1·3	9467	8994	8545	8125	7729	7353	6991	6637	6286	5922
1·4	9515	9063	8639	8240	7862	7500	7151	6810	6474	6115
1·5	9552	9131	8704	8356	7989	7759	7314	6985	6656	6313

VALUES OF σ_{na}/σ_a FOR POSITIVE VALUES OF k, WHEN $n = 3{\cdot}0$

	$k=$									
h/a	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
0	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000
0·1	0·9946	0·9871	0·9766	0·9615	0·9395	0·9066	0·8559	0·7737	0·6298	0·3330
0·2	9779	9516	9201	8822	8361	7796	7096	6208	5039	3334
0·3	9572	9114	8624	8092	7512	6876	6173	5287	4486	3334
0·4	9394	8797	8202	7600	6983	6349	5688	4992	4240	3335
0·5	9250	8552	7889	7251	6627	6010	5392	4764	4106	3342
0·6	9146	8377	7673	7016	6395	5798	5215	4634	4037	3361
0·7	9075	8261	7530	6862	6243	5650	5102	4556	4005	3396
0·8	9035	8195	7450	6779	6173	5592	5052	4532	4012	3450
0·9	9018	8167	7421	6751	6141	5579	5053	4550	4053	3524
1·0	9021	8174	7429	6763	6161	5608	5092	4602	4123	3617
1·1	9032	8201	7465	6807	6213	5669	5163	4683	4216	3728
1·2	9062	8245	7522	6877	6292	5756	5257	4786	4329	3855
1·3	9094	8299	7595	6962	6388	5862	5371	4906	4458	3995
1·4	9130	8363	7678	7061	6499	5982	5500	5043	4601	4146
1·5	9169	8431	7769	7168	6620	6113	5639	5190	4754	4307

VALUES OF ρ_{na}/ρ_a FOR NEGATIVE VALUES OF k, WHEN $n = 1{\cdot}5$

	$k=$									
h/a	−0·1	−0·2	−0·3	−0·4	−0·5	−0·6	−0·7	−0·8	−0·9	−1·0
0	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000
0·1	0·9981	0·9967	0·9958	0·9951	0·9945	0·9942	0·9941	0·9941	0·9942	1·0000
0·2	9881	9778	9688	9608	9532	9452	9356	9210	8870	0·3333
0·3	9683	9374	9059	8726	8357	8486	8065	7406	6072	0·0885
0·4	9669	9327	8967	8575	8130	7599	6925	5998	4547	1834
0·5	9603	9188	8748	8268	7733	7119	6389	5486	4302	2651
0·6	9573	9128	8659	8156	7609	7004	6321	5534	4605	3473
0·7	9569	9123	8597	8170	7650	7091	6486	5934	5078	4240
0·8	9583	9156	8717	8240	7764	7285	6755	6191	5585	4932
0·9	9608	9214	8804	8388	7960	7518	7051	6579	6076	5548
1·0	9638	9273	8903	8529	8145	7757	7357	6946	6532	6085
1·1	9675	9338	9005	8670	8340	7988	7640	7287	6926	6433
1·2	9701	9402	9104	8805	8528	8203	7900	7594	7284	6970
1·3	9730	9459	9196	8930	8672	8399	8139	7862	7597	7328
1·4	9758	9514	9280	9043	8807	8574	8346	8106	7872	7639
1·5	9782	9568	9355	9145	8936	8729	8522	8317	8113	7908

VALUES OF ρ_{na}/ρ_a FOR NEGATIVE VALUES OF k, WHEN $n = 2{\cdot}0$

	$k=$									
h/a	−0·1	−0·2	−0·3	−0·4	−0·5	−0·6	−0·7	−0·8	−0·9	−1·0
0	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000
0·1	0·9971	0·9951	0·9936	0·9926	0·9918	0·9914	0·9912	0·9913	0·9913	
0·2	9836	9700	9584	9484	9395	9308	9212	9078	8792	
0·3	9674	9373	9080	8785	8471	8106	7634	6910	5455	0442
0·4	9521	9048	8568	8061	7506	6863	6066	4990	3338	0280
0·5	9405	8807	8192	7546	6847	6065	5156	4055	2637	0685
0·6	9333	8658	7968	7250	6488	5664	4755	3729	2532	1100
0·7	9298	8592	7875	7139	6373	5567	4711	3786	2771	1643
0·8	9294	8588	7876	7153	6396	5650	4855	4022	3142	2202
0·9	9310	8627	7936	7244	6544	5832	5099	4356	3582	2778
1·0	9340	8685	8035	7385	6733	6076	5412	4765	4052	3352
1·1	9382	8761	8152	7547	6944	6341	5737	5129	4516	3898
1·2	9418	8846	8281	7722	7168	6617	6068	5519	4967	4419
1·3	9460	8931	8410	7896	7386	6885	6386	5887	5395	4904
1·4	9502	9015	8536	8057	7601	7143	6689	6240	5794	5351
1·5	9542	9095	8656	8225	7814	7384	6871	6565	6161	5762

VALUES OF ρ_{na}/ρ_a FOR NEGATIVE VALUES OF k, WHEN $n = 3{\cdot}0$

	$k=$									
h/a	−0·1	−0·2	−0·3	−0·4	−0·5	−0·6	−0·7	−0·8	−0·9	−1·0
0	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000
0·1	0·9962	0·9934	0·9915	0·9901	0·9891	0·9885	0·9883	0·9883	0·9884	
0·2	9818	9668	9543	9437	9344	9254	9158	9024	8741	
0·3	9605	9252	8922	8601	8271	7899	7429	6727	5338	
0·4	9407	8847	8301	7745	7155	6487	5675	4596	2961	0093
0·5	9239	8508	7787	7070	6288	5453	4506	3378	1950	0097
0·6	9114	8259	7418	6575	5712	4807	3834	2761	1537	0097
0·7	9032	8099	7188	6286	5380	4456	3500	2493	1412	0235
0·8	8986	8009	7062	6134	5200	4293	3362	2412	1428	0404
0·9	8967	7978	7016	6081	5164	4258	3352	2453	1538	0611
1·0	8969	7980	7029	6106	5207	4326	3458	2599	1743	0889
1·1	8991	8016	7084	6183	5309	4457	3634	2805	1998	1201
1·2	9015	8074	7170	6298	5454	4634	3835	3054	2288	1535
1·3	9051	8145	7274	6436	5625	4840	4077	3331	2606	1895
1·4	9092	8224	7339	6589	5815	5066	4339	3703	2942	2269
1·5	9136	8310	7517	6753	6016	5304	4612	3941	3288	2651

APPENDIX 3

FIGURES CONNECTING LOG ρ_a/ρ_1 WITH LOG a/h

		log ρ_a/ρ_1 *when* $k =$									
a/h	*log* a/h	−0·1	−0·2	−0·3	−0·4	−0·5	−0·6	−0·7	−0·8	−0·9	−1.0
0·1	−1·0000	0	0	−0·0001	−0·0001	−0·0001	−0·0002	−0·0002	−0·0002	−0·0002	−0·0003
0·2	−0·6990	−0·0002	−0·0005	−0·0008	−0·0010	−0·0012	−0·0014	−0·0016	−0·0019	−0·0021	−0·0023
0·3	−0·5229	−0·0008	−0·0015	−0·0023	−0·0031	−0·0038	−0·0045	−0·0052	−0·0059	−0·0066	−0·0073
0·4	−0·3979	−0·0018	−0·0040	−0·0052	−0·0069	−0·0086	−0·0102	−0·0118	−0·0134	−0·0149	−0·0164
0·5	−0·3010	−0·0029	−0·0065	−0·0096	−0·0127	−0·0155	−0·0186	−0·0216	−0·0245	−0·0273	−0·0301
0·6	−0·2218	−0·0052	−0·0103	−0·0153	−0·0203	−0·0251	−0·0299	−0·0347	−0·0393	−0·0440	−0·0486
0·7	−0·1549	−0·0075	−0·0149	−0·0223	−0·0296	−0·0367	−0·0438	−0·0508	−0·0578	−0·0647	−0·0716
0·8	−0·0969	−0·0102	−0·0204	−0·0303	−0·0403	−0·0502	−0·0600	−0·0698	−0·0796	−0·0893	−0·0991
0·9	−0·0458	−0·0131	−0·0262	−0·0392	−0·0523	−0·0652	−0·0781	−0·0911	−0·1042	−0·1173	−0·1305
1·0	0	−0·0163	−0·0325	−0·0487	−0·0650	−0·0814	−0·0979	−0·1145	−0·1313	−0·1482	−0·1654
1·1	0·0414	−0·0194	−0·0391	−0·0587	−0·0785	−0·0986	−0·1189	−0·1395	−0·1604	−0·1818	−0·2035
1·2	0·0792	−0·0227	−0·0457	−0·0689	−0·0924	−0·1163	−0·1407	−0·1656	−0·1912	−0·2174	−0·2444
1·3	0·1139	−0·0265	−0·0524	−0·0791	−0·1064	−0·1344	−0·1630	−0·1926	−0·2232	−0·2548	−0·2879
1·4	0·1461	−0·0292	−0·0589	−0·0893	−0·1204	−0·1524	−0·1857	−0·2201	−0·2560	−0·2937	−0·3335
1·5	0·1761	−0·0323	−0·0653	−0·0992	−0·1342	−0·1704	−0·2082	−0·2478	−0·2895	−0·3337	−0·3811
1·6	0·2041	−0·0353	−0·0715	−0·1088	−0·1476	−0·1881	−0·2305	−0·2754	−0·3232	−0·3745	−0·4301
1·7	0·2304	−0·0382	−0·0775	−0·1182	−0·1607	−0·2053	−0·2524	−0·3028	−0·3569	−0·4158	−0·4809
1·8	0·2553	−0·0410	−0·0831	−0·1272	−0·1732	−0·2219	−0·2738	−0·3296	−0·3903	−0·4574	−0·5328
1·9	0·2788	−0·0435	−0·0885	−0·1359	−0·1853	−0·2379	−0·2944	−0·3557	−0·4234	−0·4991	−0·5860
2·0	0·3010	−0·0459	−0·0937	−0·1438	−0·1967	−0·2532	−0·3143	−0·3813	−0·4557	−0·5406	−0·6404
2·1	0·3222	−0·0483	−0·0985	−0·1514	−0·2075	−0·2678	−0·3333	−0·4057	−0·4872	−0·5817	−0·6955
2·2	0·3424	−0·0504	−0·1031	−0·1586	−0·2178	−0·2816	−0·3514	−0·4292	−0·5177	−0·6221	−0·7513
2·3	0·3617	−0·0525	−0·1073	−0·1654	−0·2275	−0·2946	−0·3686	−0·4515	−0·5473	−0·6619	−0·8080
2·4	0·3802	−0·0544	−0·1114	−0·1717	−0·2365	−0·3068	−0·3847	−0·4728	−0·5757	−0·7009	−0·8652
2·5	0·3979	−0·0562	−0·1151	−0·1777	−0·2450	−0·3183	−0·3999	−0·4928	−0·6026	−0·7387	−0·9230

a/h	$\log a/h$	$\log \rho_a/\rho_1$ when $k =$									
		−0·1	−0·2	−0·3	−0·4	−0·5	−0·6	−0·7	−0·8	−0·9	−1·0
2·6	0·4150	−0·0579	−0·1186	−0·1833	−0·2529	−0·3290	−0·4140	−0·5118	−0·6282	−0·7755	−0·9813
2·7	0·4314	−0·0595	−0·1219	−0·1884	−0·2601	−0·3389	−0·4270	−0·5295	−0·6524	−0·8108	−1·0405
2·8	0·4472	−0·0609	−0·1249	−0·1932	−0·2670	−0·3482	−0·4397	−0·5461	−0·6755	−0·8447	−1·0996
2·9	0·4624	−0·0623	−0·1277	−0·1976	−0·2734	−0·3568	−0·4512	−0·5616	−0·6871	−0·8771	−1·1593
3·0	0·4771	−0·0635	−0·1304	−0·2018	−0·2793	−0·3649	−0·4619	−0·5760	−0·7172	−0·9080	−1·2190
3·1	0·4914	−0·0647	−0·1329	−0·2057	−0·2848	−0·3723	−0·4719	−0·5892	−0·7356	−0·9370	−1·2790
3·2	0·5051	−0·0658	−0·1353	−0·2093	−0·2899	−0·3793	−0·4810	−0·6016	−0·7530	−0·9645	−1·3501
3·3	0·5185	−0·0668	−0·1374	−0·2126	−0·2946	−0·3856	−0·4894	−0·6130	−0·7691	−0·9902	−1·4012
3·4	0·5315	−0·0678	−0·1394	−0·2158	−0·2990	−0·3914	−0·4972	−0·6234	−0·7839	−1·0141	−1·4622
3·5	0·5441	−0·0688	−0·1413	−0·2188	−0·3030	−0·3968	−0·5045	−0·6330	−0·7975	−1·0367	−1·5243
3·6	0·5563	−0·0696	−0·1430	−0·2214	−0·3068	−0·4018	−0·5110	−0·6417	−0·8103	−1·0575	−1·5867
3·7	0·5682	−0·0704	−0·1446	−0·2240	−0·3103	−0·4064	−0·5170	−0·6498	−0·8219	−1·0762	−1·6478
3·8	0·5798	−0·0712	−0·1461	−0·2262	−0·3135	−0·4107	−0·5225	−0·6572	−0·8324	−1·0937	−1·7100
3·9	0·5911	−0·0719	−0·1476	−0·2284	−0·3166	−0·4147	−0·5275	−0·6641	−0·8416	−1·1096	−1·7721
4·0	0·6021	−0·0725	−0·1490	−0·2305	−0·3193	−0·4183	−0·5324	−0·6704	−0·8502	−1·1244	−1·8356
4·1	0·6128	−0·0732	−0·1502	−0·2324	−0·3219	−0·4216	−0·5369	−0·6762	−0·8582	−1·1379	−1·8996
4·2	0·6232	−0·0738	−0·1514	−0·2342	−0·3243	−0·4248	−0·5407	−0·6813	−0·8659	−1·1500	−1·9626
4·3	0·6335	−0·0744	−0·1524	−0·2358	−0·3265	−0·4276	−0·5444	−0·6859	−0·8726	−1·1612	−2·0223
4·4	0·6435	−0·0749	−0·1536	−0·2373	−0·3285	−0·4304	−0·5477	−0·6902	−0·8784	−1·1713	−2·0862
4·5	0·6532	−0·0754	−0·1545	−0·2387	−0·3305	−0·4330	−0·5508	−0·6942	−0·8838	−1·1805	−2·1549
4·6	0·6628	−0·0758	−0·1554	−0·2401	−0·3323	−0·4352	−0·5538	−0·6982	−0·8887	−1·1884	−2·2147
4·7	0·6721	−0·0763	−0·1563	−0·2413	−0·3340	−0·4372	−0·5564	−0·7015	−0·8934	−1·1959	−2·2757
4·8	0·6812	−0·0768	−0·1571	−0·2426	−0·3356	−0·4393	−0·5589	−0·7046	−0·8975	−1·2027	−2·3372
4·9	0·6902	−0·0772	−0·1578	−0·2437	−0·3369	−0·4412	−0·5613	−0·7073	−0·9014	−1·2090	−2·4089
5·0	0·6990	−0·0775	−0·1585	−0·2448	−0·3384	−0·4429	−0·5633	−0·7100	−0·9049	−1·2147	−2·4685

a/h	$\log a/h$	$\log \rho_a/\rho_1$ when $k =$									
		0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
0·1	−1·0000	0	0·0001	0·0001	0·0001	0·0002	0·0002	0·0003	0·0003	0·0003	0·0004
0·2	−0·6990	0·0003	0·0005	0·0007	0·0010	0·0012	0·0015	0·0019	0·0022	0·0025	0·0029
0·3	−0·5229	0·0008	0·0015	0·0024	0·0033	0·0043	0·0052	0·0063	0·0072	0·0083	0·0097
0·4	−0·3979	0·0018	0·0035	0·0055	0·0075	0·0095	0·0118	0·0140	0·0163	0·0189	0·0217
0·5	−0·3010	0·0032	0·0066	0·0101	0·0137	0·0175	0·0214	0·0254	0·0296	0·0341	0·0390
0·6	−0·2218	0·0052	0·0107	0·0161	0·0219	0·0277	0·0337	0·0401	0·0464	0·0535	0·0612
0·7	−0·1549	0·0075	0·0153	0·0234	0·0315	0·0399	0·0482	0·0572	0·0666	0·0765	0·0873
0·8	−0·0969	0·0103	0·0209	0·0315	0·0423	0·0534	0·0648	0·0765	0·0888	0·1018	0·1159
0·9	−0·0458	0·0132	0·0266	0·0402	0·0539	0·0679	0·0821	0·0970	0·1124	0·1286	0·1463
1·0	0	0·0160	0·0328	0·0492	0·0659	0·0829	0·1002	0·1180	0·1365	0·1560	0·1773
1·1	0·0414	0·0194	0·0389	0·0584	0·0782	0·0981	0·1184	0·1393	0·1608	0·1834	0·2083
1·2	0·0792	0·0226	0·0450	0·0677	0·0903	0·1132	0·1364	0·1601	0·1845	0·2104	0·2387
1·3	0·1139	0·0257	0·0512	0·0767	0·1022	0·1278	0·1539	0·1804	0·2078	0·2366	0·2684
1·4	0·1461	0·0288	0·0572	0·0854	0·1139	0·1423	0·1709	0·2001	0·2303	0·2619	0·2969
1·5	0·1761	0·0318	0·0630	0·0939	0·1249	0·1558	0·1870	0·2189	0·2516	0·2862	0·3243
1·6	0·2041	0·0349	0·0685	0·1024	0·1354	0·1689	0·2026	0·2367	0·2721	0·3093	0·3504
1·7	0·2304	0·0373	0·0738	0·1098	0·1456	0·1812	0·2173	0·2540	0·2916	0·3313	0·3754
1·8	0·2553	0·0399	0·0788	0·1169	0·1551	0·1930	0·2311	0·2701	0·3101	0·3524	0·3994
1·9	0·2788	0·0422	0·0834	0·1240	0·1642	0·2042	0·2444	0·2854	0·3277	0·3724	0·4222
2·0	0·3010	0·0443	0·0879	0·1305	0·1728	0·2147	0·2570	0·3001	0·3445	0·3914	0·4440
2·1	0·3222	0·0466	0·0921	0·1367	0·1808	0·2246	0·2689	0·3139	0·3603	0·4095	0·4649
2·2	0·3424	0·0485	0·0960	0·1426	0·1887	0·2341	0·2801	0·3269	0·3754	0·4268	0·4849
2·3	0·3617	0·0505	0·0999	0·1478	0·1955	0·2429	0·2907	0·3394	0·3898	0·4434	0·5040
2·4	0·3802	0·0522	0·1033	0·1531	0·2023	0·2513	0·3009	0·3512	0·4034	0·4591	0·5223
2·5	0·3979	0·0541	0·1066	0·1578	0·2086	0·2593	0·3103	0·3625	0·4166	0·4742	0·5401

a/h	$\log a/h$	$\log \rho_a/\rho_1$ when $k =$ 0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
2·6	0·4150	0·0556	0·1096	0·1625	0·2147	0·2668	0·3195	0·3732	0·4289	0·4887	0·5570
2·7	0·4314	0·0571	0·1125	0·1668	0·2205	0·2740	0·3280	0·3833	0·4408	0·5025	0·5734
2·8	0·4472	0·0585	0·1152	0·1709	0·2258	0·2807	0·3363	0·3931	0·4523	0·5158	0·5890
2·9	0·4624	0·0597	0·1178	0·1748	0·2308	0·2872	0·3439	0·4023	0·4631	0·5286	0·6032
3·0	0·4771	0·0610	0·1202	0·1783	0·2357	0·2933	0·3514	0·4111	0·4736	0·5408	0·6189
3·1	0·4914	0·0621	0·1225	0·1817	0·2403	0·2991	0·3587	0·4196	0·4836	0·5527	0·6333
3·2	0·5051	0·0633	0·1247	0·1850	0·2447	0·3046	0·3653	0·4277	0·4918	0·5641	0·6470
3·3	0·5185	0·0643	0·1268	0·1880	0·2488	0·3098	0·3718	0·4354	0·5025	0·5759	0·6604
3·4	0·5315	0·0653	0·1287	0·1909	0·2526	0·3148	0·3780	0·4430	0·5113	0·5858	0·6734
3·5	0·5441	0·0662	0·1304	0·1937	0·2565	0·3197	0·3838	0·4501	0·5199	0·5960	0·6859
3·6	0·5563	0·0671	0·1322	0·1963	0·2601	0·3241	0·3895	0·4570	0·5282	0·6059	0·6982
3·7	0·5682	0·0679	0·1338	0·1989	0·2635	0·3286	0·3950	0·4635	0·5362	0·6155	0·7101
3·8	0·5798	0·0686	0·1353	0·2013	0·2666	0·3327	0·4001	0·4699	0·5439	0·6249	0·7217
3·9	0·5911	0·0693	0·1368	0·2035	0·2697	0·3368	0·4051	0·4761	0·5513	0·6340	0·7330
4·0	0·6021	0·0700	0·1382	0·2055	0·2727	0·3405	0·4098	0·4819	0·5586	0·6427	0·7439
4·1	0·6128	0·0707	0·1396	0·2075	0·2756	0·3441	0·4146	0·4876	0·5655	0·6512	0·7546
4·2	0·6232	0·0712	0·1408	0·2095	0·2783	0·3478	0·4190	0·4932	0·5722	0·6595	0·7650
4·3	0·6335	0·0719	0·1420	0·2113	0·2808	0·3511	0·4232	0·4984	0·5787	0·6675	0·7753
4·4	0·6435	0·0724	0·1431	0·2132	0·2832	0·3544	0·4273	0·5036	0·5851	0·6754	0·7853
4·5	0·6532	0·0730	0·1443	0·2148	0·2857	0·3575	0·4314	0·5086	0·5911	0·6828	0·7951
4·6	0·6628	0·0734	0·1454	0·2164	0·2879	0·3605	0·4352	0·5133	0·5971	0·6903	0·8046
4·7	0·6721	0·0739	0·1462	0·2180	0·2900	0·3633	0·4388	0·5180	0·6030	0·6975	0·8139
4·8	0·6812	0·0744	0·1471	0·2194	0·2921	0·3662	0·4424	0·5224	0·6086	0·7046	0·8231
4·9	0·6902	0·0748	0·1480	0·2209	0·2942	0·3688	0·4459	0·5268	0·6139	0·7114	0·8321
5·0	0·6990	0·0752	0·1488	0·2223	0·2962	0·3714	0·4492	0·5311	0·6192	0·7181	0·8408

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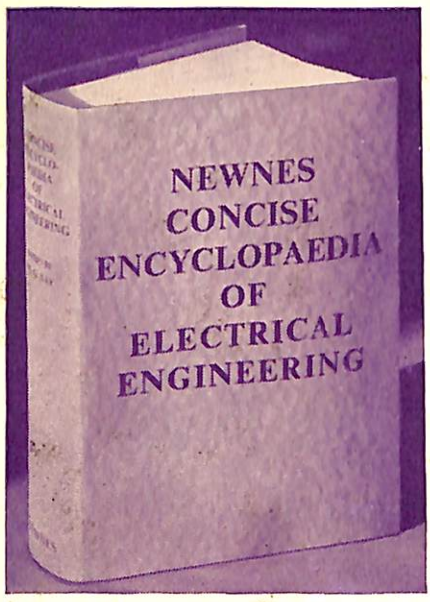

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